APPROACHES TO VALIDATING METHODOLOGIES AND MODELS WITH INSURANCE APPLICATIONS

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AGENDA

QCRM to Certify VaR model

ODP Bootstrap Chain-Ladder Risk Model

Approach to Validating Risk Model

QCRM to Certify VaR Model

Basel VaR Model

Limitations of the Basel Model

QCRM hypothesis test

Power of QCRM Test

The Problem

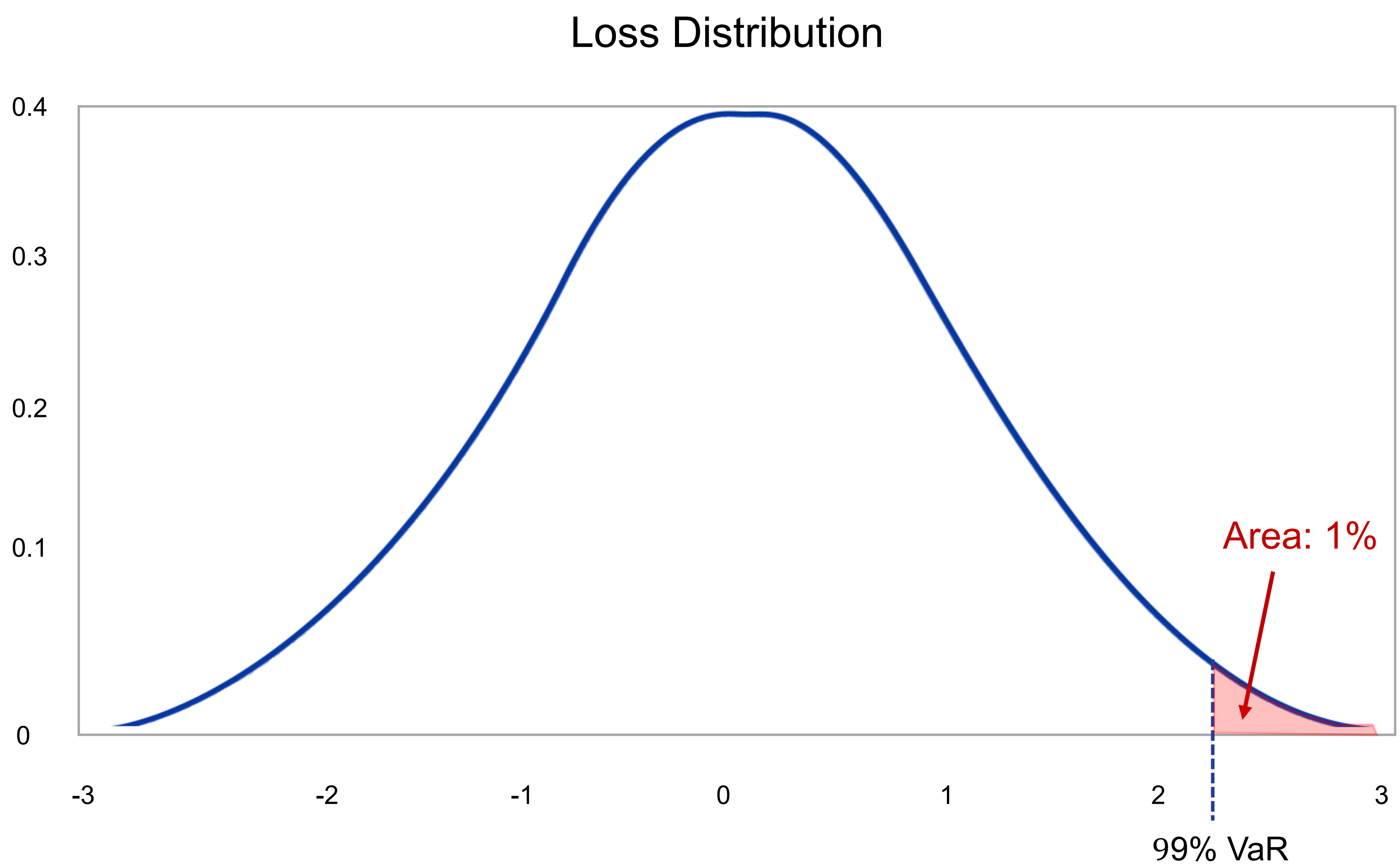
Regulators and Risk managers have to decide a course of action: accept or reject a bank's risk model:

Model correct vs. Model incorrect

VaR Backtesting: Compare loss with VaR model-based risk measures

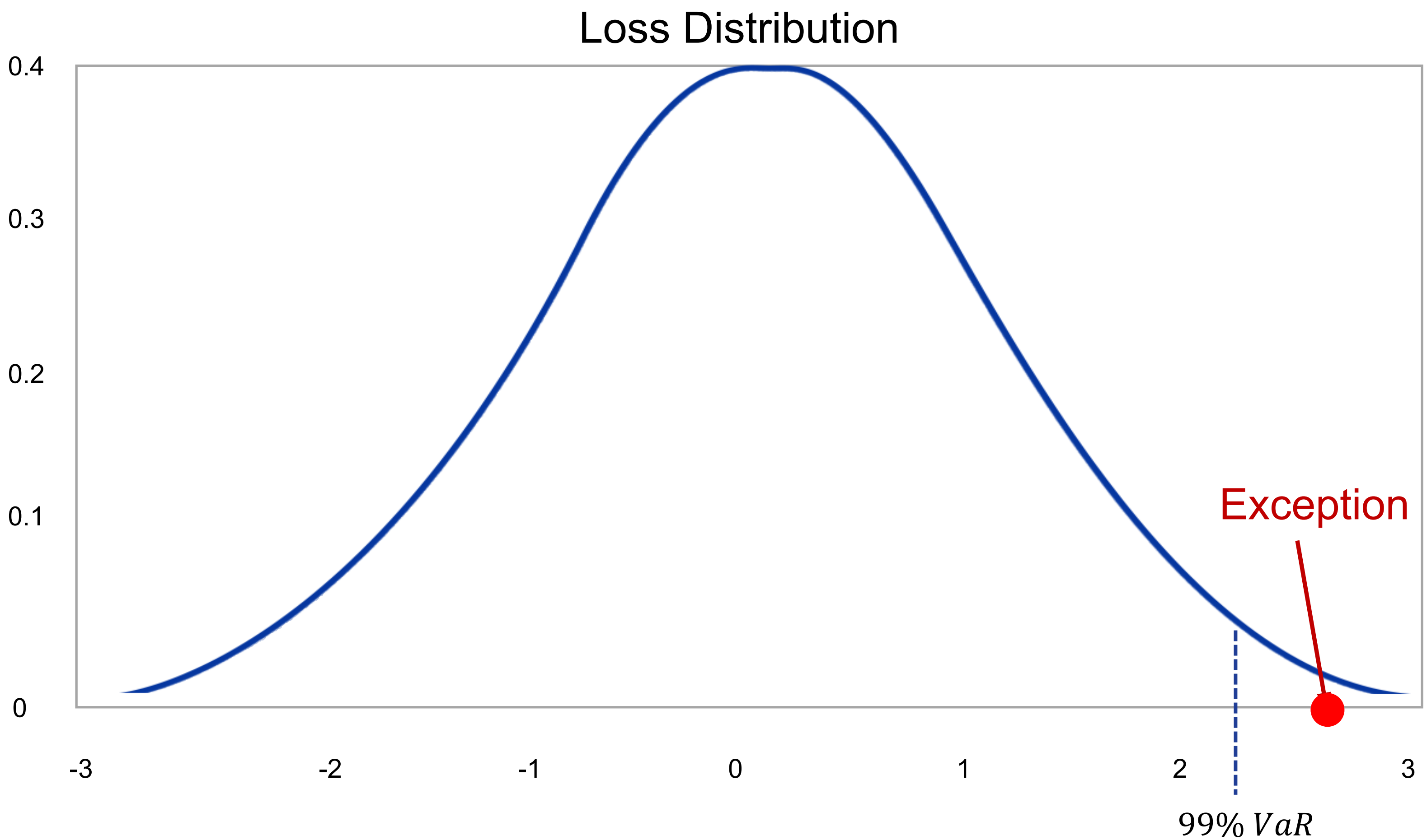
Value at Risk: refreshment

The $(1-\alpha)*100\%$ VaR is the percentile $(1-\alpha)$ of the distribution of the Portfolio losses

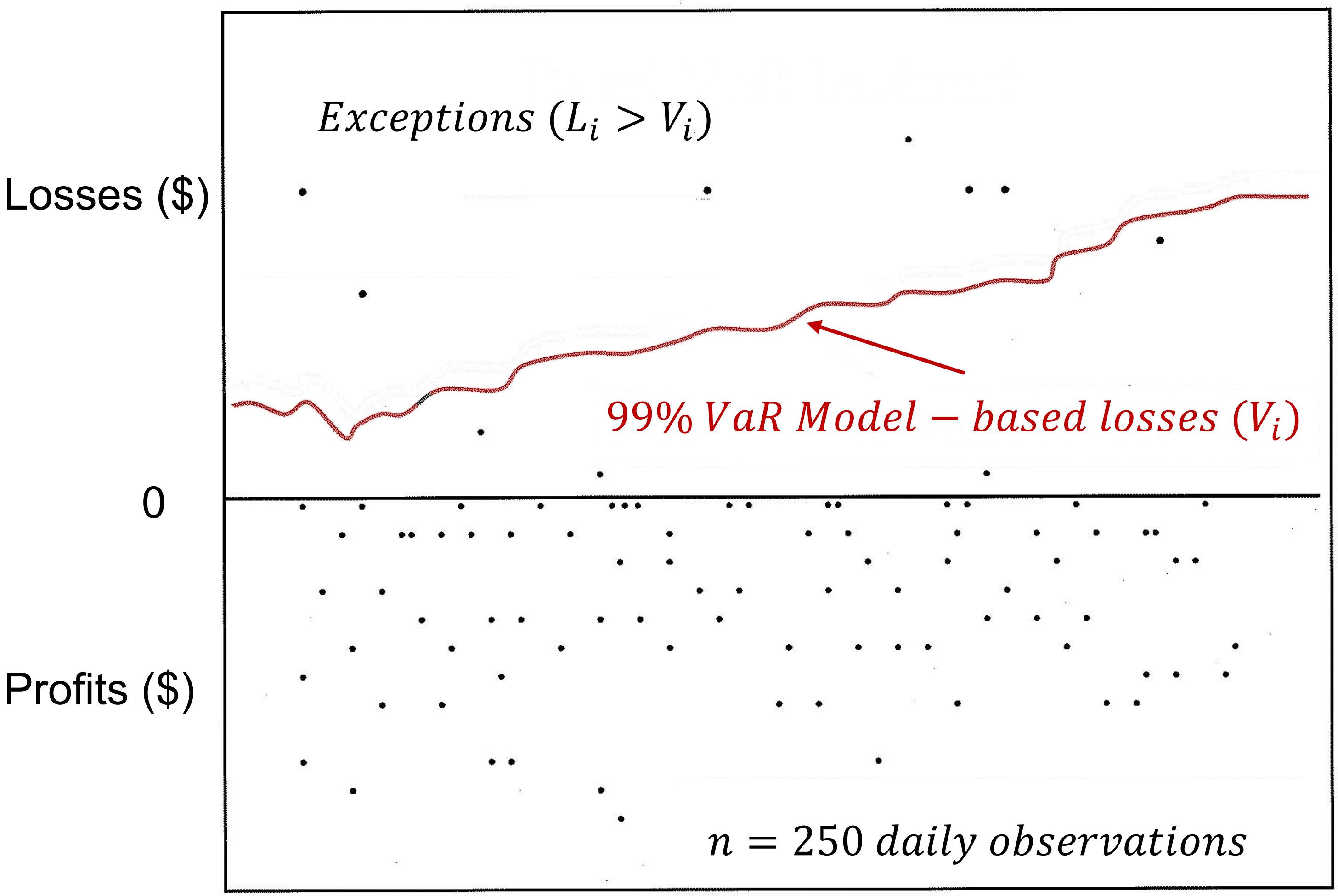


Exception (model failure)

The event that the portfolio loss exceeds the corresponding VaR predicted for a trading day



Basel VaR backtest



Basel VaR Model

acceptance and rejection regions

Zone	# of exception	
Green	0-4	Model is deeded accurate
Yellow	5-9	Additional Info before taking action
Red	≥10	Model is deeded inaccurate

type I error = Pr(# of exceptions ≥ 10 | p_0 =0.01) = α=0.025%

The probability of rejecting the correct VaR model is 0.025%.

Limitation of Basel VaR Model

"The Committee of course recognizes that tests of this type are limited in their power to distinguish an accurate model from an inaccurate model"[1]

Alternative Coverage Level:

Coverage	98%	97%	96%
5	43.9%	12.8%	2.7%
6	61.6%	23.7%	6.3%

[1]Basel Committee on Banking Supervision (Basel), page 5 of "Supervisory Framework for the use of "Back Testing" in conjunction with the internal models approach to Market Risk Capital requirements", January 1996

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Remark

Note on Statistical Hypothesis

a) Not rejecting a statistical hypothesis is not (in general) equivalent to accepting it
b) It is valid to reject a statistical hypothesis when there is overwhelming probability against it

Remark

As a consequence

a) Not rejecting that $p=0.01$ ($p \leq 0.01$) is not equivalent to accepting that $p=0.01$ ($p \leq 0.01$)
b) It is valid to reject the hypothesis $p > 0.01$ against $p \leq 0.01$ when there is overwhelming probability against it

Change of hypotheses

QCRM hypothesis:

H_0: VaR Model incorrect vs. H_A: VaR Model correct

Accepting H_0 implies rejecting the VaR Model

Rejecting H_0 implies accepting the VaR Model

Type I error (of QCRM)=Pr(Accept VaR Model | VaR incorrect)

= Type II Error (of Basel)

New hypothesis test

Assume p is the true probability of having one exception (unknown), QCRM tests:

$$H_0 : p > p_1 \ (\geq 0.01) \text{ vs. } H_A : p \leq p_o \ (= 0.01)$$

This is the quality control problem: control the probability p_1 (and setting α to a small level) of accepting an wrong model.

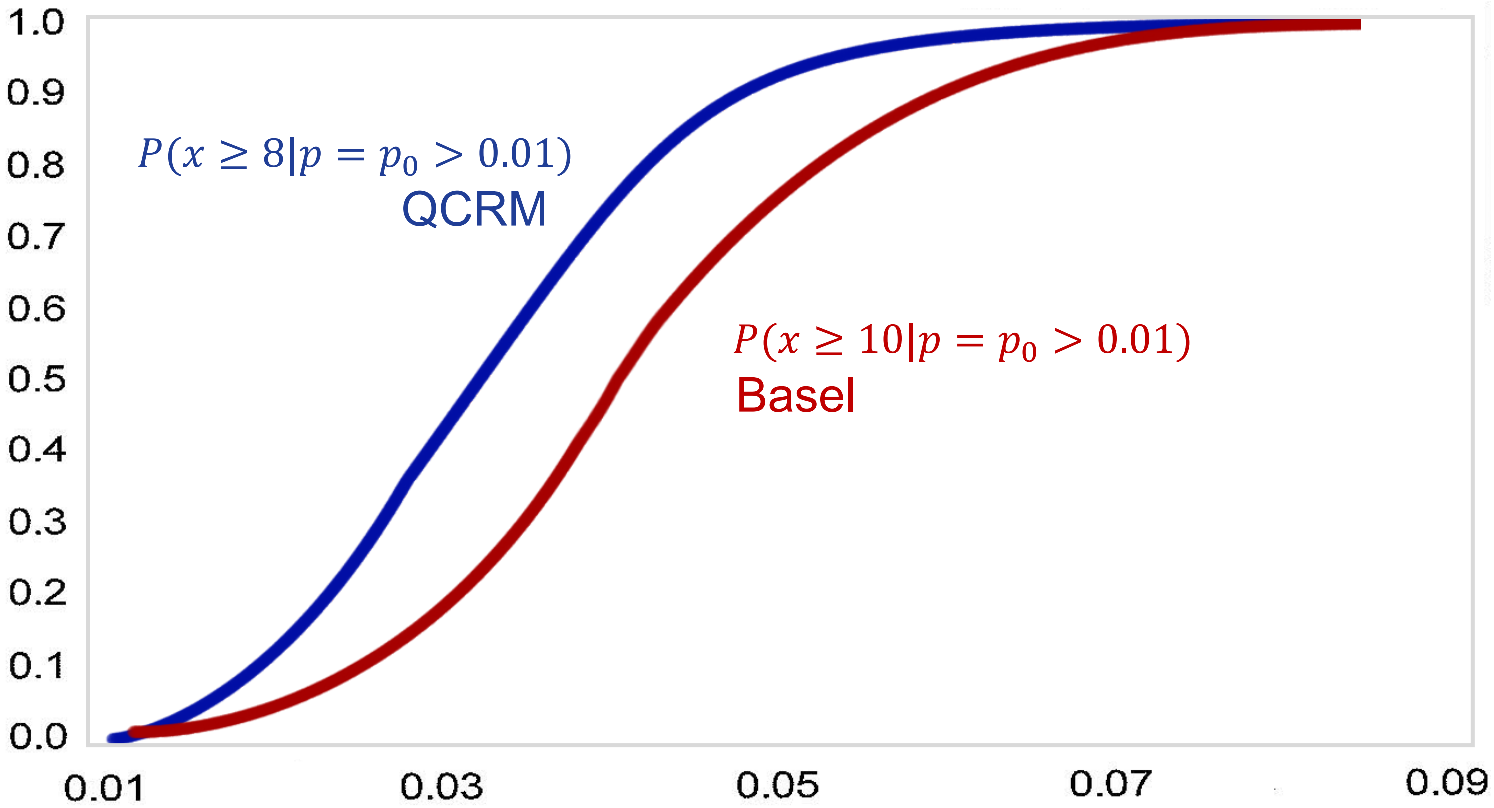
New acceptance and rejection regions

Zone	# of exception	p_0 one-side confident interval
Green	0-5	$p \in (p_L(X,0.05),1]$
Yellow	6-7	$p \in (p_L(X,0.01),1]$ $p \notin (p_L(X,0.05),1]$
Red	≥8	$p \notin (p_L(X,0.05),1]$ $p \notin (p_L(X,0.01),1]$

Powers of QCRM and Basil tests

	Probability of rejecting the model when it is	
Test	**Correct**	**incorrect**
Basil	<0.025%	$P(X \geq 10 \mid p > 0.01)$
QCRM	<0.4%	$P(X \geq 8 \mid p > 0.01)$

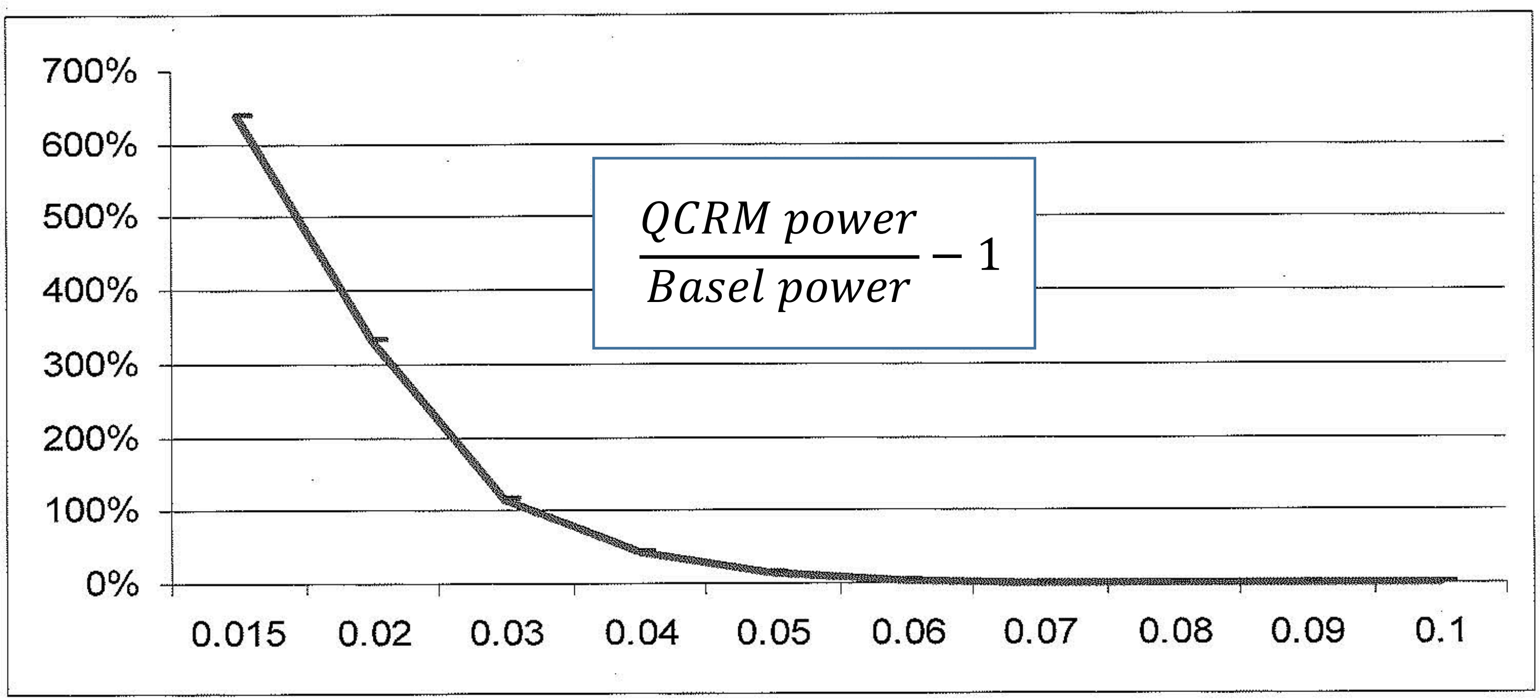
Probability of rejecting a wrong model



X-axis: different values of alternative hypotheses p

Power rate curve

Percentage gains of QCRM over Basel in the probability of rejecting the wrong model



ODP Chain-Ladder Risk Model

ODP Chain-Ladder Risk Model

Bootstrap ODP Chain-Ladder Risk Model

Wang Transform Adjustment

ODP Chain-Ladder Model

Steps in ODP Chain-Ladder Model:

1. Cumulative loss data by AY and DY – Upper Triangle
2. Estimate Development factors by DY
3. Estimate a fitted cumulated loss (upper triangle)
4. Calculate ODP scale parameter ϕ and Adjusted Pearson Residuals

ODP Chain-Ladder Model

Data - Cumulative loss d_{ij} upper triangle

Cumulative loss d_{ij} upper triangle

AY	1	2	3	4	5	6	7	8	9	10
1994	34,254	57,579	63,827	65,817	66,589	66,964	67,037	67,054	67,043	67,067
1995	39,744	63,192	69,380	71,640	72,254	72,486	72,745	72,748	72,756	
1996	42,783	66,602	73,550	76,471	77,394	77,835	78,002	78,027		
1997	43,494	67,870	75,909	78,578	79,933	80,223	80,358			
1998	44,373	68,267	76,507	79,515	81,079	81,502				
1999	44,066	67,425	76,490	78,662	79,916					
2000	45,555	69,961	79,024	81,436						
2001	49,557	76,180	84,956							
2002	52,028	80,804								
2003	55,868									

ODP Chain-Ladder Model

Estimate Development Factors

Step1: Company A, paid loss & ALAE, net of reinsurance as of 12/2003										
AY	1	2	3	4	5	6	7	8	9	10
1994	34,254	57,579	63,827	65,817	66,589	66,964	67,037	67,054	67,043	67,067
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2002	52,028	80,804								
2003	55,868									

Sum( ) / Sum( ) = 1.12

1.56	**1.12**	**1.03**	**1.01**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

ODP Chain-Ladder Model

Re-estimate past Cumulative triangle, use the LDFs to fit the original data

Step1: Company A, paid loss & ALAE, net of reinsurance as of 12/2003										
AY	1	2	3	4	5	6	7	8	9	10
1994	34,254	57,579	63,827	65,817	66,589	66,964	67,037	67,054	67,043	67,067
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2003	55,868									

1.56	**1.12**	**1.03**	**1.01**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

ODP CHAIN-LADDER MODEL

Unscaled Residuals										
AY	1	2	3	4	5	6	7	8	9	10
1994	-11.39	20.24	-4.62	-3.45	-5.60	3.64	-5.82	0.85	0.00	
1995	1.07	8.57	-11.80	-1.52	-12.82	-5.73	8.39	-3.10	0.00	
1996	1.88	0.26	-8.67	8.37	-5.30	4.17	0.09	2.21		
1997	-0.84	-0.75	1.10	1.80	6.64	-4.28	-2.74			
1998	-0.06	-6.35	1.88	7.58	12.20	2.28				
1999	1.63	-7.45	12.49	-8.05	3.59					
2000	1.68	-5.93	9.31	-4.95						
2001	3.66	-4.35	-0.94							
2002	1.14	-1.52								
2003										

$$r = \frac{c - \hat{c}}{\sqrt{\hat{c}}}$$

C is AIL
Ĉ is EIL

$$\phi = \frac{\sum r^2}{DoF}$$

Adjusted Pearson Residuals										
AY	1	2	3	4	5	6	7	8	9	10
1994	-14.08	25.02	-5.71	-4.27	-6.92	4.50	-7.20	1.05	0.00	
1995	1.32	10.59	-14.58	-1.88	-15.85	-7.08	10.37	-3.84	0.00	
1996	2.33	0.33	-10.71	10.34	-6.55	5.15	0.11	2.73		
1997	-1.04	-0.93	1.36	2.22	8.21	-5.29	-3.39			
1998	-0.08	-7.85	2.32	9.37	15.08	2.81				
1999	2.01	-9.20	15.44	-9.95	4.44					
2000	2.07	-7.34	11.51	-6.12						
2001	4.52	-5.38	-1.16							
2002	1.41	-1.88								
2003										

$$r_p = r \cdot \sqrt{\frac{n}{DoF}}$$

ODP Chain-Ladder Model

Steps in ODP Chain-Ladder Model:

1. Cumulative loss data by AY and DY – Upper Triangle
2. Estimate Development factors by DY
3. Estimate a fitted cumulated loss (upper triangle)

Calculate ODP scale parameter ϕ and Adjusted Pearson Residuals.

4. Calculated the unscaled Pearson Residual r
5. Calculated the (ODP) Scale Parameter
6. Calculate Adjust unscaled Pearson Residuals r_p.

Note that: r_p is an upper triangle matrix.

Steps in Bootstrap ODP Chain-Ladder Model:

1. Sample the Adjusted Pearson Residual r_p (Upper Triangle) with replacement
2. Calculate the (upper) triangle of sampled incremental loss (*EIL*): $C = \hat{c} + r_p \cdot \sqrt{\hat{c}}$
3. Project the future IL (or cumulative loss) (lower Triangle)
4. Include process variance by simulating each *future IL* from a Gamma distribution (approximate ODP distribution)

   mean = future IL

   Variance = mean × scale parameter ϕ
5. Calculate Ultimate Loss (*UL*)
6. Obtain UL Distribution by repeat 1-5 (for example, 10,000).

Bootstrap ODP Chain-Ladder Model-Steps

(Data - Cumulative or Incremental Loss by AY and DY):

1. Estimate Development factors by DY
2. Estimate a fitted cumulated loss (upper triangle)
3. Calculate ODP scale parameter ϕ and Adjusted Pearson Residuals r_p (Upper Triangle)
4. Sample the Adjusted Pearson Residual r_p with replacement
5. Calculate the (upper) triangle of sampled incremental loss (EIL): $C = \hat{c} + r_p \cdot \sqrt{\hat{c}}$
6. Project the future IL (lower Triangle)
7. Simulating each future IL ≈ Gamma (EIL, EIL×ϕ)
8. Calculate Ultimate Loss (UL)
9. Obtain UL distribution by repeat 4-8 (for example, 10,000).

Backtest ODP Chain-Ladder Model

The percentile of the actual Loss should be uniform distributed.

Backtesting an Accident Year (AI) as of mmyyyy (ex, AI 2003 as of December 2012):

1. Create a distribution of the UL by Bootstrap ODP Chain-Ladder Method
2. Percentile of the actual unpaid for each company
3. Test the uniformness of the percentiles

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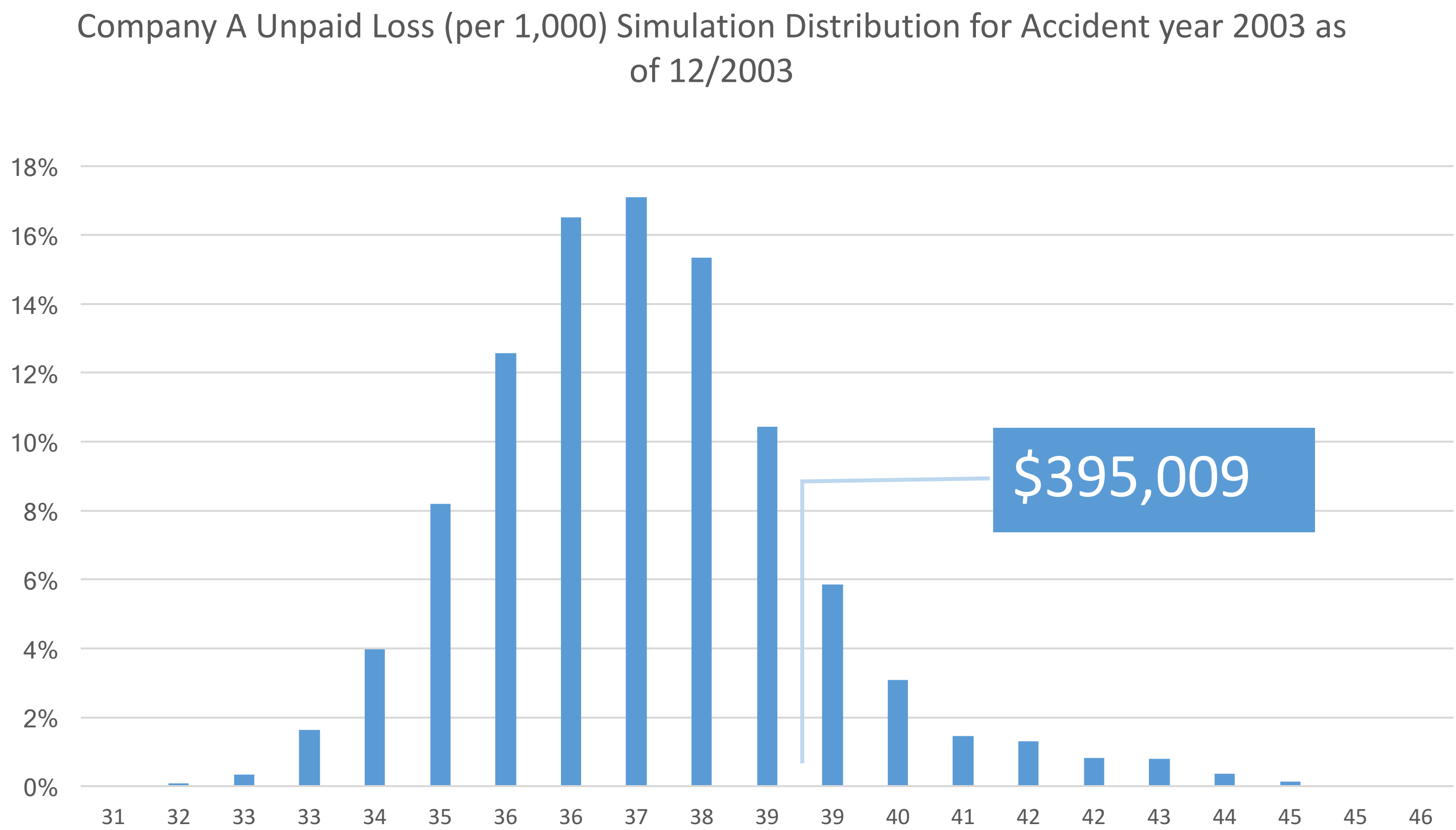
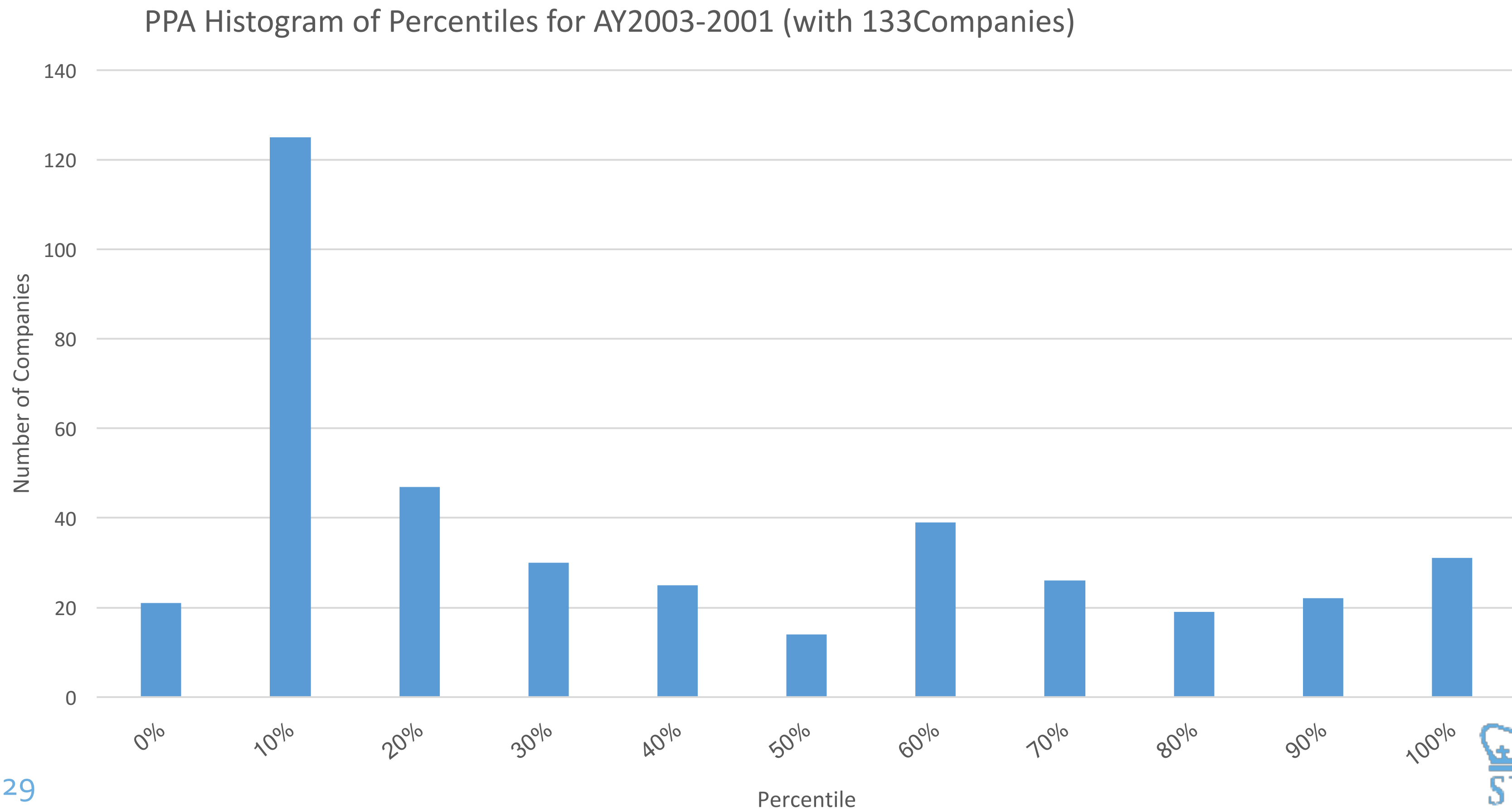
Backtest ODP Chain-Ladder Model



This is 88.63% percentile in the simulated Chain Ladder model.

Percentile Distribution of Bootstrap Model for 133 companies from AY2003 to AY2001



Wang Transform adjustment

Wang et al showed that the chain-ladder reserving method has systemic error and moreover the systemic error *are highly correlated with the reserve cycle*.

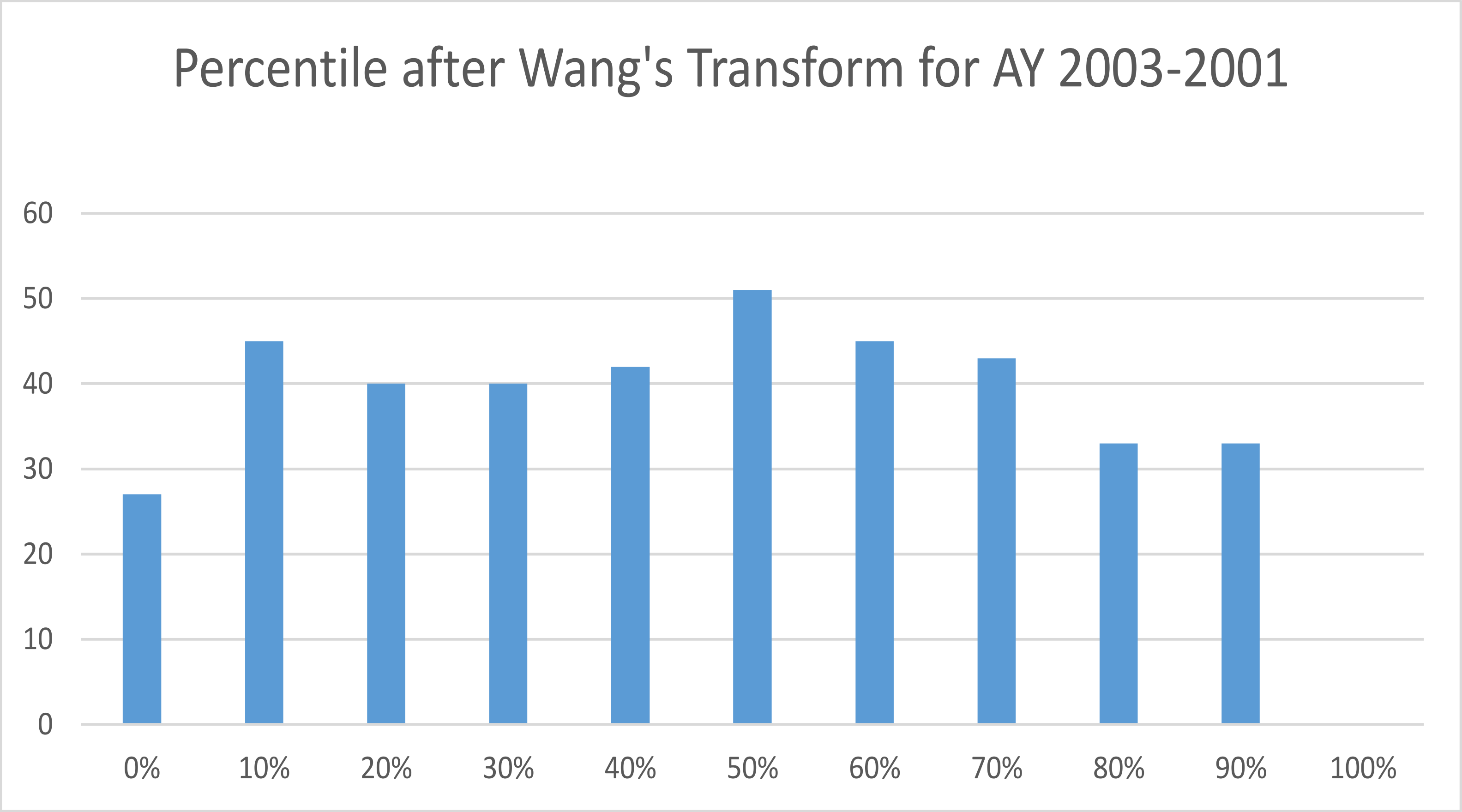
The contemporary correlation between the estimation error and the reserve development is .64 for the chain-ladder method. More noticeably the one-year lag correlation is 0.91. The estimation error leads to the loss reserve development by one year.

Wang Transform adjustment

Wang transform adjustment method tried to catch the systemic over course of reserve cycle.

Wang Transform method will first adjust the variability of the loss reserve and then give the distribution a shift.

Wang Transform Adjustment



Backtest ODP Chain-Ladder Model-Steps

(Data - Cumulative or Incremental Loss by AY and DY):

1. Estimate the (upper) triangle of sampled incremental loss (EIL): $C = \hat{c} + r_p \cdot \sqrt{\hat{c}}$
2. Project the future IL (lower Triangle)
3. Simulating each future IL ≈ Gamma (EIL, EIL×ϕ)
4. Calculate Ultimate Loss (UL)
5. Obtain UL distribution by repeat 4-8 (for example, 10,000)
6. Backtest uniformity of distribution
7. Adjust the reserve distribution by Wang Transform

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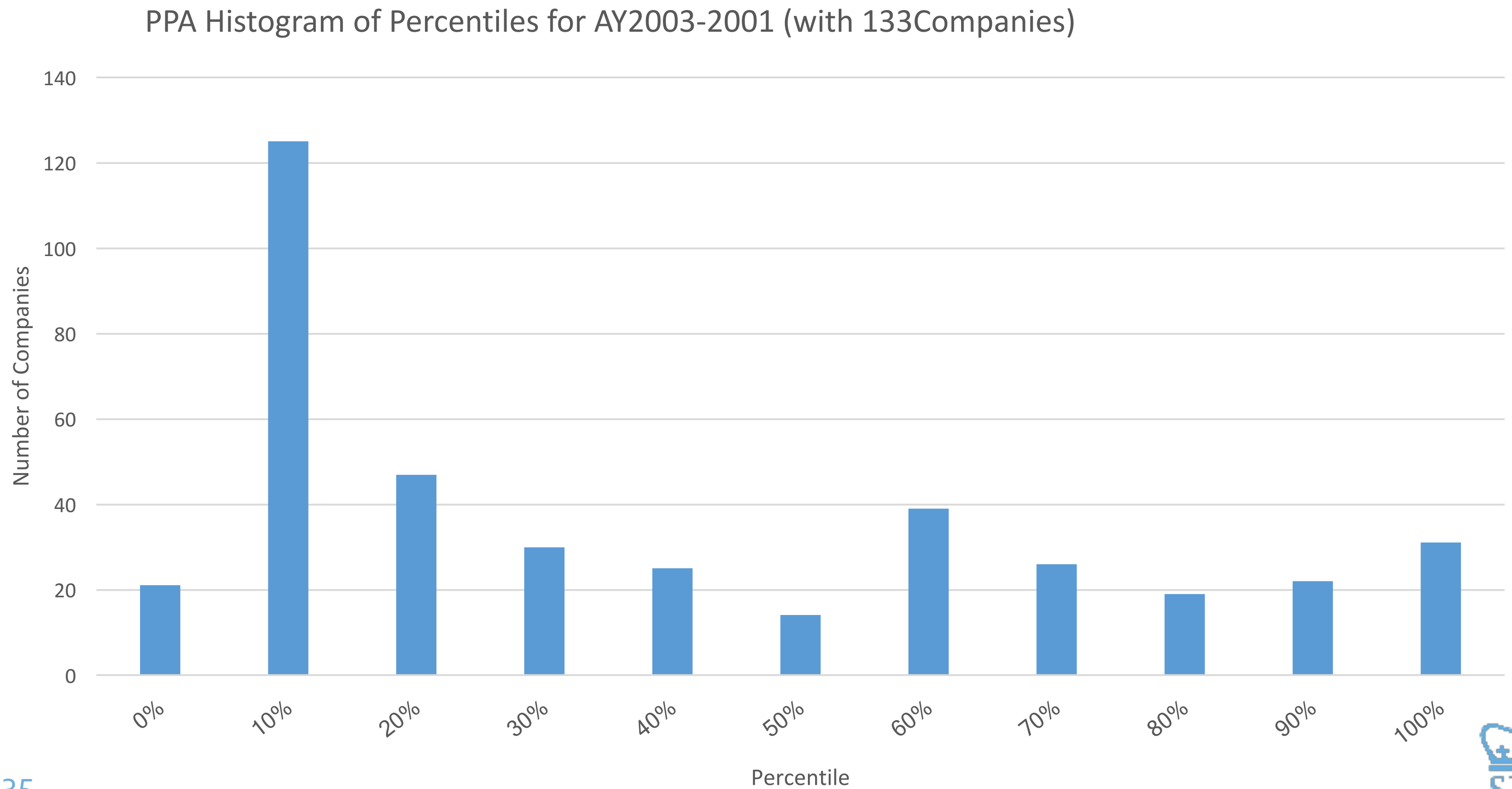
Approaches to Validating Risk Model Using QCRM

ODP Bootstrap Chain-Ladder Model

QCRM Hypothesis Test to ODP Chain-Model

Assertion Zones

Percentile Distribution of Bootstrap Model for 133 companies from AY2003 to AY2001



QCRM Hypothesis Test

New hypothesis test

Assume p is the true probability of having one exception (unknown), QCRM tests:

$$H_0 : p > p_1 \ (\geq 0.01) \text{ vs. } H_A : p \leq p_o \ (= 0.01)$$

Intuitively, we are expecting 1% of the time the actual unpaid percentiles will be above the 99th percentile of the bootstrap distribution if the model is correct.

Definition: an exception is when the actual unpaid percentile of the simulated unpaid loss bootstrap model is greater than or equal to 99th VaR.

Validating VaR Model using QCRM

$p_L(X,\alpha)$ for 399 trials (133 companies for three accident years)

		95%	99%
Green			
	k=1	0.00090	0.00038
	k=2	0.0021	0.0011
	k=3	0.0034	0.0021
	k=4	0.0049	0.0032
	k=5	0.0066	0.0045
	k=6	0.0083	0.0058
Yellow			
	k=7	0.0100	0.0072
	k=8	0.0118	0.0087
Red			
	k=9	0.0136	0.0103
	k=10	0.0155	0.0120

VALIDATING VAR MODEL USING QCRM

The assertion zones for 399 trials:

Zone	#of exception	Decision
Green	≤6	accept the bootstrap model
Yellow	btwn 7 and 8	model is questionable
Red	≥9	reject the bootstrap model

There are 10 exceptions *before* Wang transform Therefore, the bootstrap model is rejected.

There are 4 exceptions *after* Wang transform Therefore, the bootstrap model is accepted.

Q&A

Appendix

Backtest ODP Chain-Ladder Model

Chain-Ladder Techniques

ODP Chain-Ladder Model

Bootstrap ODP Chain-Ladder Model

Backtest ODP bootstrap Chain-Ladder Model

Chain-Ladder Technique

1. Cumulative Triangle

    incremental Loss: $\{c_{ij} : i = 1, 2, .., n;\ j = 1, 2, ..., \text{n - i} + 1\}$

    cumulative loss: $$d_{ij} = \sum_{k=1}^{j} c_{ik}$$

2. Calculate Development factors $\{\lambda_i : i = 1, 2, .., n\}$

$$\lambda_j = \frac{\sum_{i=1}^{n-j+1} d_{ij}}{\sum_{i=1}^{n-j+1} d_{i,j-1}}$$

3. Project future cumulative loss D_{ik}

$$D_{i,n-i+2} = d_{i,n-i+1} \cdot \lambda_{n-i+2}$$

$$D_{i,k} = D_{i,k-1} \cdot \lambda_k \qquad k = n - i + 3,\ n - i + 4,\ ...,\ \text{n.}$$

Chain-Ladder Model

Data - Cumulative loss d_{ij} upper triangle

Cumulative loss d_{ij} upper triangle										
AY	1	2	3	4	5	6	7	8	9	10
1994	34,254	57,579	63,827	65,817	66,589	66,964	67,037	67,054	67,043	67,067
1995	39,744	63,192	69,380	71,640	72,254	72,486	72,745	72,748	72,756	
1996	42,783	66,602	73,550	76,471	77,394	77,835	78,002	78,027		
1997	43,494	67,870	75,909	78,578	79,933	80,223	80,358			
1998	44,373	68,267	76,507	79,515	81,079	81,502				
1999	44,066	67,425	76,490	78,662	79,916					
2000	45,555	69,961	79,024	81,436						
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2002	52,028	80,804								
2003	55,868									

Chain-Ladder Model

Estimate Development Factors based on Cumulative loss d_{ij} upper triangle

Step1: Company A, paid loss & ALAE, net of reinsurance as of 12/2003										
AY	1	2	3	4	5	6	7	8	9	10
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2002	52,028	80,804								
2003	55,868									

Sum( ) / Sum( ) = 1.56

1.56	**1.12**	**1.03**	**1.01**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

Chain-Ladder Model

Estimate Development Factors

Step1: Company A, paid loss & ALAE, net of reinsurance as of 12/2003										
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2002	52,028	80,804								
2003	55,868									

Sum( ) / Sum( ) = 1.12

1.56	**1.12**	**1.03**	**1.01**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

Chain-Ladder Model

Estimate Development Factors

Step1: Company A, paid loss & ALAE, net of reinsurance as of 12/2003										
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Sum( ) / Sum( ) = 1.03

1.56	**1.12**	**1.03**	**1.01**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

Chain-Ladder Model

Estimate Development Factors

Step1: Company A, paid loss & ALAE, net of reinsurance as of 12/2003										
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2003	55,868									

Same logic to get the rest LDF

1.56	1.12	1.03	1.01	1.00	1.00	1.00	1.00	1.00

ODP Chain-Ladder Model

Steps in ODP Chain-Ladder Model:

1. Cumulative loss data by AY and DY – Upper Triangle
2. Estimate Development factors by DY
3. Estimate a fitted cumulated loss (upper triangle)
4. Calculate ODP scale parameter ϕ and Adjusted Pearson Residuals

ODP Chain-Ladder Model

Estimate fitted incremental loss C for the upper triangle

i. Estimate the cumulative loss $\hat{d}_{ik}$ upper triangle by:

$$\hat{d}_{i,k} = \begin{cases} \lambda_j \cdot d_{i,i} & \mathrm{k = n - i - 1} \\ \lambda_{k+1} \cdot m_{i,k-1} & \mathrm{k = n - i - 2, ..., 1} \end{cases}$$

ii. Calculate Pearson residual and Scale parameter ϕ

a. Calculate unscaled Pearson residual

b. Calculate degree of freedom (DoF)

c. Calculate the adjusted Pearson residual

$$r = \frac{c - \hat{c}}{\sqrt{\hat{c}}}$$

$$r_p = r \cdot \sqrt{\frac{n}{DoF}} \qquad \phi = \frac{\sum r^2}{DoF} \qquad (l)$$

iii. Calculate the fitted incremental loss $C = \hat{c} + r_p \cdot \sqrt{\hat{c}}$ (2)

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ODP Chain-Ladder Model

Re-estimate past Cumulative triangle, use the LDFs to fit the original data

Step1: Company A, paid loss & ALAE, net of reinsurance as of 12/2003

AY	1	2	3	4	5	6	7	8	9	10
1994	34,254	57,579	63,827	65,817	66,589	66,964	67,037	67,054	67,043	67,067
1995	39,744	63,192	69,380	71,640	72,254	72,486	72,745	72,748	72,756	
1996	42,783	66,602	73,550	76,471	77,394	77,835	78,002	78,027		
1997	43,494	67,870	75,909	78,578	79,933	80,223	80,358			
1998	44,373	68,267	76,507	79,515	81,079	81,502				
1999	44,066	67,425	76,490	78,662	79,916					
2000	45,555	69,961	79,024	81,436						
2001	49,557	76,180	84,956							
2002	52,028	80,804								
2003	55,868									

1.56	**1.12**	**1.03**	**1.01**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**

ODP CHAIN-LADDER MODEL

Unscaled Residuals										
AY	1	2	3	4	5	6	7	8	9	10
1994	-11.39	20.24	-4.62	-3.45	-5.60	3.64	-5.82	0.85	0.00	
1995	1.07	8.57	-11.80	-1.52	-12.82	-5.73	8.39	-3.10	0.00	
1996	1.88	0.26	-8.67	8.37	-5.30	4.17	0.09	2.21		
1997	-0.84	-0.75	1.10	1.80	6.64	-4.28	-2.74			
1998	-0.06	-6.35	1.88	7.58	12.20	2.28				
1999	1.63	-7.45	12.49	-8.05	3.59					
2000	1.68	-5.93	9.31	-4.95						
2001	3.66	-4.35	-0.94							
2002	1.14	-1.52								
2003										

$$r = \frac{c - \hat{c}}{\sqrt{\hat{c}}}$$

C is AIL
Ĉ is EIL

$$\phi = \frac{\sum r^2}{DoF}$$

Adjusted Pearson Residuals										
AY	1	2	3	4	5	6	7	8	9	10
1994	-14.08	25.02	-5.71	-4.27	-6.92	4.50	-7.20	1.05	0.00	
1995	1.32	10.59	-14.58	-1.88	-15.85	-7.08	10.37	-3.84	0.00	
1996	2.33	0.33	-10.71	10.34	-6.55	5.15	0.11	2.73		
1997	-1.04	-0.93	1.36	2.22	8.21	-5.29	-3.39			
1998	-0.08	-7.85	2.32	9.37	15.08	2.81				
1999	2.01	-9.20	15.44	-9.95	4.44					
2000	2.07	-7.34	11.51	-6.12						
2001	4.52	-5.38	-1.16							
2002	1.41	-1.88								
2003										

$$r_p = r \cdot \sqrt{\frac{n}{DoF}}$$

SPS

Recap ODP Chain-Ladder Model

Steps in ODP Chain-Ladder Model:

1. Cumulative loss data by AY and DY – Upper Triangle
2. Estimate Development factors by DY
3. Estimate a fitted cumulated loss (upper triangle)

Calculate ODP scale parameter ϕ and Adjusted Pearson Residuals.

4. Calculated the unscaled Pearson Residual r
5. Calculated the (ODP) Scale Parameter
6. Calculate Adjust unscaled Pearson Residuals r_p.

Note that: r_p is an upper triangle matrix.

ODP Bootstrap Chain-Ladder Model

Beginning with the estimates from ODP Chain-Ladder Model, r_p, C, and φ, the bootstrap is to repeat the iterative N (in our case 10,000) times:

1. Sample the adjusted Pearson residuals r_p from formula (1) with replacement;
2. Calculate the sampled incremental loss C using formula (2)
3. Project the future incremental loss using the sampled triangle in 2. using Chain-Ladder method
4. Include process variance by simulating each incremental future loss from a Gamma distribution (approximation to ODP distribution):
5. Calculate the ultimate loss

Bootstrap ODP Chain-Ladder Model

Steps in Bootstrap ODP Chain-Ladder Model:

1. Sample the Adjusted Pearson Residual r_p (Upper Triangle) with replacement
2. Calculate the (upper) triangle of sampled incremental loss (EIL): $C = \hat{c} + r_p \cdot \sqrt{\hat{c}}$
3. Project the future IL (or cumulative loss) (lower Triangle)
4. Include process variance by simulating each future IL from a Gamma distribution (approximate ODP distribution)

   mean = future IL

   Variance = mean × scale parameter ϕ
5. Calculate Ultimate Loss (UL)
6. Obtain UL Distribution by repeat 1-5 (for example, 10,000).

SPS

Bootstrap ODP Chain-Ladder Model

Adjusted Pearson Residual

AY	1	2	3	4	5	6	7	8	9	10
1994	-14.08	25.02	-5.71	-4.27	-6.92	4.50	-7.20	1.05	0.00	
1995	1.32	10.59	-14.58	-1.88	-15.85	-7.08	10.37	-3.84	0.00	
1996	2.33	0.33	-10.71	10.34	-6.55	5.15	0.11	2.73		
1997	-1.04	-0.93	1.36	2.22	8.21	-5.29	-3.39			
1998	-0.08	-7.85	2.32	9.37	15.08	2.81				
1999	2.01	-9.20	15.44	-9.95	4.44					
2000	2.07	-7.34	11.51	-6.12						
2001	4.52	-5.38	-1.16							
2002	1.41	-1.88								
2003										

1

Re-calculate the past incremental loss triangle

AY	1	2	3	4	5	6	7	8	9	10
1994	39306	20764	7851	1786	723	347	159	0	0	25
1995	36733	21323	7359	2288	730	207	284	8	0	
1996	40970	24088	7237	2555	1176	187	223	58		
1997	44616	24330	7970	2721	1224	413	200			
1998	47640	26712	7987	2760	1148	260				
1999	43748	24966	8411	1866	1133					
2000	46160	25052	9157	1849						
2001	51038	27188	10318							
2002	57461	30633								
2003	54233									

2

Bootstrap ODP Chain-Ladder Model

Project future cumulative loss

AY	1	2	3	4	5	6	7	8	9	10
1994	38448	58005	64637	66630	67291	67604	67734	67744	67744	67763
1995	40415	64887	72458	74924	76092	76602	76659	76674	76674	76695
1996	47547	71759	79858	82477	83315	83716	83908	83928	83928	83952
1997	43889	67256	76119	78764	79906	80177	80323	80339	80339	80361
1998	44673	70000	78101	80358	81587	81825	81965	81981	[illegible]	82004
1999	42221	66077	74431	76722	78698	79049	79183	[illegible]	[illegible]	79221
2000	45488	70540	78117	80507	81735	82099	82239	82256	82256	82279
2001	49207	75843	83699	86362	87679	88070	88221	88238	88238	88263
2002	51842	81634	91090	93987	95421	95847	96010	96029	96029	96056
2003	56358	87385	97507	100609	102143	102599	102774	102795	102795	102823

Obtain the future incremental loss and therefore UL

AY	1	2	3	4	5	6	7	8	9	10
1994	38448	19557	6633	1993	661	313	130	10	0	19
1995	40415	24472	7572	2465	1169	509	57	15	0	30
1996	47547	24212	8099	2619	839	401	192	20	0	4
1997	43889	23367	8863	2645	1143	271	146	1	0	12
1998	44673	25327	8100	2257	1229	239	184	4	0	43
1999	42221	23857	8354	2291	1975	408	141	24	0	12
2000	45488	25052	7576	2390	1169	339	96	11	[illegible]	21
2001	49207	26637	7856	2543	1169	426	103	10	[illegible]	46
2002	51842	29792	9946	2855	1459	458	143	69	0	12
2003	56358	31917	9836	3100	1481	512	366	6	0	26

Bootstrap ODP Chain-Ladder Model-Steps

(Data - Cumulative or Incremental Loss by AY and DY):

1. Estimate Development factors by DY
2. Estimate a fitted cumulated loss (upper triangle)
3. Calculate ODP scale parameter ϕ and Adjusted Pearson Residuals r_p (Upper Triangle)
4. Sample the Adjusted Pearson Residual r_p with replacement
5. Calculate the (upper) triangle of sampled incremental loss (EIL): $C = \hat{c} + r_p \cdot \sqrt{\hat{c}}$
6. Project the future IL (lower Triangle)
7. Simulating each future IL ≈ Gamma (EIL, EIL×ϕ)
8. Calculate Ultimate Loss (UL)
9. Obtain UL distribution by repeat 4-8 (for example, 10,000).

Backtest ODP Chain-Ladder Model

Backtesting an Accident Year (AI) as of mmyyyy (ex, AI 2003 as of December 2012):

1. Create a distribution of the UL by Bootstrap ODP Chain-Ladder Method
2. Percentile of the actual unpaid for each company
3. Test the uniformness of the percentiles

Backtest ODP Chain-Ladder Model

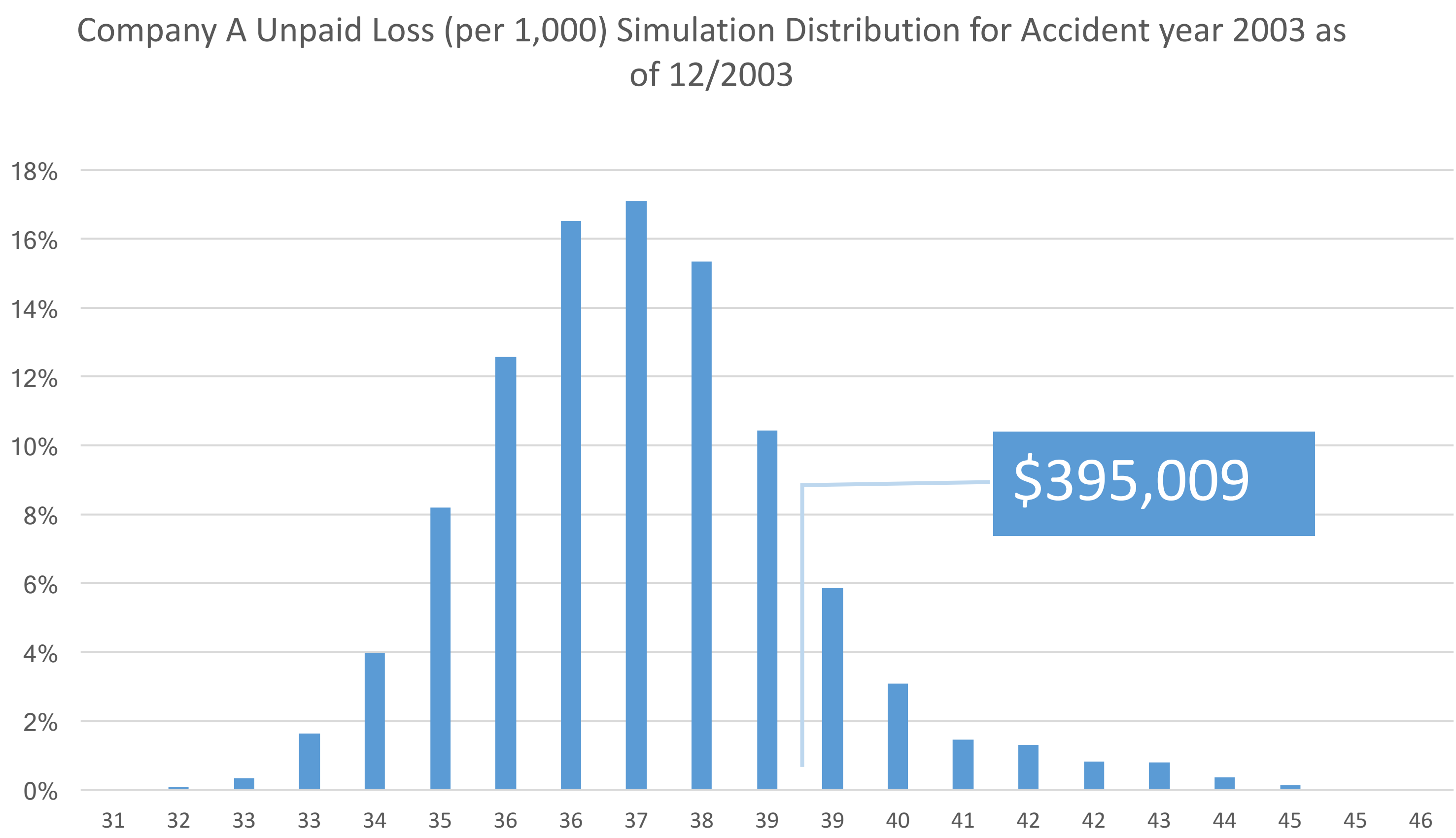
Percentile for a company for AI 2003 as of December 2012:

i. Create a distribution of Ultimate Loss (UL) by using Bootstrap method as of 12/2003

ii. Isolate the distribution of UL for the single year 2003

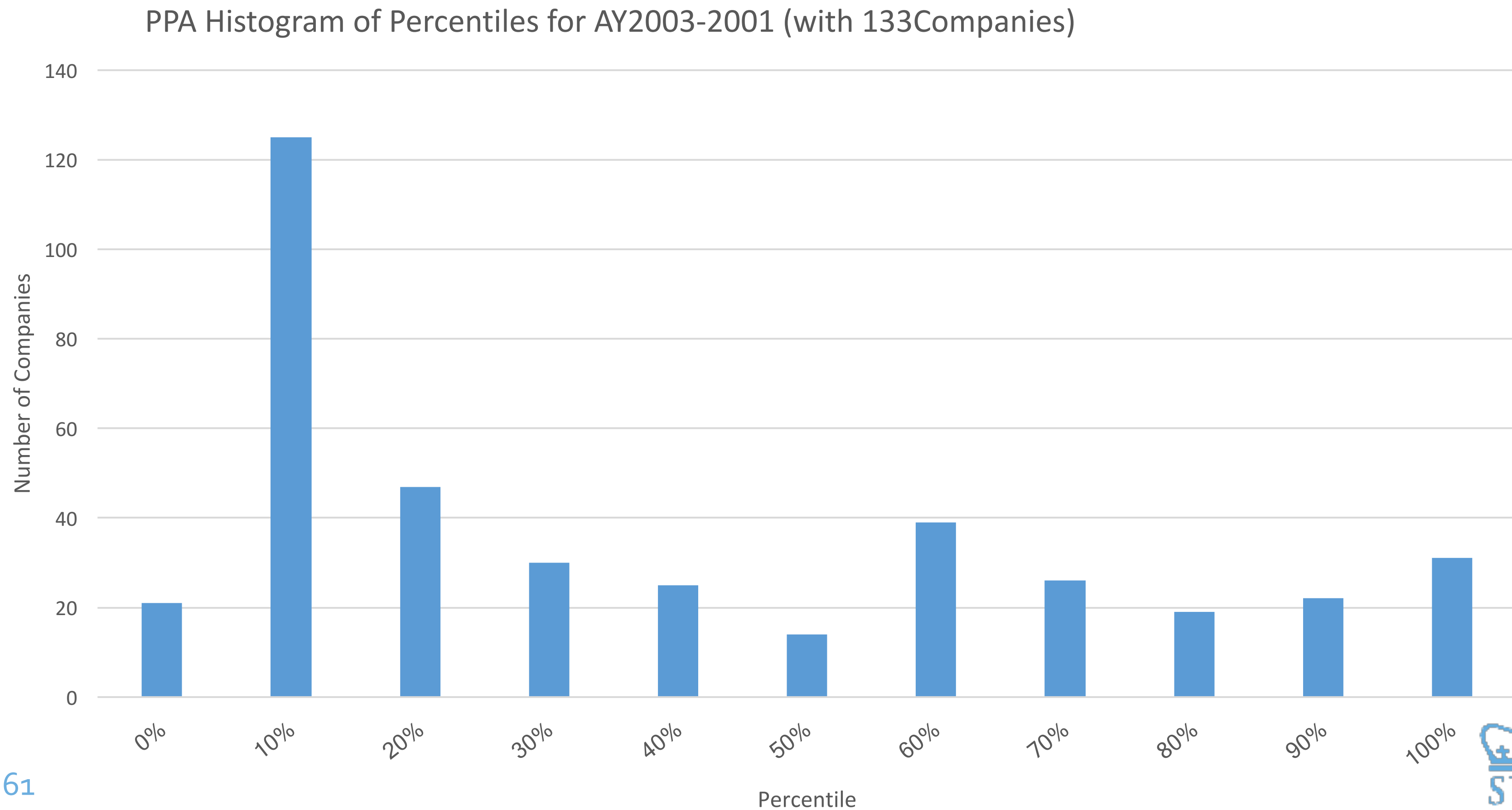
iii. Percentile of the actual unpaid in the distribution in ii. above.

Backtest ODP Chain-Ladder Model



This is 88.63% percentile in the simulated Chain Ladder model.

Percentile Distribution of Bootstrap Model for 133 companies from AY2003 to AY2001



Backtest ODP Chain-Ladder Model

Why the distribution is not uniform?

Wang et al and many papers analyzed the results;
concluded that ODP Chain-ladder method didn't catch systematic risk;
Wang transform adjustment can be used for this purpose.

Wang Transform adjustment

Wang et al showed that the chain-ladder reserving method has systemic error and moreover the systemic error *are highly correlated with the reserve cycle*.

The contemporary correlation between the estimation error and the reserve development is .64 for the chain-ladder method. More noticeably the one-year lag correlation is 0.91. The estimation error leads to the loss reserve development by one year.

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Wang Transform adjustment

Wang transform adjustment method tried to catch the systemic over course of reserve cycle.

Wang Transform method will first adjust the variability of the loss reserve and then give the distribution a shift.

WANG TRANSFORM ADJUSTMENT - PROCEDURES

1. Widen the reserve distribution. Apply the ratio of double exponential over normal to after bootstrap chain-ladder loss triangle:

$$x^{*} = (x - \mu) \times \text{Ratio}(q) + \mu \qquad \text{Ratio}(q) = \text{Exponential}^{-1}(q) / \phi^{-1}(q)$$

2. Calculated β the correlation between each company and industry
3. Wang transform is applied to adjust the mean of the reserve distribution:

$$F_2(x) = \phi\left[\phi^{-1}\left(F_1(x)\right) + \beta * \lambda\right]$$

Note: $F_1(x)$ is reported reserve's percentile in the reserve distribution after the above adjustment; and λ is changed so that back-testing results in the most uniformly distributed percentiles as measured by a chi-square test

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Wang Transform Adjustment

1. Widen the reserve distribution.

$\Phi \approx N(0,1)$;

Exponential is a double exponential distribution with pdf

$$f(x) = 0.5\lambda e^{-\lambda|x|}, \quad -\infty < x < \infty$$

q is the quantile of each simulated reserve;

μ is the median of 10,000 simulated reserves;

x is the simulated reserve;

x* is the reserve after adjustment.

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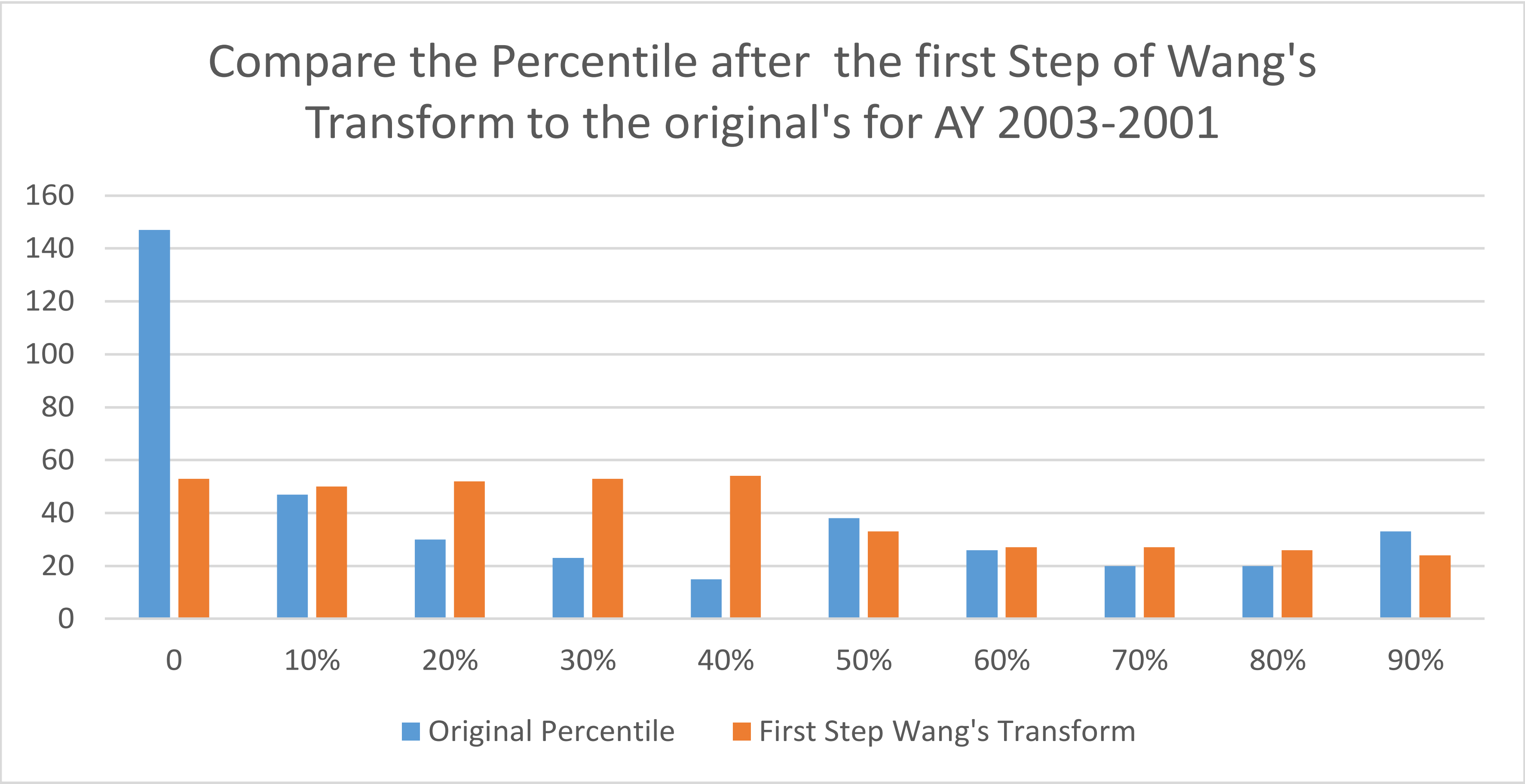
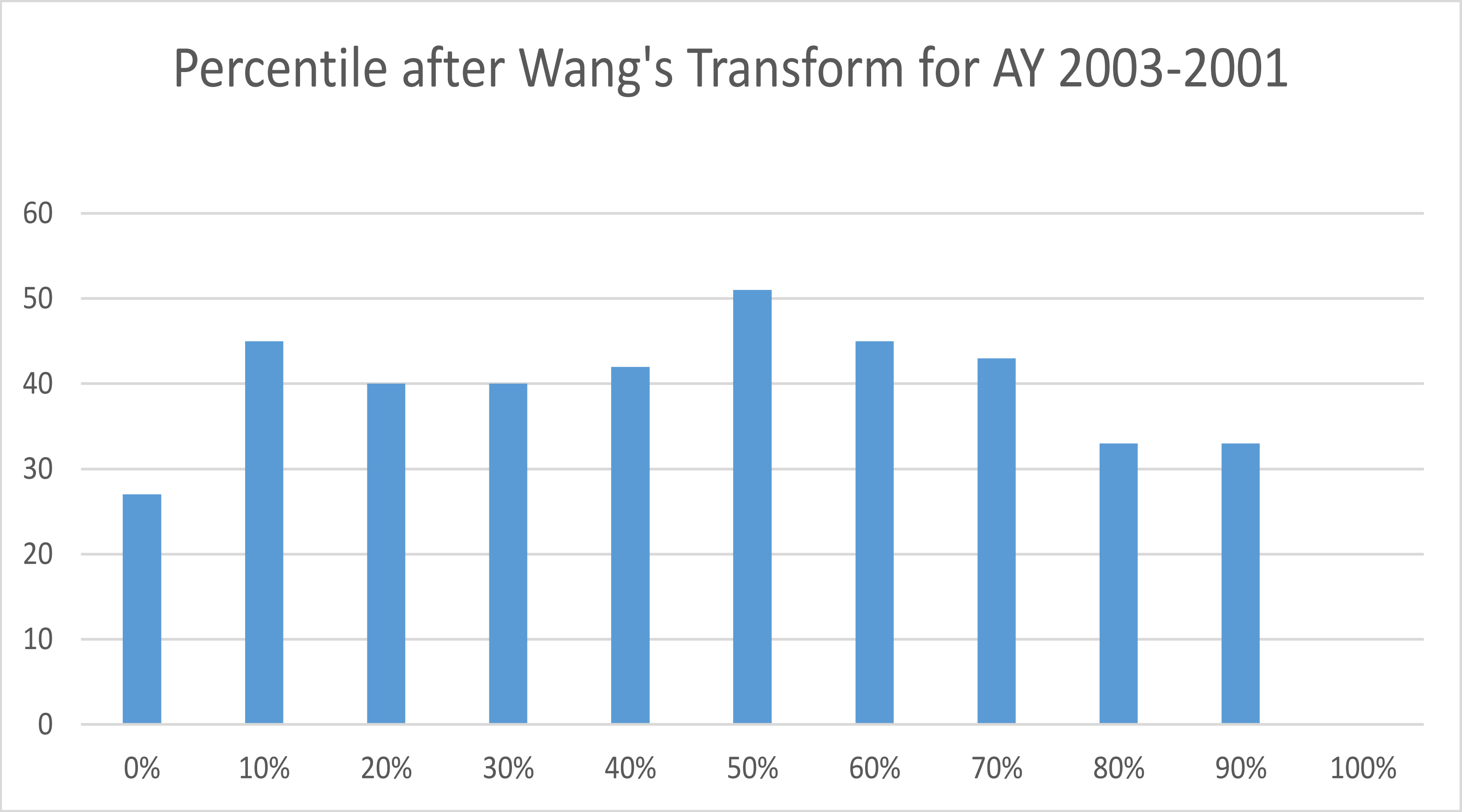
Backtest ODP Chain-Ladder Model

Percentile Distribution of Bootstrap Model for 133 companies from AY2003 to AY2001



Wang Transform Adjustment



Backtest ODP Chain-Ladder Model-Steps

(Data - Cumulative or Incremental Loss by AY and DY):

1. Estimate the (upper) triangle of sampled incremental loss (EIL): $C = \hat{c} + r_p \cdot \sqrt{\hat{c}}$
2. Project the future IL (lower Triangle)
3. Simulating each future IL ≈ Gamma (EIL, EIL×ϕ)
4. Calculate Ultimate Loss (UL)
5. Obtain UL distribution by repeat 4-8 (for example, 10,000)
6. Backtest uniformity of distribution
7. Adjust the reserve distribution by Wang Transform

End