



## Introduction

The goal of this work is to investigate the importance of internal tide-mesoscale eddy interactions in the ocean.

## Motivation

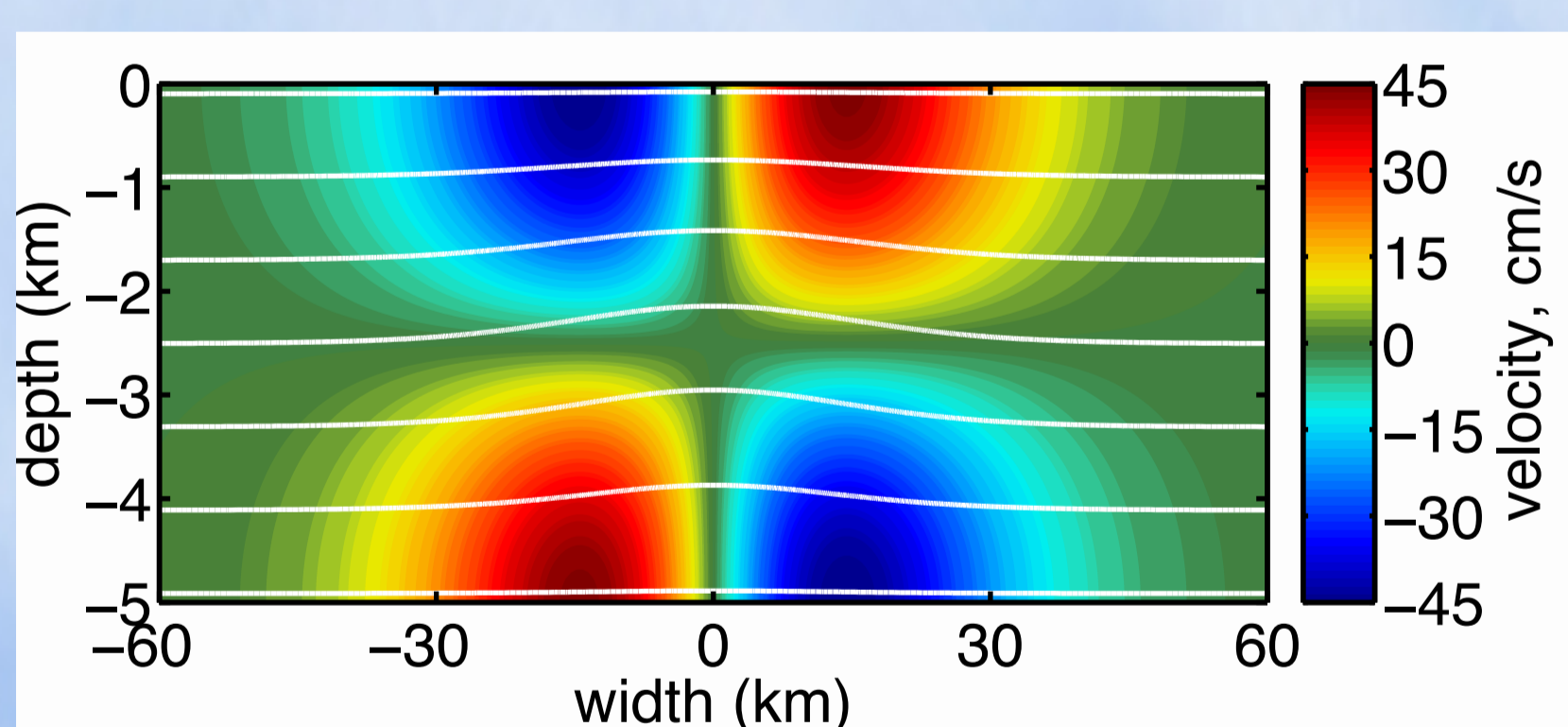
Lelong and Riley (1991) showed that resonant triads exist between two periodic internal wave modes and one periodic vortex mode. This leads us to ask the question: Are these wave-vortex interactions important in the ocean? Do they enhance the internal wave energy cascade, and if so, is the enhancement significant? We design idealised numerical experiments wherein wave-eddy interactions take place to address these questions.

## Wave and eddy selection

Strong generation sites in the ocean continuously produce the internal tide with a large mode-one content and we choose mode-one as the forced wave. We choose an isolated eddy model for the vortex, and have conducted experiments with both barotropic and baroclinic eddies. The baroclinic results are the most interesting. The baroclinic isolated eddy model is chosen to be mode-one with a streamfunction of

$$\Psi = \psi(r)\phi(z) = \frac{-5^{\frac{5}{2}}}{64} U L \operatorname{sech}^4\left(\frac{3r}{L}\right) \cos\left(\frac{\pi z}{5000}\right)$$

A cross section of eddy velocity is shown below for  $L=90$  km and  $U=45$  cm/s with white density contours. The eddy diameter is roughly  $L$ .



## The Numerical Model

We use the numerical model MITgcm to solve the 3D hydrostatic Boussinesq equations of motion with a rigid lid.

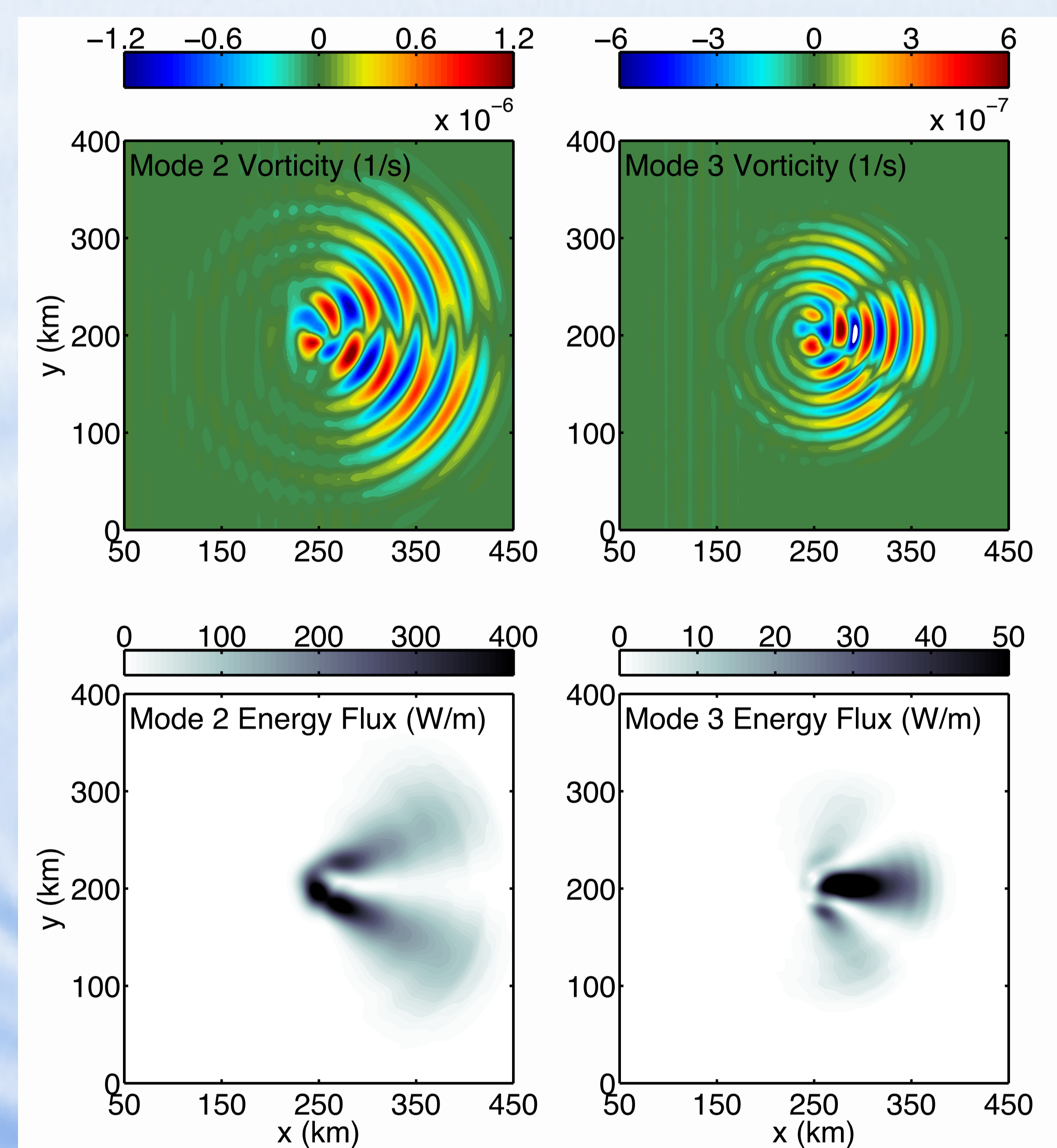
- The domain is a (720x400x5) km box with  $(dx,dy,dz) = (1\text{km},1\text{km},50\text{m})$ . The North-South boundary is periodic, the East closed, and the West forced by a mode-one M2 tide with energy flux 4775 W/m. Constant  $N$  and  $f$ -plane,  $N = 1 \times 10^{-3} \text{s}^{-1}$ ,  $f = 0.5 \times 10^{-4} \text{s}^{-1}$  ( $20^\circ N$ )
- We initialise with an eddy at (250,200) km and adjust for 5 tidal periods. The forcing is then switched on and the mode-one waves propagate eastward, interacting with the eddy.
- A suite of experiments are carried out with four velocities ( $U=15, 30, 45$  and  $60$  cm/s) and nine diameters ( $L=45, 60, \dots, 165$  km).

## Results

The results show the production of coherent internal tides at mode-two and higher which propagate out from the eddy centre. We illustrate with a representative case where  $L=90$  km and  $U=45$  cm/s

- A vertical vorticity snapshot is projected onto normal modes two and three. (below, top row).
- The energy flux is projected onto normal modes two and three, vertically integrated, and averaged over a tidal period (below, bottom row).

The mode-two plots show that two beams of waves are produced and they propagate out of the eddy travelling roughly northeast and southeast, the southeast beam being stronger. At mode three, three beams are produced, and here the due-east beam is the most energetic. In both cases, radial spreading is evident.



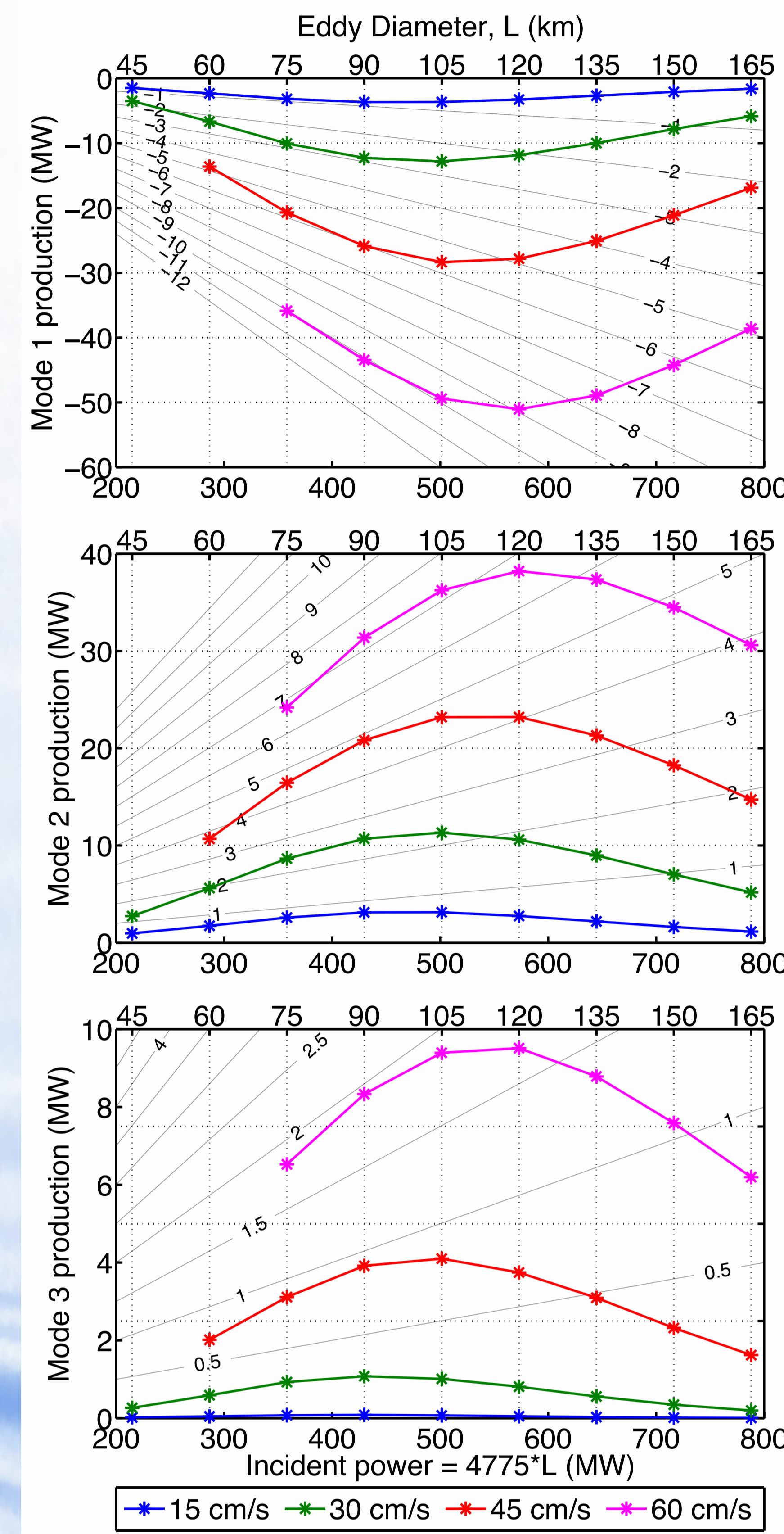
## Resonant Triad Expectation

The isolated eddy does not have a single horizontal wavenumber, rather it has a continuous spectrum of wavenumbers. However we can use the resonant triad equations to find an estimate of the optimal eddy size:

- Frequency condition: The forced wave has the M2 frequency, and the eddy has zero frequency, thus the wave that completes the triad must have M2 frequency as well.
- Vertical wavenumber condition: The forced wave and the eddy are both mode-one, we find the second wave must be mode-two.
- Horizontal wavenumber condition: The dispersion relation governs the horizontal wavenumber of the waves. Mode-one is 75.3 km, mode-two is 37.7 km, and we find the vortex's horizontal wavenumber should be 64.7 km.

## Energy Budget

The model flow fields satisfy the global energy budget to within 1%. To quantify energy flow between modes, a per-mode energy budget is constructed. This constitutes projecting each term in the energy budget onto  $M$  normal modes yielding  $M$  energy budgets. The residuals at each mode represent the net source/sink of energy at each mode.



The plots show a peak in absolute conversion at  $L=120$  km and a peak in incident conversion at  $L=90$  km. These values are a bit larger than the resonant triad expectation for the eddy's horizontal wavelength.

## Conclusions

The results of this investigation show that

- A mode-one eddy can interact with a mode-one internal tide and produce an internal tide at modes two and higher.
  - Of the power lost at mode-one, roughly 75% is found at mode-two, roughly 20% at mode-three and the rest at mode-four and higher.
  - Absolute conversion rates peak for an eddy diameter of 120 km and reach 50 MW.
  - Eddies of diameter 90 km are most efficient at scattering power from mode-one to higher modes, incident power conversion reaches 10%.
  - The mechanism of energy transfer is wave-wave-vortex resonant triad interaction.
- Implications for the energy cascade are enhancement: the presence of a vortex activates more triads than would be available in its absence.
- Future work will involve expanding this work to (realistic) non-linear stratification. A couple outstanding questions are
- Is the scattering more or less intense with a realistic stratification?
  - Can a coherent mode-two signal be observed from satellite altimetry near an eddy?

## References

M. Pascale Lelong and James J. Riley (1991). Internal wave-vortical mode interactions in strongly stratified flows. *Journal of Fluid Mechanics*, 232, pp 1-19

## Conversion rates

At the left we show conversion rates computed from the numerical experiments. The upper axis is eddy diameter and the lower is incident power (incoming energy flux times eddy diameter). The contours show the percentage of incident power converted.

- At mode one (top), conversion is always negative and we see that losses reach 50 megawatts (MW) for the fastest eddy. For a fixed velocity, power conversion from mode one is more efficient in smaller eddies and reaches 10% in a few cases.
- At mode two (middle) we see positive curves similar in shape to those at mode-one. Roughly 75% of the power lost from mode-one is accounted for at mode-two.
- Mode-three (bottom) accounts for roughly 20% of the power lost from mode-one.
- The remainder of the energy is scattered to mode-four and higher (not shown here).