Drivers of Mortality Dynamics: Identifying Age/Period/Cohort Components of Historical U.S Mortality Improvements

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About the Project

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Project Team and Oversight Group

- Project title: "Components of Historical Mortality Improvement"
- Project team: Johnny Li (Waterloo), Rui Zhou (Manitoba) and Yanxin Liu (Nebraska-Lincoln)
- Project oversight group: Jennifer Haid (Chair), Jean-Marc Fix, Zach Granovetter, George Graziani, Alla Kleyner, Dale Hall, Bob Howard, Al Klein, Andy Peterson, Larry Pinzur, Erika Schulty, and Larry Stern

Objective

- to compare and contrast methodologies for allocating historical gender-specific mortality improvement/deterioration experience in the United States into four separate components: age, period, year-of-birth cohort, and residual

Literature Review

- *"Literature Review and Assessment of Mortality Improvement Rates in the U.S. Population: Past Experience and Future Long-Term Trends"* by Rosner et al. (2013)
- *"A Quantitative Comparison of Stochastic Mortality Models Using Data from England and Wales and The United States"* by Cairns et al. (2009)
- *"Mortality Improvement Rates: Modelling and Parameter Uncertainty"* by Hunt and Villegas (2017)
- CMI Working Papers 38, 39, 74, 77, 97, 98 and 99

Exploratory Data Analyses

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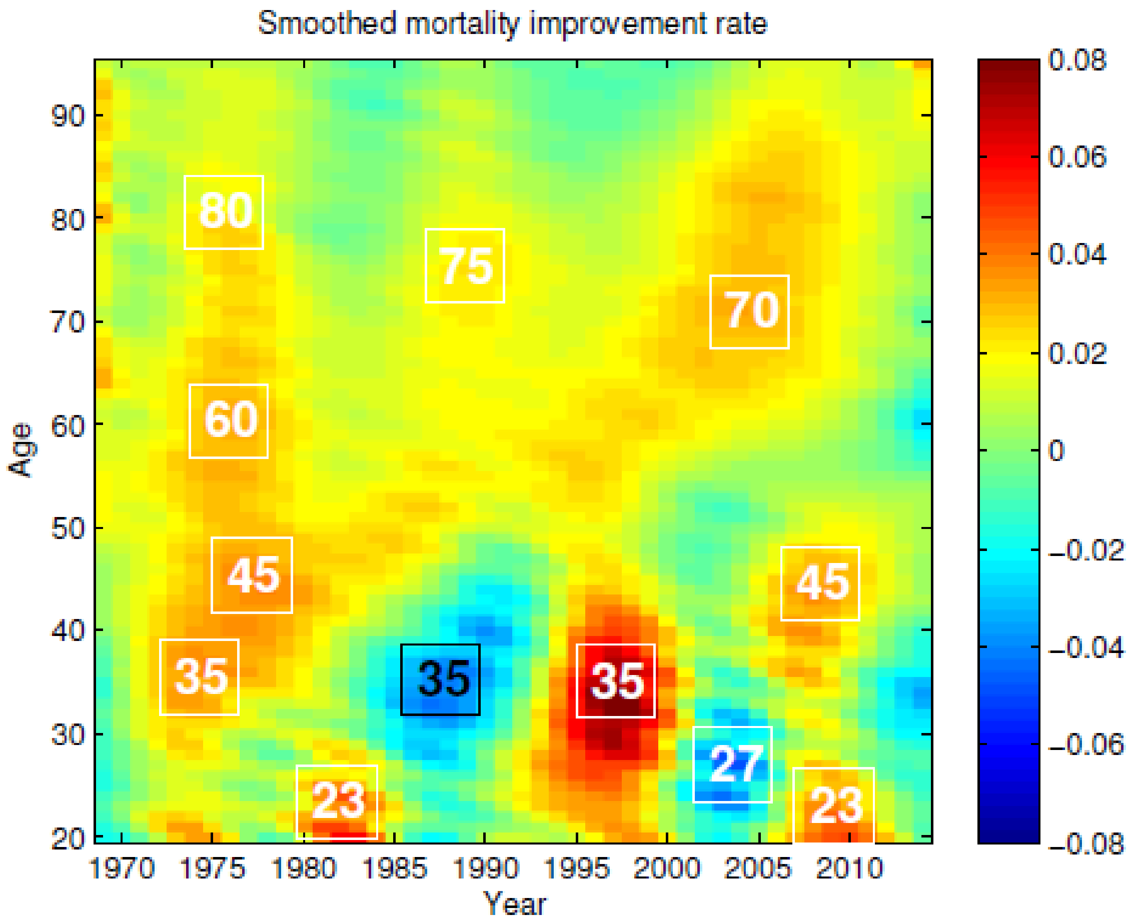
Data Sources

- Human Mortality Database (HMD) and the US Social Security Administration (SSA)
- Age range: 20-95; Sample period: 1968-2014
- The SSA data may be more reliable for the following reasons (Goss et al., 2015):
  - Age accuracy
  - Representation of almost the entire Social security area population
  - Both death and exposure counts are obtained from the same source
- Conclusions drawn are based on the SSA data.

Smoothed Historical Mortality Improvement Rates (US males)



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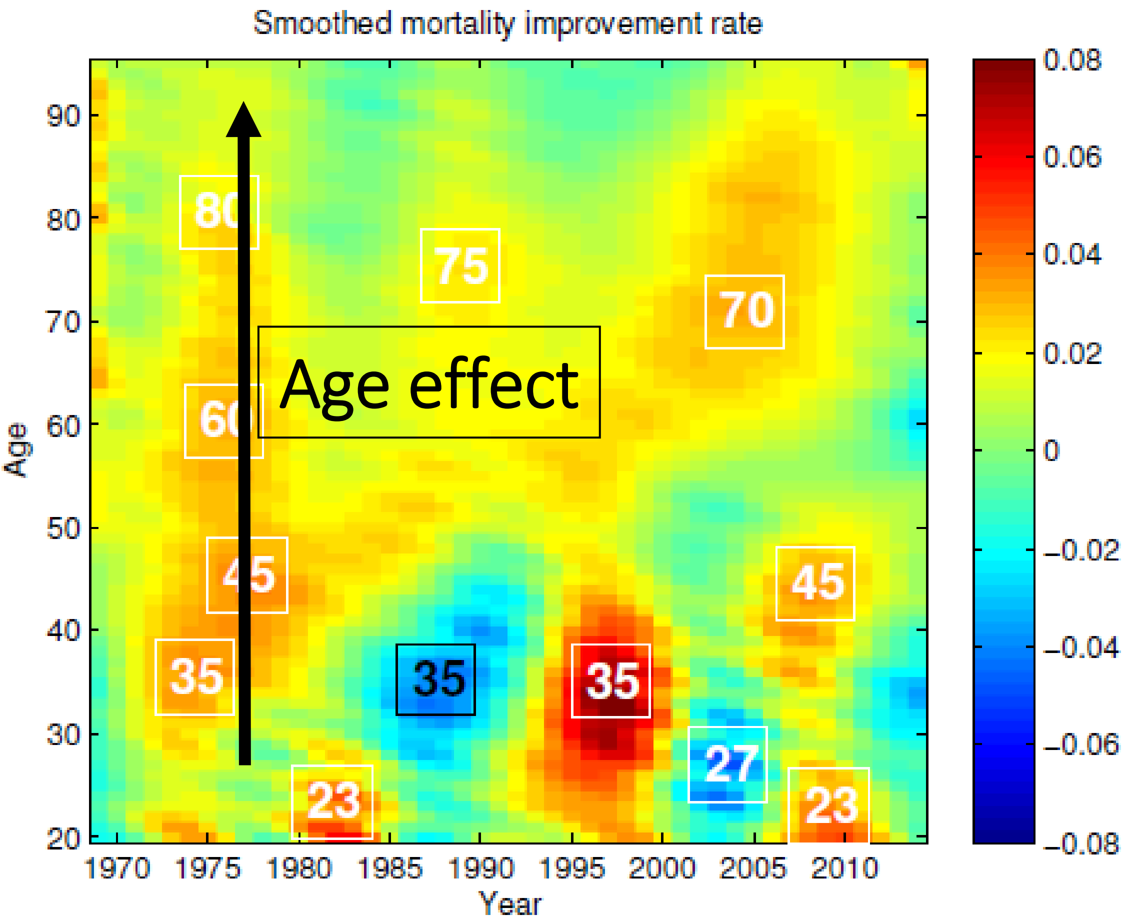

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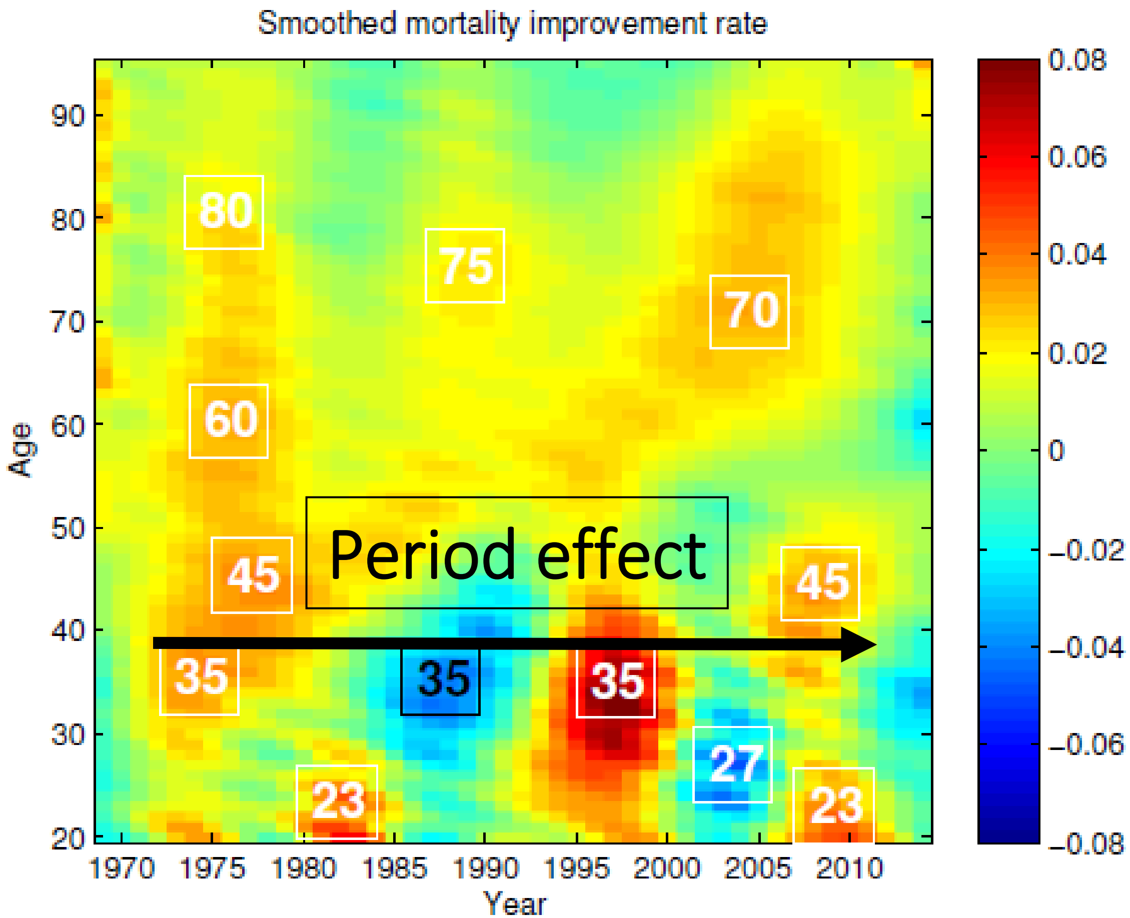

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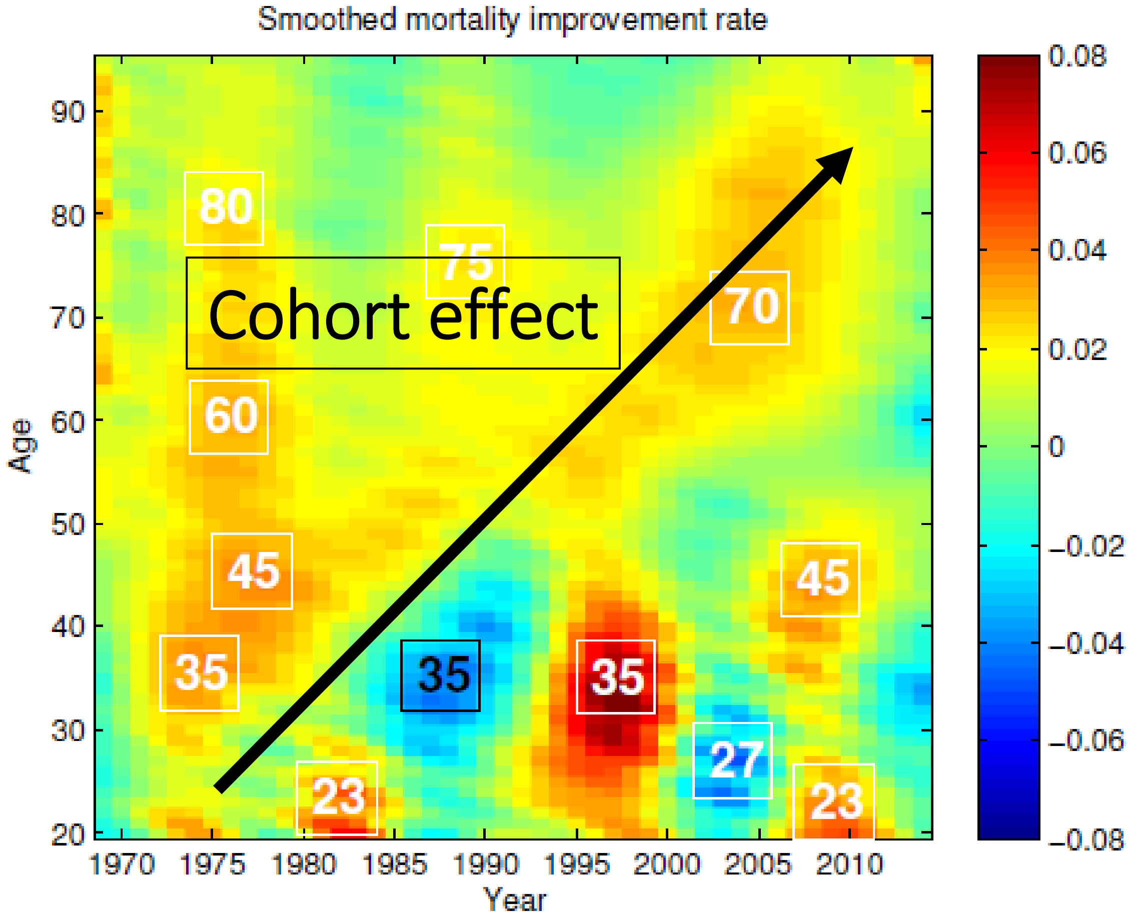
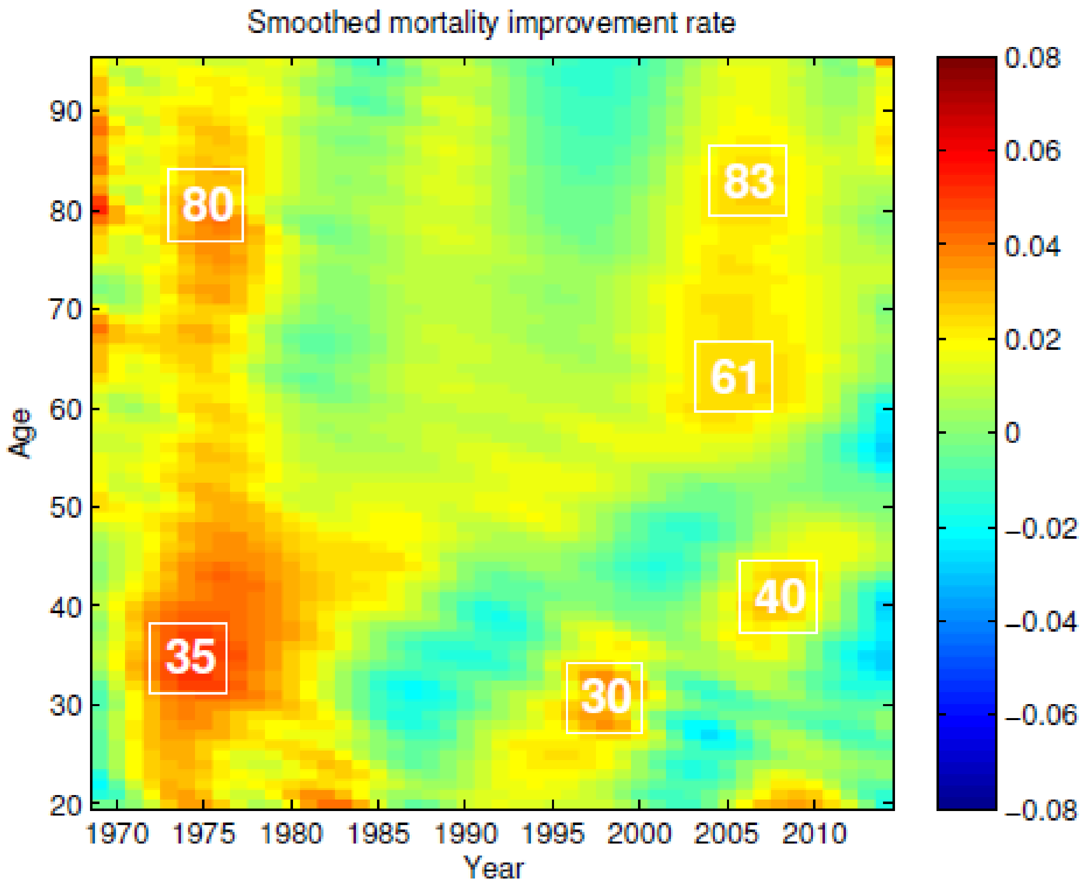


Smoothed Historical Mortality Improvement Rates (US females)



An Overview of the Methodology

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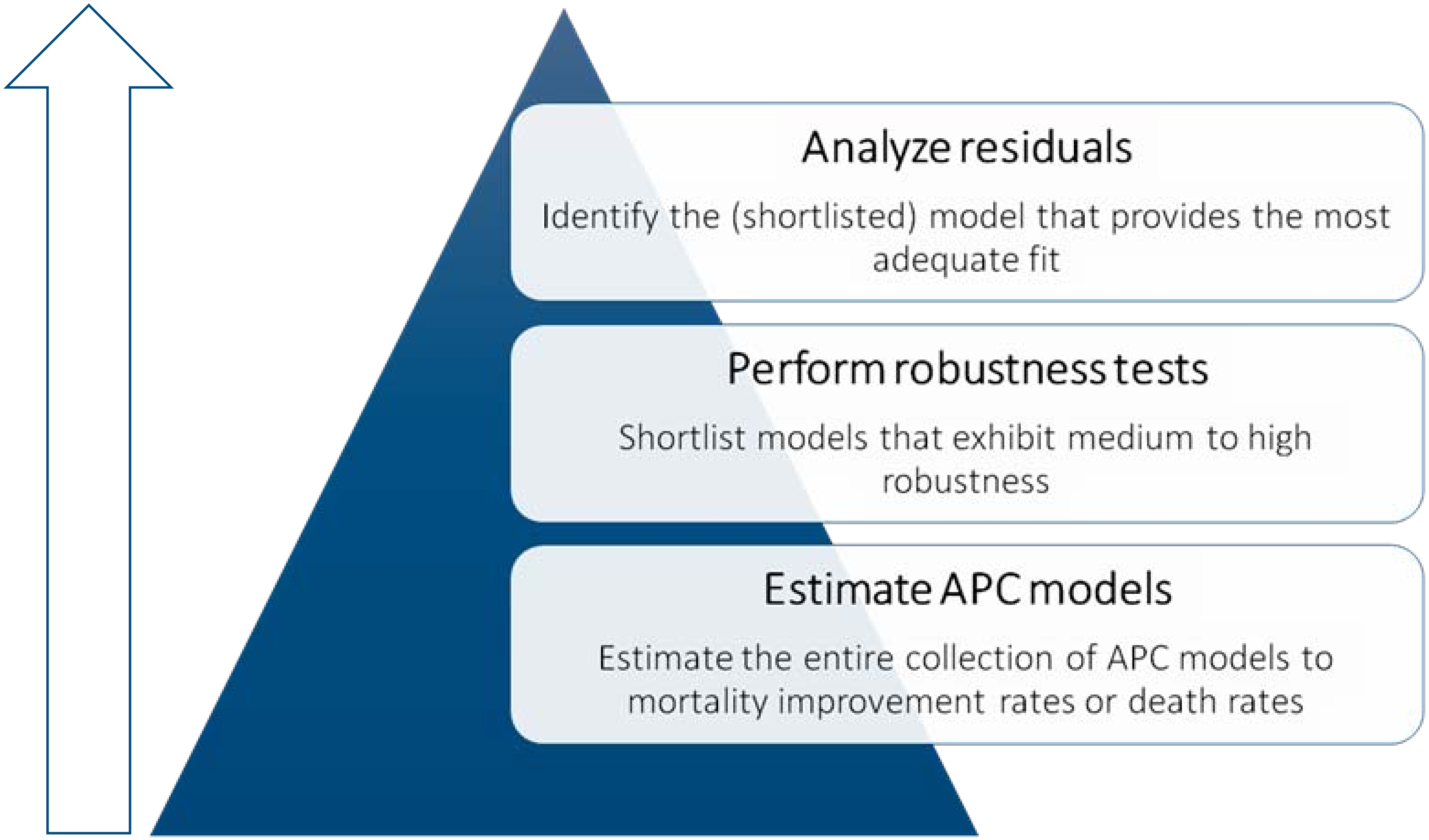
Two Pathways

- Route A
  - An APC model structure is applied directly to historical mortality improvement rates.
  - The estimated model parameters are the age, period and cohort components of historical mortality improvements.
- Route B
  - An APC model structure is applied to historical death rates.
  - The estimated model parameters are then "differentiated" to obtain the age, period and cohort components of historical mortality improvements.

Model Selection



Route A

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Notation

- $Z_{x,t} = 1 - \frac{q_{x,t}}{q_{x,t-1}}$: mortality improvement rate
- $c = t - x$: year of birth
- $\beta_x^{(i)}, i = 1,2,3$: age-specific parameters
- $\kappa_t^{(1)}, \kappa_t^{(2)}, \kappa_t^{(3)}$: time-varying parameters
- γ_c : cohort-related parameter
- $e_{x,t}$: residual

The Candidate Models

- M2: The Renshaw-Haberman model

$$Z_{x,t} = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_c + e_{x,t}$$

- M3: The age-period-cohort model

$$Z_{x,t} = \beta_x^{(1)} + \kappa_t^{(2)} + \gamma_c + e_{x,t}$$

- M6: The CBD model with a cohort effect

$$Z_{x,t} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_c + e_{x,t}$$

- M7: The CBD model with quadratic age and cohort effects

$$Z_{x,t} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \kappa_t^{(3)}\left((x - \bar{x}) - \hat{\sigma}_c^2\right) + \gamma_c + e_{x,t}$$

The Candidate Models

- M8: The CBD model with an age-dependent cohort effect

$$Z_{x,t} = \kappa_t^{(1)} + \kappa_t^{(2)}(x - \bar{x}) + \gamma_c(x_c - x) + e_{x,t}$$

- The full Plat model

$$Z_{x,t} = \beta_x^{(1)} + \kappa_t^{(1)} + \kappa_t^{(2)}(\bar{x} - x) + \kappa_t^{(3)}(\bar{x} - x)_+ + \gamma_c + e_{x,t}$$

- The simplified Plat model

$$Z_{x,t} = \beta_x^{(1)} + \kappa_t^{(1)} + \kappa_t^{(2)}(\bar{x} - x) + \gamma_c + e_{x,t}$$

Robustness Tests

- We examine the robustness of the candidate models with respect to:
  1. Changes in the tolerance value used in optimizing model parameters
  2. Changes in the calibration window
  3. Changes in the age range used
  4. The choice of parameter constraints
  5. Exclusion of the oldest and newest cohorts

Quantifying Robustness

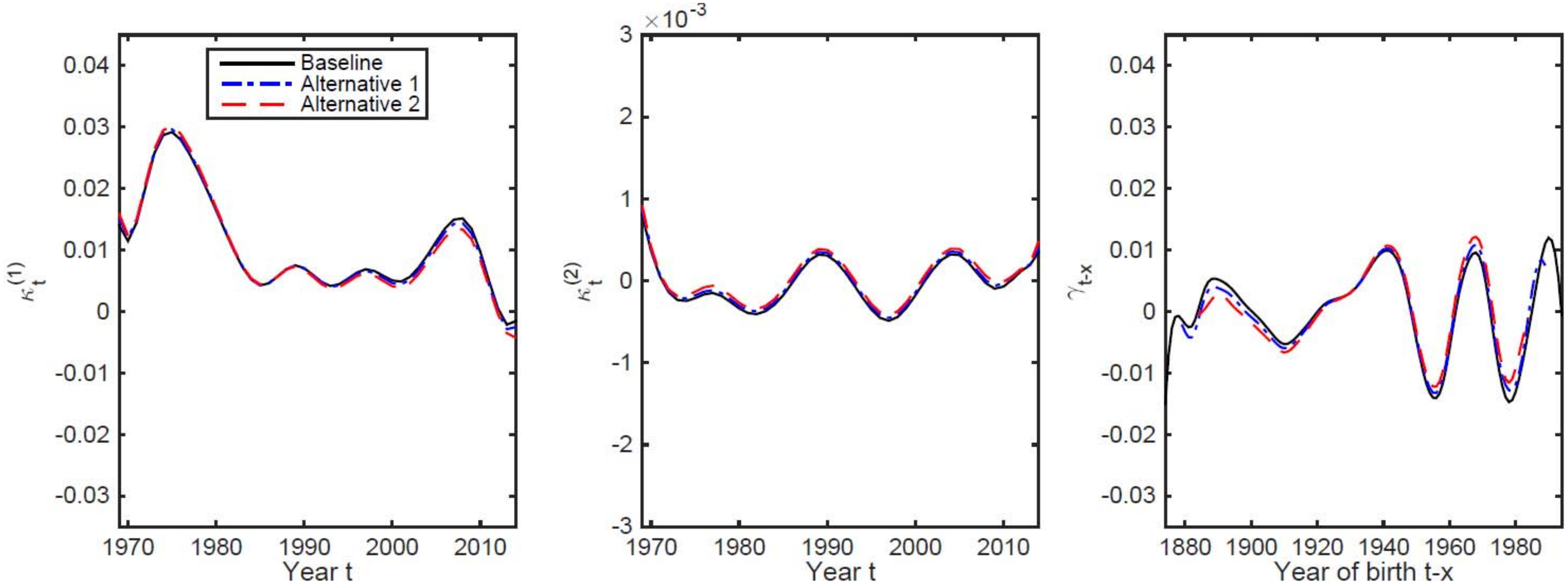
- We define the following measure of robustness:

$$\text{robustness} = \frac{\max_i(\text{maximum absolute change in the } i^{\text{th}} \text{ model term})}{\max_{x,t}(Z_{x,t}) - \min_{x,t}(Z_{x,t})}$$

  - "the i^{th} model term" refers to the i^{th} term on the right-hand-side of the equation specifying $Z_{x,t}$.
  - The denominator $\max(Z_{x,t}) - \min(Z_{x,t})$ "standardizes" the robustness measure by considering the variability of the data being fed into the model.

- Classification:
  - High robustness: $0 \le$ robustness measure $\le 10\%$
  - Medium robustness: $10\% <$ robustness measure $\le 20\%$
  - Low robustness: robustness measure $> 20\%$

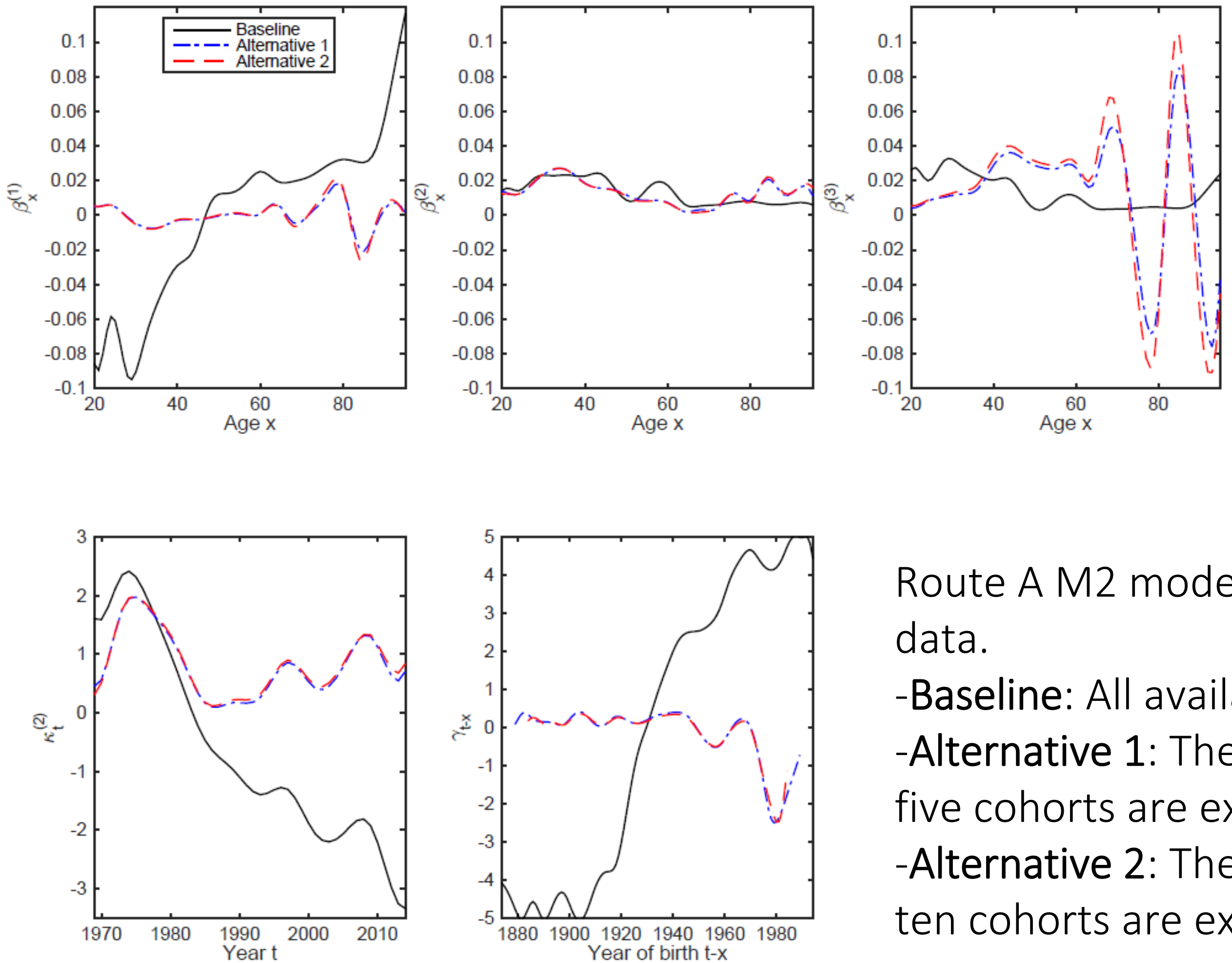
An Example of High Robustness



Route A M6 model fitted to SSA female data.

- **Baseline**: All available data are used.
- **Alternative 1**: The oldest and youngest five cohorts are excluded.
- **Alternative 2**: The oldest and youngest ten cohorts are excluded.

An Example of Low Robustness



Route A M2 model fitted to SSA female data.

-**Baseline**: All available data are used.

-**Alternative 1**: The oldest and youngest five cohorts are excluded.

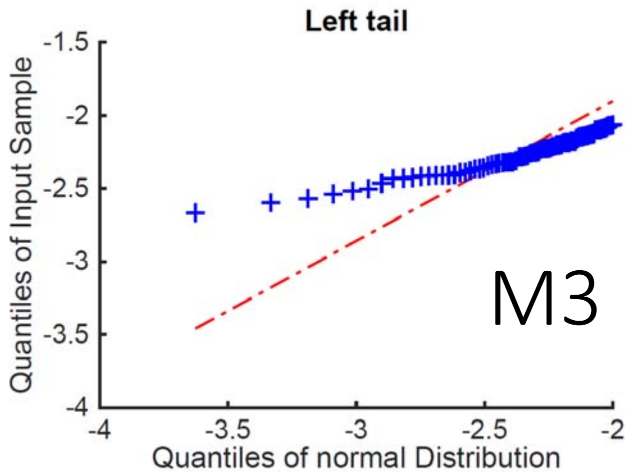
-**Alternative 2**: The oldest and youngest ten cohorts are excluded.

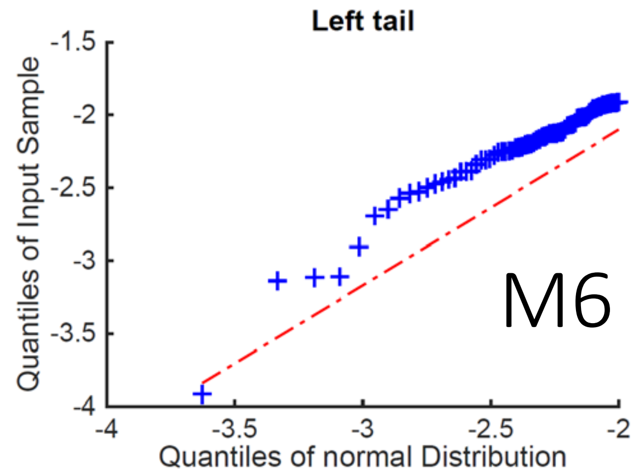
Shortlisting

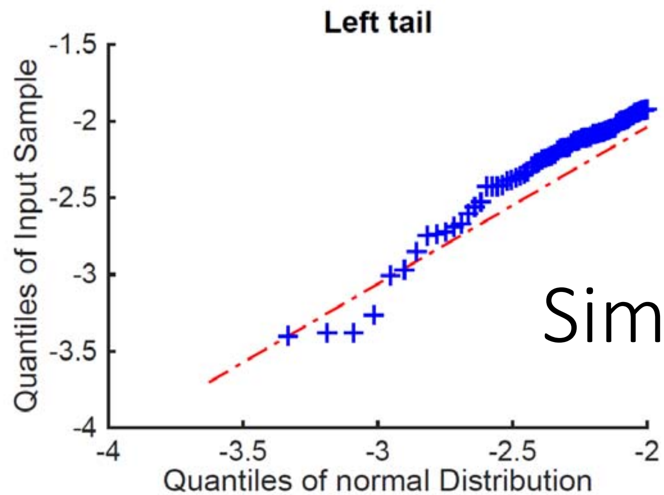
Robustness Test	M2	M3	M6	M7	M8	Full Plat	Simplified Plat
Tolerance value	Medium (M) Low (F)	High	High	High	High (M) Medium (F)	High (M) Medium (F)	High
Calibration window	Low	Medium (M) High (F)	High	Low (M) Medium (F)	High (M) Low (F)	Low	Medium (M) High (F)
Age range	Low	Medium	Medium	Medium	Medium (M) Low (F)	Low	Medium
Parameter constraints	Low	High	High	Medium (M) High (F)	High (M) Medium (F)	High (M) Medium (F)	High
Exclusion of cohorts	Low	High	High	Medium (M) High (F)	High (M) Medium (F)	High (M) Medium (F)	High

M3, **M6** and **the simplified Plat model** consistently show medium to high levels of robustness. They are shortlisted for further consideration

Analyzing Standardized Residuals





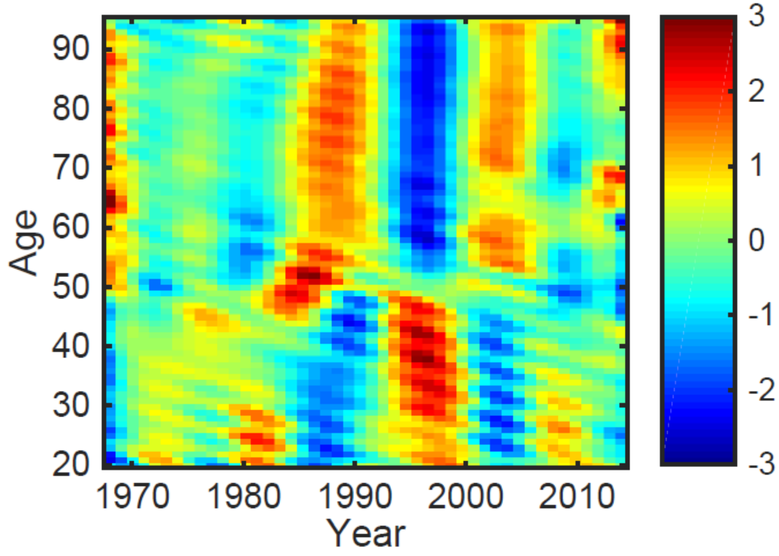


Simplified Plat

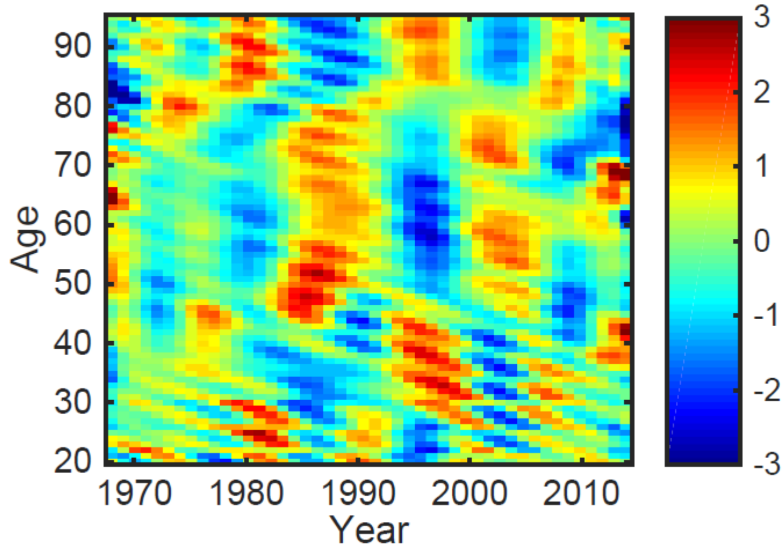
Data: US male (SSA)

Analyzing Standardized Residuals

M3



Simplified Plat



Data: US male (SSA)

Conclusion for Route A

- The simplified Plat model is chosen for Route A:
  - It performs the best in the visual inspection of the standardized residuals (q-q plots and heat maps).
  - It passes the Anderson-Darling normality test at a 5% level of significance (with the lowest p-value among the three shortlisted models).
  - Its standardized residuals are the most in line with the standard normal distribution in terms of the first four moments.

Route B

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The Candidate Models

- M2: The Renshaw-Haberman model

$$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)}\kappa_t^{(2)} + \beta_x^{(3)}\gamma_c$$

- M3: The age-period-cohort model

$$\ln(m_{x,t}) = \beta_x^{(1)} + \kappa_t^{(2)} + \gamma_c$$

- M6: The CBD model with a cohort effect

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \gamma_c$$

- M7: The CBD model with quadratic age and cohort effects

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \kappa_t^{(3)}\left((x-\bar{x}) - \hat{\sigma}_c^2\right) + \gamma_c$$

The Candidate Models

- M8: The CBD model with an age-dependent cohort effect

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_t^{(1)} + \kappa_t^{(2)}(x-\bar{x}) + \gamma_c(x_c - x)$$

- The full Plat model

$$\ln(m_{x,t}) = \beta_x^{(1)} + \kappa_t^{(1)} + \kappa_t^{(2)}(\bar{x}-x) + \kappa_t^{(3)}(\bar{x}-x)_+ + \gamma_c$$

- The simplified Plat model

$$\ln(m_{x,t}) = \beta_x^{(1)} + \kappa_t^{(1)} + \kappa_t^{(2)}(\bar{x}-x) + \gamma_c$$

- The APCI Model

$$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)}(t-\bar{t}) + \kappa_t^{(1)} + \gamma_c$$

The Candidate Models

The CMI-17 method
(CMI Working Papers 97, 98 and 99)

- The APCI Model

$$\ln(m_{x,t}) = \beta_x^{(1)} + \beta_x^{(2)}(t - \bar{t}) + \kappa_t^{(1)} + \gamma_c$$

The Implied Components of Historical Mortality Improvements

- To obtain an A/P/C decomposition of mortality improvement experience, some of the model parameters are "differentiated".
- Notation:
  - $MI_{x,t} = \begin{cases} \ln\left(\frac{q_{x,t-1}}{1-q_{x,t-1}}\right) - \ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right), & \text{Model M6, M7} \\ \ln(m_{x,t-1}) - \ln(m_{x,t}), & \text{All other models} \end{cases}$
  - $K_t^{(i)} = \kappa_{t-1}^{(i)} - \kappa_t^{(i)}, i = 1,2,3$
  - $G_c = \gamma_{c-1} - \gamma_c$
  - $B_x^{(K)} = \beta_x^{(2)}, B_x^{(G)} = \beta_x^{(3)}, B_x^{(\cdot)} = -\beta_x^{(2)}$

The Implied Components of Historical Mortality Improvements

- M2: $MI_{x,t} = B_x^{(K)} K_t^{(2)} + B_x^{(G)} G_c$
- M3: $MI_{x,t} = K_t^{(1)} + G_c$
- M6: $MI_{x,t} = K_t^{(1)} + K_t^{(2)}(x - \bar{x}) + G_c$
- M7: $MI_{x,t} = K_t^{(1)} + K_t^{(2)}(x - \bar{x}) + K_t^{(3)}((x - \bar{x})^2 - \hat{\sigma}_x^2) + G_c$
- M8: $MI_{x,t} = K_t^{(1)} + K_t^{(2)}(x - \bar{x}) + G_c(x_c - x)$
- The full Plat: $MI_{x,t} = K_t^{(1)} + K_t^{(2)}(\bar{x} - x) + K_t^{(3)}(\bar{x} - x)_+ + G_c$
- The simplified Plat: $MI_{x,t} = K_t^{(1)} + K_t^{(2)}(\bar{x} - x) + G_c$
- APCI: $MI_{x,t} = B_x^{(\cdot)} + K_t^{(1)} + G_c$

Shortlisting

Robustness Test	M2	M3	M6	M7	M8	Full Plat	Simplified Plat	APCI
Tolerance value	Low	High	High	High	High	High	High	High
Calibration window	High	High	High (M) Medium (F)	High	High	High (M) Medium (F)	High	High
Age range	Low	High	Medium	High (M) Medium (F)	Low	High (M) Medium (F)	High	High
Parameter constraints	Medium (M) Low (F)	High	High	High	High (M) Medium (F)	High (M) Medium (F)	High	High
Exclusion of cohorts	Low	High	High	High	High (M) Medium (F)	High (M) Medium (F)	High	High

M3, M6 , M7, the full Plat model, the simplified Plat model, and **the APCI model** consistently show medium to high levels of robustness. They are shortlisted for further consideration

Conclusion for Route B

- Analyses of standardized residuals:
  - M6 and M7 perform the worst in the residual analysis. Large horizontal clusters are found in the heat maps produced from these models.
  - Residuals clustering is significant in M3 and the APCI model for males. Large vertical clusters are observed between mid-80s and mid-90s.
  - The full Plat model performs the best in the residual analysis.
  - The simplified Plat model is the close second best.
- Chosen model for Route B: <u>The simplified Plat model</u>

Summary and Conclusion

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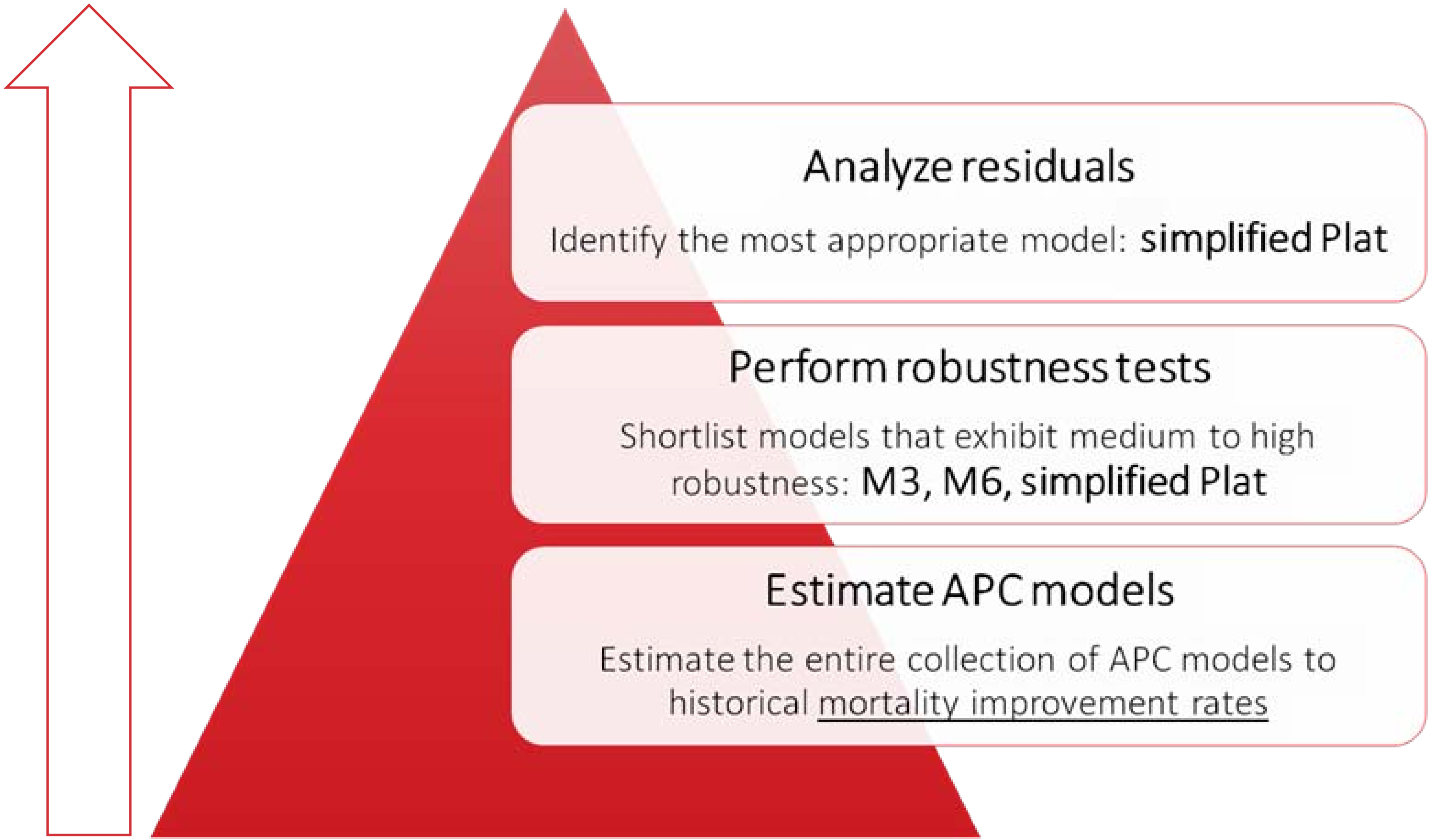
Model Selection for Route A



Model Selection for Route B

Analyze residuals

Identify the most appropriate model: simplified Plat

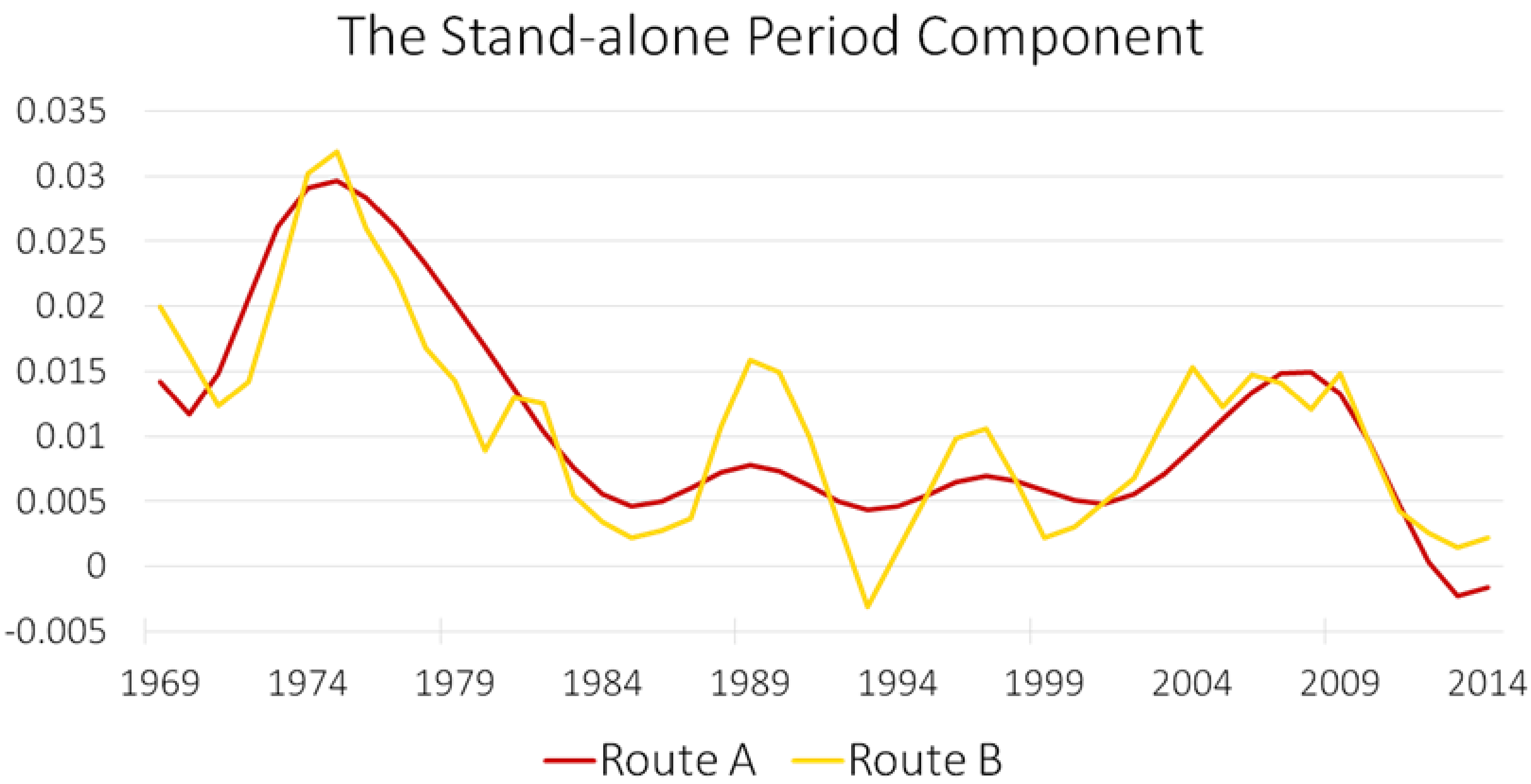
Perform robustness tests

Shortlist models that exhibit medium to high robustness: M3, M6, M7, full/simplified Plat, APCI

Estimate APC models

Estimate the entire collection of APC models to historical death rates

Route A vs. Route B



Recommendation

- We recommend Route A simplified Plat model for decomposing historical US mortality improvements.
- Caveats:
  - When focusing only on pensionable ages (say 60 and above), the advantage of the chosen model over other (simpler) models becomes less apparent.
  - The conclusion drawn pertains only to the data set (US gender-specific) considered in this study.

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