MATH 136 Midterm Self-Assessment

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- There is no substitute to working through Chapters 1–4, attempting all of the relevant Practice Problems, re-doing any Quiz or WA problems you got wrong, and carefully studying the solutions to all of the Quiz and WA problems (even for problems you got right—since the posted solution might offer a different approach).

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- This document can be used to help you identify some areas that you need to review or study more deeply.

Chapter 1: Vectors

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- I can perform vector addition and scalar multiplication with vectors in $\mathbb{F}^n$.
- I know the algebraic properties that vector addition and scalar multiplication satisfy.
- I understand vector addition and scalar multiplication geometrically.

Chapter 1: Vectors

How many of these sentences can you truthfully state about your current state of understanding?

- I can perform vector addition and scalar multiplication with vectors in $\mathbb{F}^n$.
- I know the algebraic properties that vector addition and scalar multiplication satisfy.
- I understand vector addition and scalar multiplication geometrically.
- I know what the standard basis vectors $\vec{e}_1, \ldots, \vec{e}_n$ in $\mathbb{F}^n$ are.

Chapter 1: Vectors – Dot Product

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- I can compute the dot product of two vectors in $\mathbb{R}^n$.
- I know the algebraic properties that the dot product satisfies.
- I can determine if two vectors in $\mathbb{R}^n$ are orthogonal.
- More generally, I can determine the angle between any two non-zero vectors in $\mathbb{R}^n$.

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- I know what a unit vector is.

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- I can compute the norm (or length) of a vector in $\mathbb{R}^n$.
- I know the algebraic properties that the norm satisfies.
- I understand what the norm measures geometrically.
- I know what a unit vector is.
- I know the relationship between the norm and the dot product.

Chapter 1: Vectors – Projection

- Given $\vec{v}, \vec{u} \in \mathbb{R}^n$, I can determine $\text{proj}_{\vec{u}}(\vec{v})$ and $\text{perp}_{\vec{u}}(\vec{v})$.

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- Given $\vec{v}, \vec{u} \in \mathbb{R}^n$, I can determine $\text{proj}_{\vec{u}}(\vec{v})$ and $\text{perp}_{\vec{u}}(\vec{v})$.
- I know how to visualize $\text{proj}_{\vec{u}}(\vec{v})$ and $\text{perp}_{\vec{u}}(\vec{v})$, at least if $\vec{v}$ and $\vec{u}$ are in $\mathbb{R}^2$ or $\mathbb{R}^3$.

Chapter 1: Vectors – Cross Product

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- I know how to compute the cross product of two vectors in $\mathbb{R}^3$.
- I know the algebraic properties that the cross product satisfies.
- I understand the geometric significance of the cross product.
- I know how to use the cross product to find a vector in $\mathbb{R}^3$ that is orthogonal to two given vectors.

Chapter 2: Linear Combinations

- I can define what it means for a vector to be a "linear combination of $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{F}^n$".

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- I can define what it means for a vector to be a "linear combination of $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{F}^n$".
- I can determine if a given vector is or is not a linear combination of some other given vectors.

Chapter 2: Span

- I can define the span of $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{F}^n$.

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- I can define the span of $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{F}^n$.
- I know the difference between Span$\{\vec{v}_1, \ldots, \vec{v}_k\}$ and a linear combination of $\vec{v}_1, \ldots, \vec{v}_k$.

Chapter 2: Span

- I can define the span of $\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{F}^n$.
- I know the difference between $\text{Span}\{\vec{v}_1, \ldots, \vec{v}_k\}$ and a linear combination of $\vec{v}_1, \ldots, \vec{v}_k$.
- I know how to determine if a given vector $\vec{u} \in \mathbb{F}^n$ is in $\text{Span}\{\vec{v}_1, \ldots, \vec{v}_k\}$.

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- I know the difference between Span$\{\vec{v}_1, \ldots, \vec{v}_k\}$ and a linear combination of $\vec{v}_1, \ldots, \vec{v}_k$.
- I know how to determine if a given vector $\vec{u} \in \mathbb{F}^n$ is in Span$\{\vec{v}_1, \ldots, \vec{v}_k\}$.
- I know how to determine if a given *set* $A \subseteq \mathbb{F}^n$ is equal to Span$\{\vec{v}_1, \ldots, \vec{v}_k\}$. (In particular, I can determine if $\mathbb{F}^n = \text{Span}\{\vec{v}_1, \ldots, \vec{v}_k\}$.)

Chapter 2: Lines

- I know how to represent a line $\mathcal{L}$ in $\mathbb{R}^n$ algebraically (using a vector equation and/or using parametric equations).

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- I know how to represent a line $\mathcal{L}$ in $\mathbb{R}^n$ algebraically (using a vector equation and/or using parametric equations).
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- I know how to find equations for a line $\mathcal{L}$ given a point on $\mathcal{L}$ and a direction vector for $\mathcal{L}$.

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- I can determine when a given point in $\mathbb{R}^n$ lies on a given line.

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- I can determine when a given point in $\mathbb{R}^n$ lies on a given line.
- I can determine when two lines intersect.

Chapter 2: Planes

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- I know how to represent a plane $\mathcal{P}$ in $\mathbb{R}^n$ algebraically (using a vector equation and/or a scalar equation (in $\mathbb{R}^3$)).
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- I know how to represent a plane $\mathcal{P}$ in $\mathbb{R}^n$ algebraically (using a vector equation and/or a scalar equation (in $\mathbb{R}^3$)).
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- I know how to find equations for a plane $\mathcal{P}$ given three points that lie on $\mathcal{P}$.
- I know how to find equations for a plane $\mathcal{P}$ (in $\mathbb{R}^3$) given a point on $\mathcal{P}$ and a normal vector for $\mathcal{P}$.

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- I know how to find equations for a plane $\mathcal{P}$ (in $\mathbb{R}^3$) given a point on $\mathcal{P}$ and a normal vector for $\mathcal{P}$.
- I can determine when a given point in $\mathbb{R}^n$ lies on a given plane.

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- I can determine when a given point in $\mathbb{R}^n$ lies on a given plane.
- I can determine when two planes intersect.

Chapter 3: Systems of Linear Equations

- I can determine when a vector is a solution to a system of equations.

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- I can express a system of equations in augmented matrix form $[A \mid \vec{b}]$ and using matrix–vector multiplication $A\vec{x} = \vec{b}$.

Chapter 3: Systems of Linear Equations

- I can determine when a vector is a solution to a system of equations.
- I can express a system of equations in augmented matrix form $[A \mid \vec{b}]$ and using matrix–vector multiplication $A\vec{x} = \vec{b}$.
- I know how to multiply an $m \times n$ matrix A with a vector $\vec{x} \in \mathbb{F}^n$ to get the vector $A\vec{x} \in \mathbb{F}^m$.

Chapter 3: Gauss–Jordan

- I know how to determine when a matrix is in REF and/or in RREF.

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- I know how to determine when a matrix is in REF and/or in RREF.
- I know how to solve a system of linear equations by using elementary row operations to reduce its augmented matrix $[A \mid \vec{b}]$ to RREF.

Chapter 3: Solution Sets

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- I know what it means for a system of equations to be consistent or inconsistent.

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- I know what it means for a system of equations to be consistent or inconsistent.
- I understand that a system of linear equations can either have no solutions, only one solution ("unique solution") or infinitely many solutions.

Chapter 3: Rank and Nullity

- I know how to compute the rank of a given matrix.

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- I understand the statement of the System Rank Theorem.

Chapter 3: Rank and Nullity

- I know how to compute the rank of a given matrix.
- I know how to compute the nullity of a given matrix.
- I can give the full statement of the System Rank Theorem.
- I understand the statement of the System Rank Theorem.
- I have a conceptual understanding of how rank and nullity can be used to give information about systems of linear equations (using, e.g., the System Rank Theorem).

Chapter 3: Coefficient Matrices and Solution Sets

- Given a matrix A, I know what the homogeneous system associated to A is.

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- I know the difference between the matrix A and a system of equations with coefficient matrix A.

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- Given a matrix A, I know what the homogeneous system associated to A is.
- I know the difference between the matrix A and a system of equations with coefficient matrix A.
- Given two *consistent* systems $A\vec{x} = \vec{b}$ and $A\vec{x} = \vec{c}$, I know how their solutions sets are related. I understand this relationship both algebraically and geometrically.

Chapter 3: Null Space

- I can define the null space $\mathrm{Null}(A)$ of a given matrix A.

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- I can define the null space $\text{Null}(A)$ of a given matrix A.
- I understand the relationship between $\text{Null}(A)$ and the homogeneous system $A\vec{x} = \vec{0}$.

Chapter 3: Null Space

- I can define the null space Null(A) of a given matrix A.
- I understand the relationship between Null(A) and the homogeneous system $A\vec{x} = \vec{0}$.
- Given a vector $\vec{x}$, I can determine whether it is in Null(A).

Chapter 3: Null Space

- I can define the null space $\mathrm{Null}(A)$ of a given matrix A.
- I understand the relationship between $\mathrm{Null}(A)$ and the homogeneous system $A\vec{x} = \vec{0}$.
- Given a vector $\vec{x}$, I can determine whether it is in $\mathrm{Null}(A)$.
- Given A, I can find $\mathrm{Null}(A)$ and express it as the span of one or more vectors.

Chapter 4: Column Space

- I can define the column space $\mathrm{Col}(A)$ of a given matrix A.

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- I can define the column space $\text{Col}(A)$ of a given matrix A.
- I understand the relationship between $\text{Col}(A)$ and systems of equations with coefficient matrix A.

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- I can define the column space $\mathrm{Col}(A)$ of a given matrix A.
- I understand the relationship between $\mathrm{Col}(A)$ and systems of equations with coefficient matrix A.
- Given a vector $\vec{x}$, I can determine whether it is in $\mathrm{Col}(A)$.

Chapter 4: Column Space

- I can define the column space $\mathrm{Col}(A)$ of a given matrix A.
- I understand the relationship between $\mathrm{Col}(A)$ and systems of equations with coefficient matrix A.
- Given a vector $\vec{x}$, I can determine whether it is in $\mathrm{Col}(A)$.
- Given A, I can find $\mathrm{Col}(A)$ and express it as the span of one or more vectors.

Chapter 4: Matrix Algebra

- I know how to perform algebraic operations with matrices, including addition, subtraction, scalar multiplication, and matrix multiplication. I know when matrix multiplication is not defined.

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- I know all of the basic algebraic properties that are satisfied by the operations mentioned above.

Chapter 4: Matrix Algebra

- I know how to perform algebraic operations with matrices, including addition, subtraction, scalar multiplication, and matrix multiplication. I know when matrix multiplication is not defined.
- I know how to find the transpose of a given matrix.
- I know all of the basic algebraic properties that are satisfied by the operations mentioned above.
- I am aware of the differences between real number multiplication and matrix multiplication. I know to be careful about generalizing results from the former to the latter (e.g. I can prove that $(A+B)^2 = A^2 + 2AB + B^2$ is false for matrices).
- I know that two matrices $A, B \in M_{m\times n}(\mathbb{F})$ are equal if and only if $A\vec{x} = B\vec{x}$ for all $\vec{x} \in \mathbb{F}^n$.

Chapter 4: Elementary Matrices

- I know what an elementary matrix is.

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- I know what an elementary matrix is.
- Given a matrix, I am able to identify if it is or is not an elementary matrix.

Chapter 4: Elementary Matrices

- I know what an elementary matrix is.
- Given a matrix, I am able to identify if it is or is not an elementary matrix.
- Given an elementary matrix E and an arbitrary matrix A, I am able to compute the product EA by performing an appropriate row operation on A.

Chapter 4: Invertibility

- I know what it means for an $n \times n$ matrix to be invertible.

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- I can state several criteria that guarantee that an $n \times n$ matrix is invertible.

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- I can state several criteria that guarantee that an $n \times n$ matrix is invertible.
- Given $A \in M_{n\times n}(\mathbb{F})$, I am able to quickly test whether A is or is not invertible. If A is invertible, I am also able to find its inverse A^{-1}.

Chapter 4: Invertibility

- I know what it means for an $n \times n$ matrix to be invertible.
- I can state several criteria that guarantee that an $n \times n$ matrix is invertible.
- Given $A \in M_{n\times n}(\mathbb{F})$, I am able to quickly test whether A is or is not invertible. If A is invertible, I am also able to find its inverse A^{-1}.
- If A is 2×2, I am aware that there is a particularly quick test of invertibility, and a particularly simple formula for the inverse (when the matrix is invertible).