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The long road to enlightenment

Loss reserving models from the past, with some speculation on the future

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Overview

- The Landscape
- The Jurassic period
  – Unformulated models
- The Cretaceous period – seed-bearing organisms appear
  – Stochastic models
- The Paleogene – increased diversity in the higher forms
  – Evolutionary models
  – Parameter reduction
  – Granular (micro-) models
- The Anthropocene period – intelligent beings intervene
  – Artificial intelligence
- The later Anthropocene – the future

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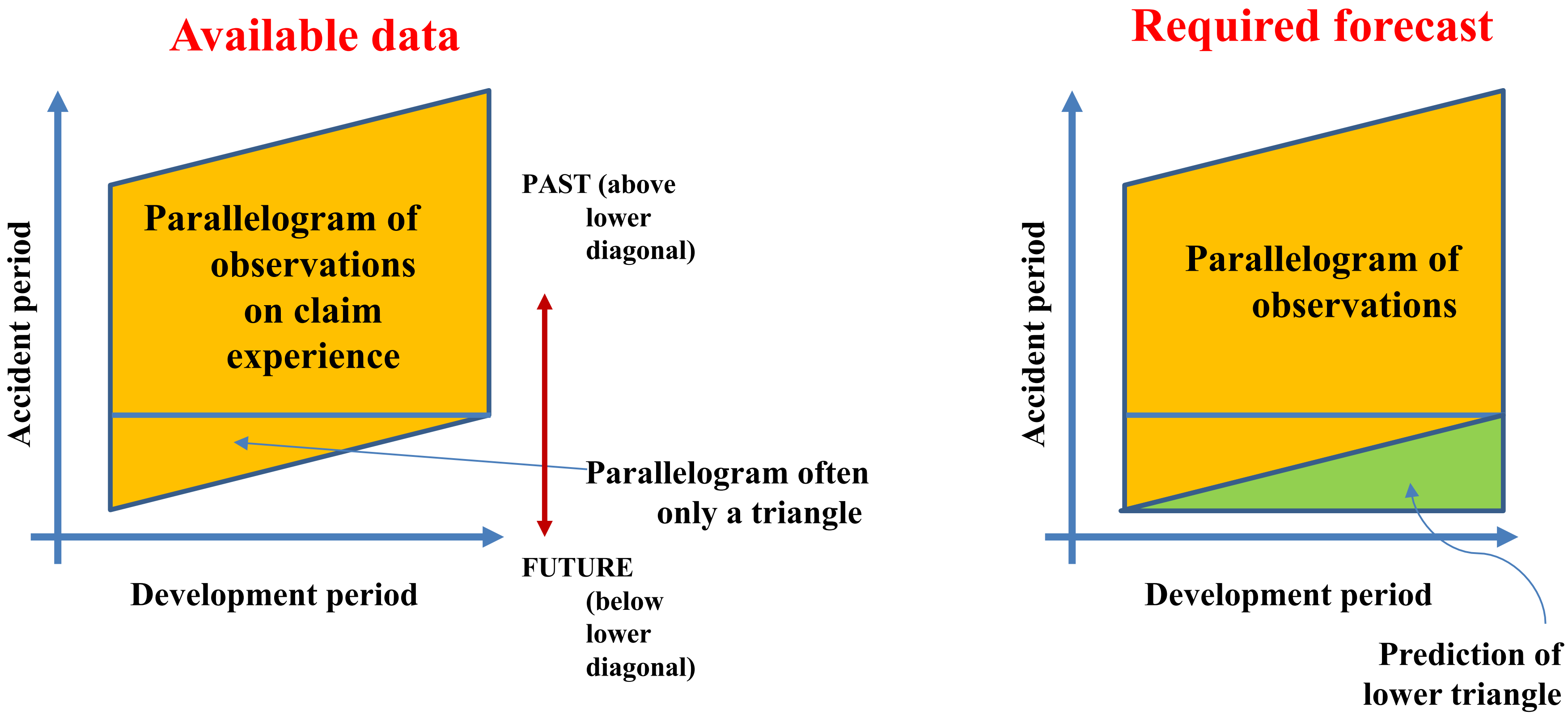
The Landscape

- Loss reserving
  - A problem in forecasting



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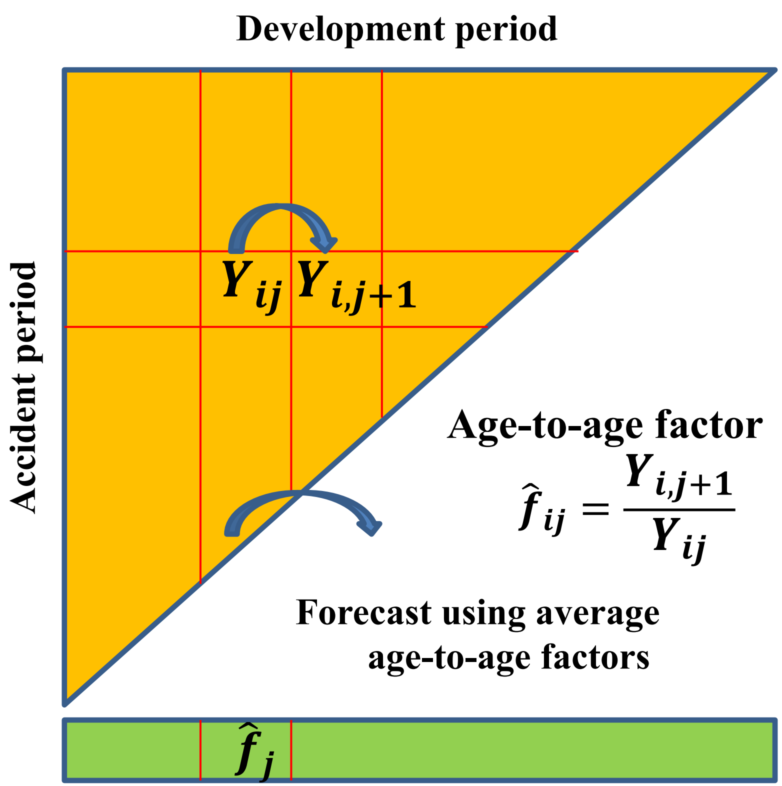
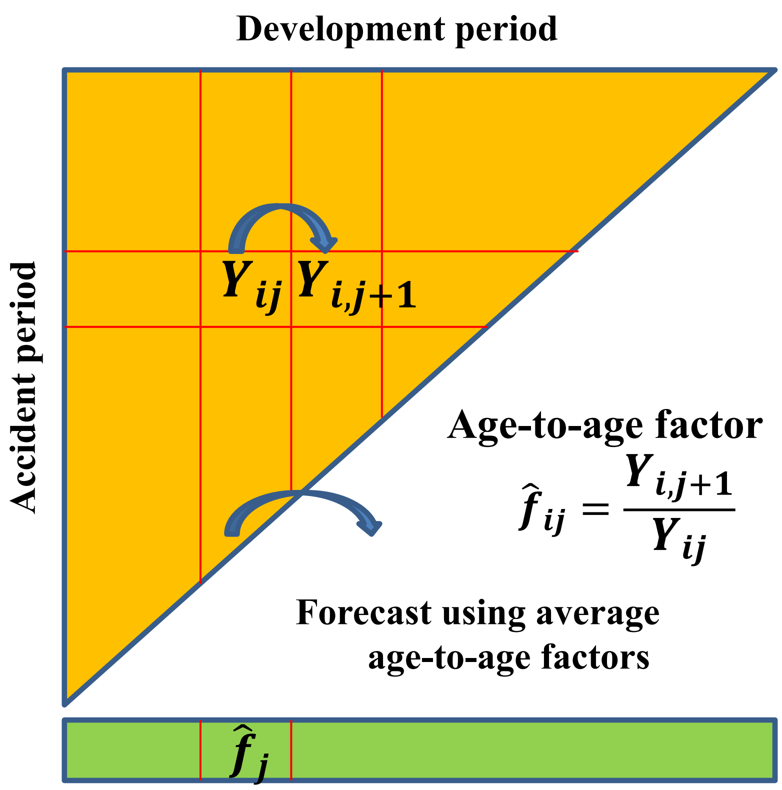
Earliest “models”



Some form of averaging of age-to age factors over each row

- Observations organized by row and column
- An accident period (row) “develops” from one column to the next
- Forecast is $Y_{i,j+1} = \hat{f}_j Y_{ij}$
- These “models” include (Taylor, 1986, 2000; Wüthrich & Merz, 2008):
  - Chain ladder
  - Separation method
  - and all their derivatives:
    - Bornhuetter-Ferguson
    - Cape Cod;
    - etc.

Properties of earliest "models"



Some form of averaging of age-to age factors over each row

- No model formulated
  - No stochastics introduced
  - Actually a "procedure" or "algorithm" rather than model
- Assumption of same age-to-age factors for each row
- Parameter estimation carried out by row, column or diagonal averaging
- In statistical parlance, include "row, column and/or diagonal effects"
- Over-parameterized
  - For an $n \times n$ triangle, chain ladder involves $2n - 1$ parameters
  - This increases prediction error

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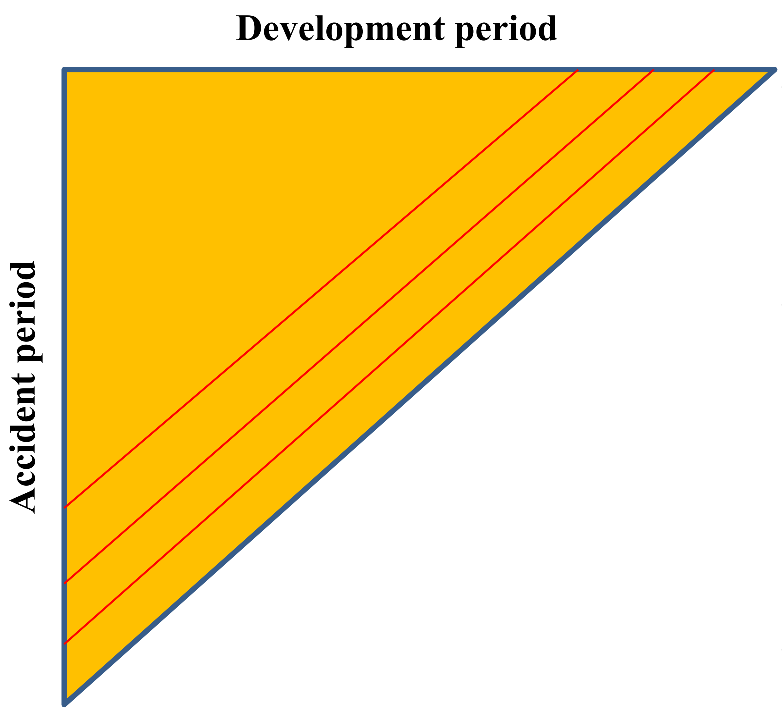
Unfitness of Jurassic denizens



- Most early models include row and column effects
- What if there is a need to include diagonal effects also?
  - e.g. variable inflation
- What if rate of claim settlement changes from one row to another (Fisher & Lange, 1973)?
  - Age-to-age factors vary from row to row
- Such features:
  - Increase parameterization
  - Are difficult to parameterize by row/column/ diagonal manipulation (Taylor, 2000)

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The appearance of stochastic models

- The Jurassic models were unformulated, but essentially took the form:

Observation set
Parameter set

$$Y_{ij} = f(Y, \alpha)$$

- This is not stochastic, but can easily be made so:

$$Y_{ij} = f(Y, \alpha) + \varepsilon_{ij}, \qquad E[\varepsilon_{ij}] = 0$$

- With some restriction on f and ε_{ij}, one arrives at a GLM

$$Y_{ij} \sim F(\mu_{ij}, \varphi / w_{ij}), \qquad \mu_{ij} = E[Y_{ij}], \qquad h(\mu_{ij}) = x_{ij}^T \beta,$$

$$F \in EDF$$

Exponential dispersion family

Scale parameter

Link function

Covariate vector

Parameter vector

- Chain ladder example ($Y_{i,j+1} = \hat{f}_j Y_{ij}$)

$$F = ODP, h = \ln, \quad x_{i,j+1} = [0, \ldots 0, Y_{ij}, 0 \ldots 0], \beta = [f_1, f_2, \ldots]^T$$

Brief history of stochastic models

- Notably advanced creatures of the Jurassic were:
  - Stochastic claims analysis (Reid, 1978)
  - A stochastic chain ladder model (Hachemeister & Stanard, 1975)
  - An individual claim development model (Hachemeister, 1978, 1980)
- History of actuarial GLMs longer than often realized:
  - 1972: concept introduced (Nelder & Wedderburn)
  - 1977: GLIM software introduced
  - 1984: Tweedie family introduced (Tweedie, 1984)
  - 1990+: seminal actuarial papers (Wright, 1990; Brockman & Wright, 1992)
  - Note, however:
    - Early application of GLMs to pricing (Baxter, Coutts & Ross, 1979)
    - Use within my own consulting practices through the 1980's

More recent loss reserving GLMs

- Used to model claim data sets with many complex and overlapping features, e.g.
  - Taylor & McGuire (2004)
    - Auto liability
    - Rates of claim settlement vary over time
    - Superimposed inflation varies with payment quarter and operational time
    - Legislative change (accident quarter)
  - Taylor & Mulquiney (2007)
    - Mortgage insurance
    - Cascaded model with sub-models for healthy policies, in arrears, properties in possession, and claims
  - Taylor, McGuire & Sullivan (2008)
    - Medical malpractice
    - Individual claim development model with covariates such as specialty, geographic area of practice, etc.
  - Taylor & McGuire (2016) – a monograph on GLM reserving
- This type of analysis is now called **Predictive Analytics**

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Adaptation of species

- GLM model was

$$Y_{ij} \sim F\left(\mu_{ij}, \varphi/w_{ij}\right), \qquad \mu_{ij} = E\left[Y_{ij}\right], \qquad h\left(\mu_{ij}\right) = x_{ij}^T \beta$$

- Here parameter set β is constant over time
  - What if it is expected to change?
- One can introduce an **evolutionary model** in which parameters vary over time, e.g. (with **time** $t = i + j$)

$$Y_{ij} \sim F\left(\mu_{ij}^{(t)}, \varphi/w_{ij}\right), \qquad \mu_{ij}^{(t)} = E\left[Y_{ij}\right], \qquad h\left(\mu_{ij}^{(t)}\right) = x_{ij}^T \beta^{(t)}$$

$$\beta^{(t)} \sim P\left(.; \beta^{(t-1)}, \psi\right)$$

Conjugate prior

Prior dispersion structure

- See
  - Taylor (2008)
  - Taylor & McGuire (2009)

Adaptive reserving (1)

- Adaptive model

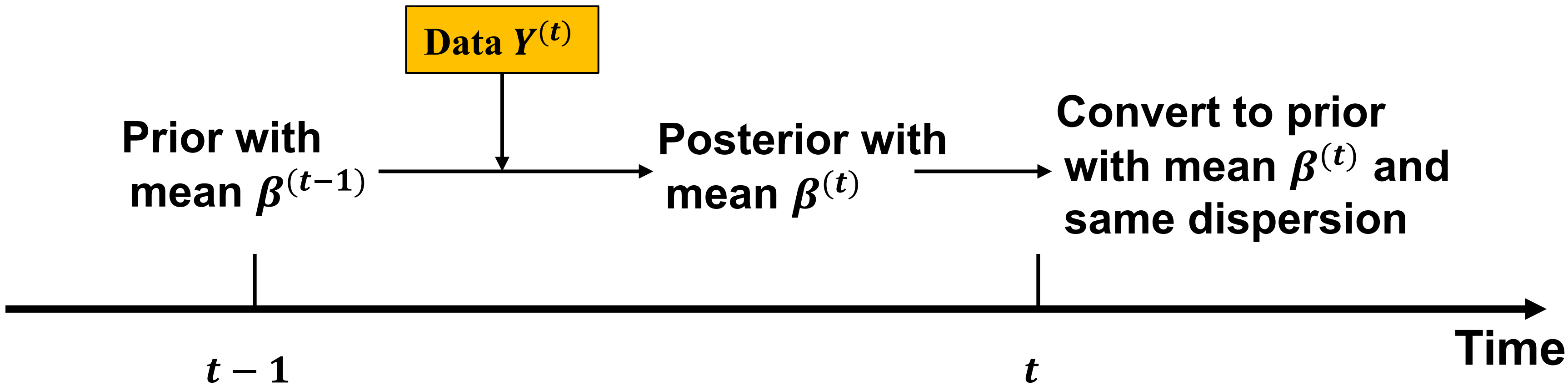
$$Y_{ij} \sim F\left(\mu_{ij}^{(t)}, \varphi/w_{ij}\right), \qquad \mu_{ij}^{(t)} = E[Y_{ij}], \qquad h\left(\mu_{ij}^{(t)}\right) = x_{ij}^T \beta^{(t)}$$

$$\beta^{(t)} \sim P\left(.; \beta^{(t-1)}, \psi\right)$$

- Reminiscent of **Kalman filter** (see Harvey, 1989), **BUT**
  - KF applies to linear models, whereas this one is **non-linear** in general
  - KF assumes normal error for observations, whereas this model assumes **non-normal**
  - The posterior likelihood at each t does not lie within the set of EDF conjugate priors
    - Must be approximated by a member of the set with same first and second order moments
    - Some stability problems

Adaptive reserving (2)

- Schematic of process from time $t-1$ to t



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Miniaturization: dimensionality reduction

- The Jurassic models were lumbering, with overblown parameter sets
- GLMs were more efficient but without much systematic attention to the issue
- A more recent approach that brings the issue into focus is **regularized regression**, and specifically the **least absolute shrinkage and selection operator (LASSO) model** (Tibshirani, 1996)

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Regularized regression

Replace squared error by GLM loss function (log-likelihood) to obtain regularized GLM

- Consider first linear regression, as opposed to GLM, and consider the loss function (in an obvious notation)

$$L(y;\beta) = \|y - X\beta\|_2^2 + \lambda\|\beta\|_p$$

where $\|.\|_p$ denotes the L_p norm and $\lambda > 0$ is a constant

This is **regularized (linear) regression**

- Note that
  - $\lambda = 0$ yields OLS regression
  - $\lambda \neq 0, p = 2$ yields Ridge regression
  - $\lambda \neq 0, p = 1$ yields the lasso

$\lambda \to 0$: no elimination of covariates

$\lambda \to \infty$: maximum elimination of covariates

- A property of the lasso is that it can force many components of β to zero
  - Thus an effective tool for elimination of covariates from a large set

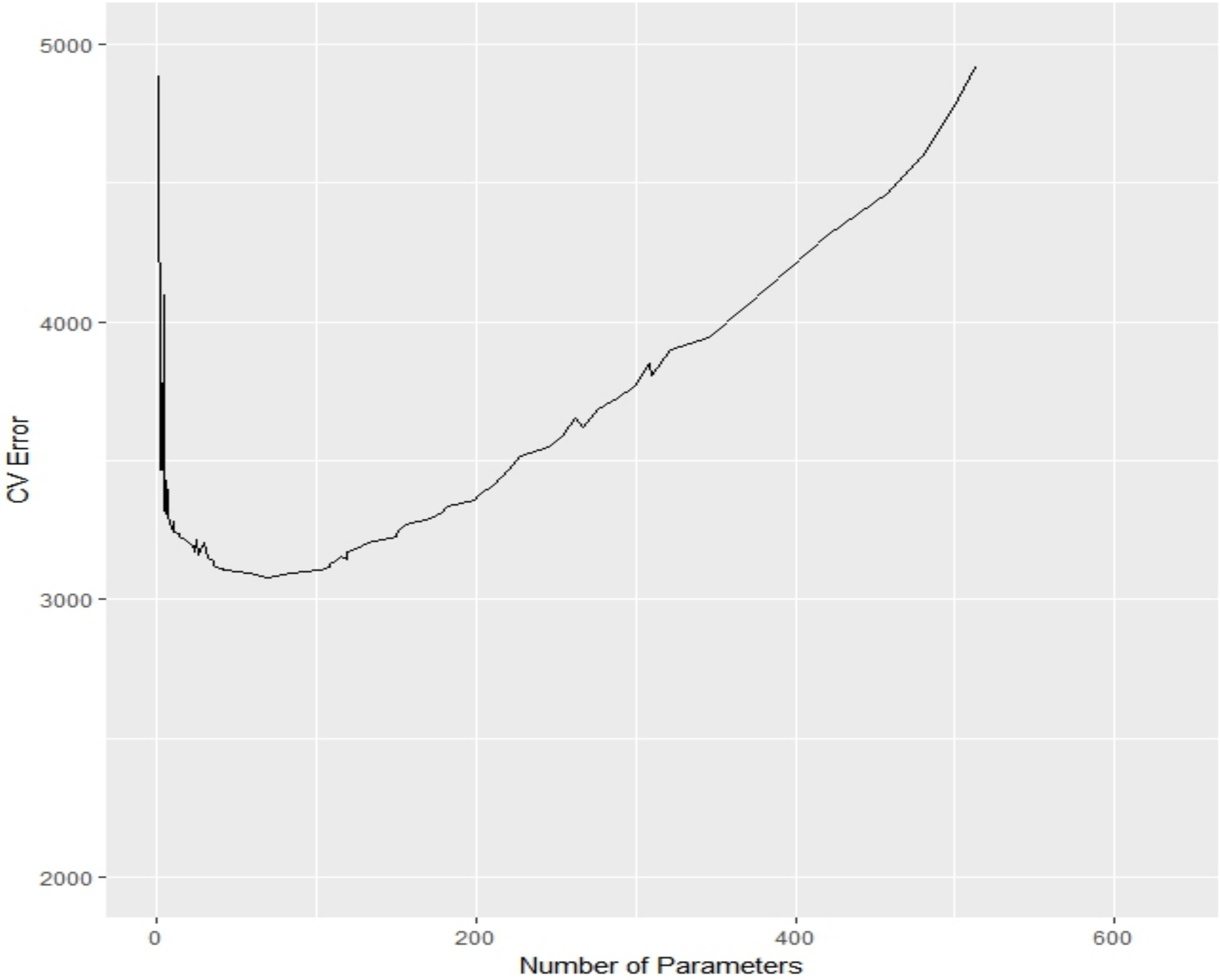
Calibration

- Consider a large set of basis functions from which all functions in the loss reserving model may be expressed as linear combinations
- Lasso used to reduce large covariate set to just the "essential" members
  - Sequence of models examined with increasing λ
  - Number of covariates decreasing
  - Model chosen to minimize cross-validation error
  - Examples
    - Venter & Şahîn (2017) – mortality
    - Gao & Meng(2017) – loss reserving
    - Taylor, McGuire & Miller (2016) – loss reserving

Cross-validation

- Randomly delete one n-th of the data set, as a test sample
- Fit the model to the remainder of the data set (training set)
- Generate fitted values for the test sample
- Compute error between test sample and fitted values (e.g. sum of squares)

Example of lasso calibration



Forecast error (1)

- Let
  - R denote the amount of unpaid losses (a random variable)
  - $\hat{R}$ denote an estimate of R (assumed unbiased)
- One wishes to know something of the distribution of $\hat{R}$, e.g.
  - The full distribution
  - Certain quantiles (risk margins, capital margins)
  - Just the mean square error of prediction (MSEP): $E\left[\left(R-\hat{R}\right)^2\right]$
- If one is not concerned with the tails of the distribution (e.g. 75-percentile risk margin), then MSEP will often provide a measure of the forecast quality

Forecast error (2)

- There are two main approaches to the estimation of forecast error
  - Bootstrap
  - Markov Chain Monte Carlo (MCMC) (Meyers, 2015)
    - Relevant to Bayesian models
- Both estimate full distribution, and therefore any property of the distribution

Parameterization and forecast error

- Beyond a certain threshold, the inclusion of additional parameters in a model will result in over-fitting and increase MSEP
- Similar considerations apply to cascaded models (i.e. those involving multiple sub-models)
- Taylor & Xu (2016) investigated, for certain data sets,
  - Chain ladder (a single model involving only paid amounts); and
  - An alternative model, incorporating reported and finalized claim count information, and comprising 3 sub-models
  - The results indicated that the alternative produced lower MSEP when the data set failed to conform with the chain ladder parametric structure by a sufficient margin

The fine detail: granular (micro-) reserving

- Models the detail of individual claims, e.g.
  - Reporting date
  - Individual payment dates
  - Amounts of individual payments
- Generally regarded as commencing with Norberg (1993, 1999), Hesselager (1994), with implementation by Pigeon, Antonio & Denuit (2013, 2014) and Antonio & Plat (2014)
  - Note, however, the earlier implementations (Hachemeister (1978,1980), Taylor & Campbell (2002))
- Distinction between aggregate and granular models is largely false
  - Any model that includes claim counts can be regarded as granular
    - It produces forecasts at an individual claim level
    - Only a question of the volume of conditioning data
  - So one should think in terms of a aggregate-granular spectrum

Applications of granular reserving

- Loss reserving at the individual claim level has an application when loss reserves are required in respect of small groups of claims and physical estimates do not exist
  - e.g. in relation to small cost centres of an organization
- Otherwise, required only if they produce a loss reserve superior to that produced by aggregate methods
  - Recall that (aggregate) chain ladder is minimum variance for ODP observations (Taylor, 2011)
  - And remember that granular models will usually be cascaded
    - With their property of inflating prediction error
- Huang, Wu & Zhou (2016) claim that micro-models outperform aggregate
  - But their calibration and forecast are essentially the same as the **Payment per Claim Finalized** aggregate model found in the literature (Taylor, 1986, 2000)
  - Just conditioned by more data than their aggregate models
- So jury still out on the value of micro-models!

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The rise of roboticus sapiens (1)

- First steps in machine learning
  - **Artificial neural nets (ANNs)**
    - Mulquiney (2006)
      - Modelled a set of claim finalizations tabulated by:
        - » Accident quarter
        - » Development quarter
        - » Payment quarter
        - » Operational time at finalization
        - » Season of finalization (calendar quarter)
      - ANN goodness-of-fit superior to GLM
      - ANN detected superimposed inflation that varied over both finalization quarter and operational time
      - ANN detected effects of a legislative change (accident quarter effect) that occurred in the midst of the claim experience

The rise of roboticus sapiens (2)

- Harej, Gächter & Jamal (2017)
  - IAA Working Group on "Individual Claim Development with Machine Learning"
  - This was an "under-powered" ANN which assumed chain ladder models for paid and incurred costs respectively for individual claims, and simply estimated the age-to-age factors
  - However, since it included both paid and incurred amounts, it managed to differentiate age-to-age factors for different claims
    - » e.g. claims with small amount paid but large amounts incurred showed high development of payments
- Wüthrich & Buser (2017) have produced a set of lecture notes on machine learning:
  - Regression trees
  - Random forests
  - Support vector machines
  - Clustering for telematics data

The watchmaker and the oracle (1)

- The tendency of micro-modelling (watchmaking) is to increase the number of cascaded sub-models
  - → individual claims
  - → individual payments, etc.
- Many parameters, with implications for prediction error
- Increases the fragility of the model
  - Increased complexity due to dependencies, e.g.
    - In Liability business, occurrence of a large payment would reduce the likelihood of another large payment
    - In Workers Compensation, a return to work from incapacity would usually lower the likelihood of immediate incapacity onset
    - Dependencies between sub-models render validation difficult
      - One may validate all sub-models internally, but then discover that the total model does not validate
- On the other hand, all aspects of the model are understood

The watchmaker and the oracle (2)

- ANN (oracle) is a model that observes all the complexity of the training data, and should accommodate it
  - By-passes all the difficulties of micro-modelling
- However, it is an extremely opaque model
  - At its core (the neurons), it consists of just a set of weighted averages
  - Individual data features (e.g. superimposed inflation) are hidden within the model
  - They may also be poorly measured
    - e.g. diagonal effects may be inaccurately measured , but compensated by measured, but actually non-existent, row effects
  - Can be difficult to validate
    - What is one's recourse in the event of validation failure?

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The future?

- Aggregate models?
- Micro-modelling?
- Machine learning?

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