Quantum Signalling with Unruh-DeWitt Detectors

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Signalling at the Quantum Limit

We study a basic model for wireless quantum communication, in the regime where quantum field fluctuations are the limiting noise source.

The quantum noise from the vacuum fluctuations of the field is fundamentally unavoidable. It even contains information about the curvature of spacetime.

The Quantum Channel

We equip Alice and Bob each with a Unruh-DeWitt detector, which couples to a scalar, massless Klein-Gordon field through the standard Hamiltonian:

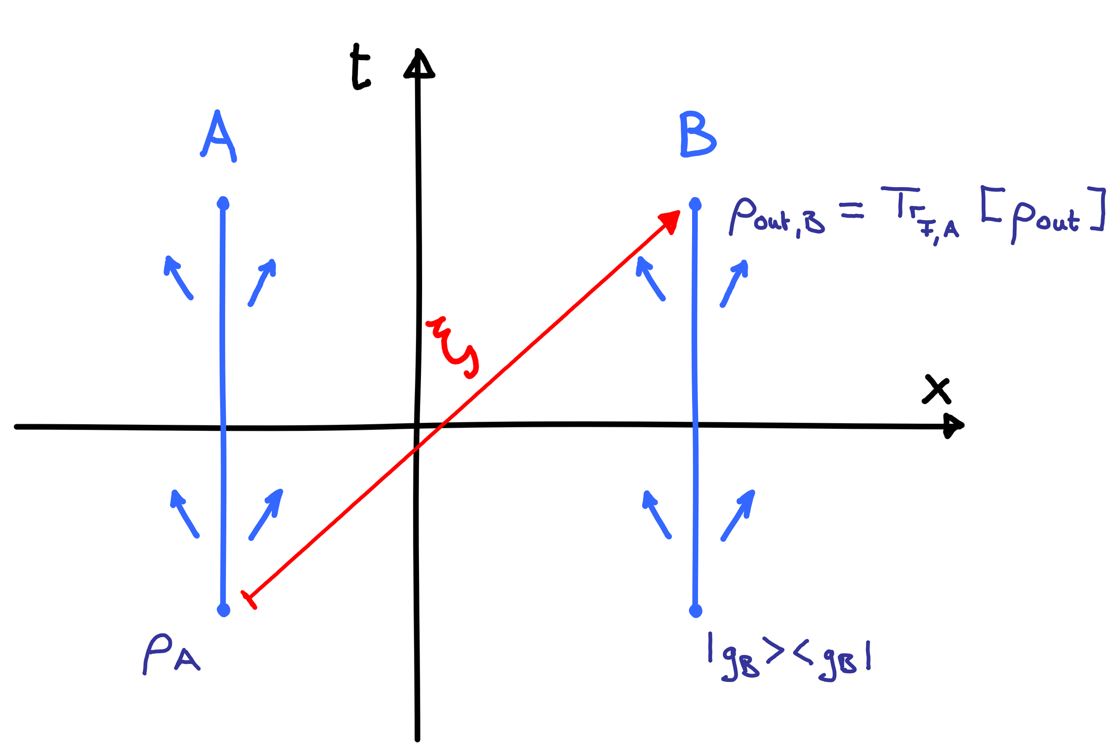
$$H_{int} = \lambda\,\eta(\tau)\left(|e\rangle\langle g|\,e^{i\Omega\tau} + |g\rangle\langle e|\,e^{-i\Omega\tau}\right)\phi(x(\tau))$$

Before the interaction is switched on, Alice encodes her message to Bob by choosing the initial state of her detector $\rho_{A,0}$. The initial total density matrix of the two detectors and the field is hence:

$$\rho_0 = \rho_{A,0} \otimes |g_B\rangle\langle g_B| \otimes |0\rangle\langle 0|$$

Then the interaction is switched on for a finite interval $t \in (0, T)$. The time evolution under the interaction is calculated perturbatively.

$$\rho_T = U(T,0)\,\rho_0\,U(T,0)^\dagger$$



The map from Alice's initial to Bob's final density matrix is a quantum channel ξ. It describes all influence Alice's detector has on Bob's detector.

$$\xi : \rho_{A,0} \mapsto \rho_{B,T} = \mathrm{Tr}_{A,f}[\rho_T] \qquad (1)$$

Bloch Sphere View of the Channel

The action of the channel in the Bloch vector picture is given by

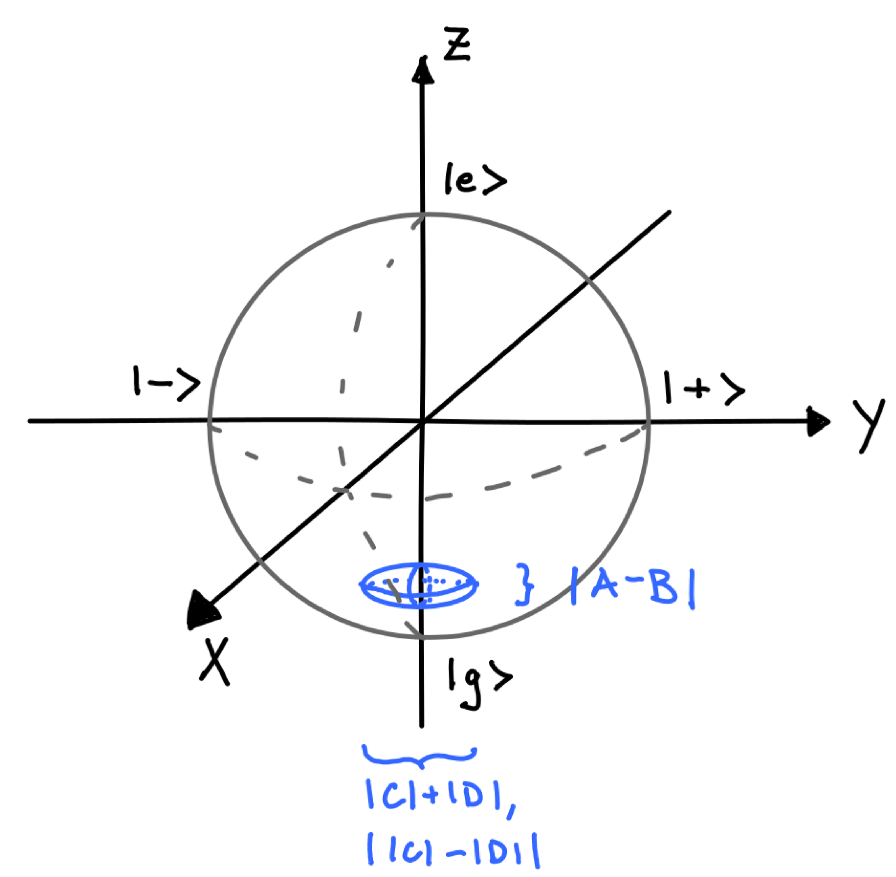
$$\vec{\xi}(\rho_{\vec{A},0}) = UDV\rho_{\vec{A},0} + \vec{t}.$$

D is the diagonal matrix

$$D = \mathrm{diag}\,(|C| + |D|, |C| - |D|, A - B)\,,$$

U and V are rotations about the $\hat{Z}$-axis and

$$\vec{t} = \big(0,\, 0,\, -1 + 2P + A + B\big)\,.$$



$P, A, B \in \mathbb{R}$ and $C, D \in \mathbb{C}$ are integrals obtained from the perturbative expansion of (1).

$$C, D, P \sim \mathcal{O}(\lambda^2) \qquad A, B \sim \mathcal{O}(\lambda^4) \qquad (2)$$

P is the *noise term* and is independent of Alice's presence. A, B, C, D are *signalling terms*, they describe Alice's influence on Bob.

Alice's best Choice of Signal

If Alice starts in the state $|e\rangle$ or $|g\rangle$, then the signals Bob receives are only of order $\mathcal{O}(\lambda^4)$.

However, for *all* other pure input states the signalling terms are of order $\mathcal{O}(\lambda^2)$!

Intuitively, the excited state has to decay first to emit a real photon which takes about a half-life time. Superposition states such as, e.g.,

$$|\pm\rangle = \frac{1}{\sqrt{2}}(|g\rangle \pm |e\rangle)$$

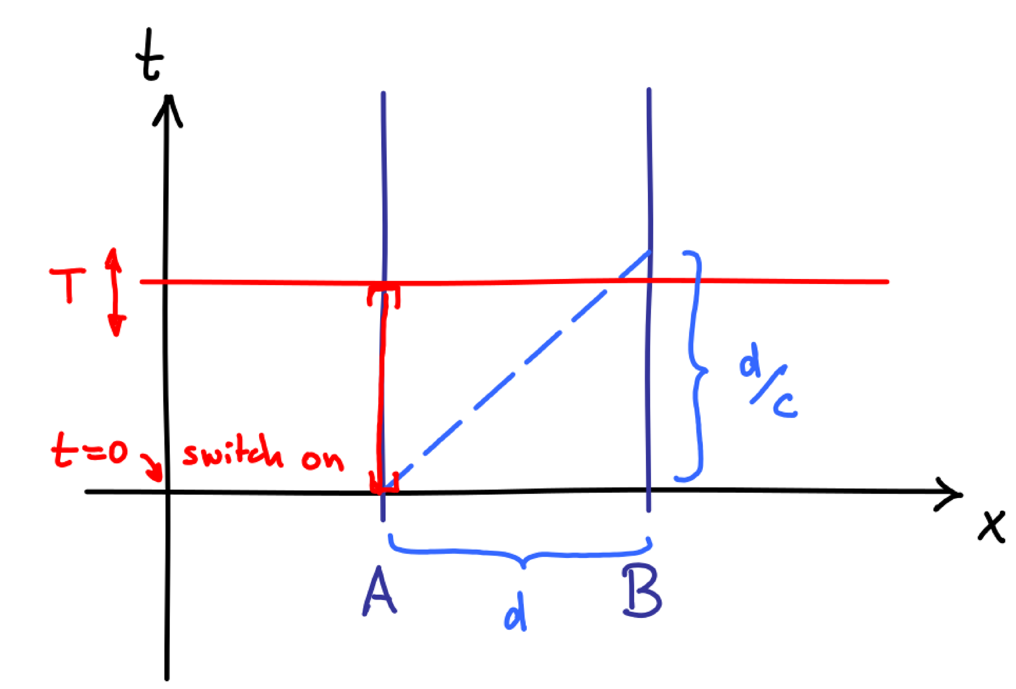
begin to exchange virtual photons with the field as soon as the interaction is switched on.

Mode Truncation in Cavity QED

Effective models often only consider a finite number of field modes, i.e., they introduce a UV cutoff:

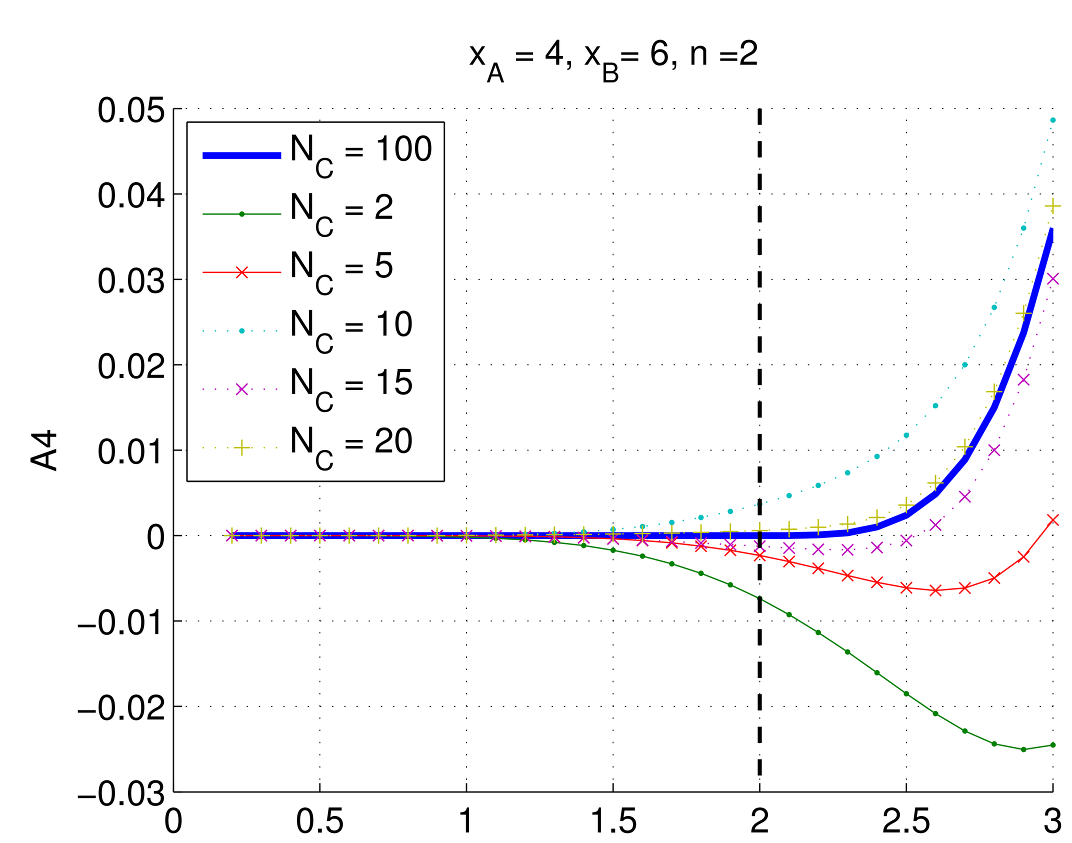
- In the *Galilean* regime, where $T \gg L$, even single-mode models are often suitable.
- In the *relativistic* regime, where $T \lesssim L$, using only a finite number of modes introduces errors in the form of superluminal signalling.

We study these errors in a setting adapted to the *Fermi problem.*



Two detectors, at a distance d, inside a cavity are switched on for $t \in (0, T)$. The signalling terms A, B, C, D in (1) have to vanish outside the light cone, i.e., for $T < d/c$.

If only a low number N_C of field modes are taken into account, the model is inconsistent with causality and the signalling terms do not vanish inside the light cone. With increasing N_C the exact limit is approached and causality is restored.



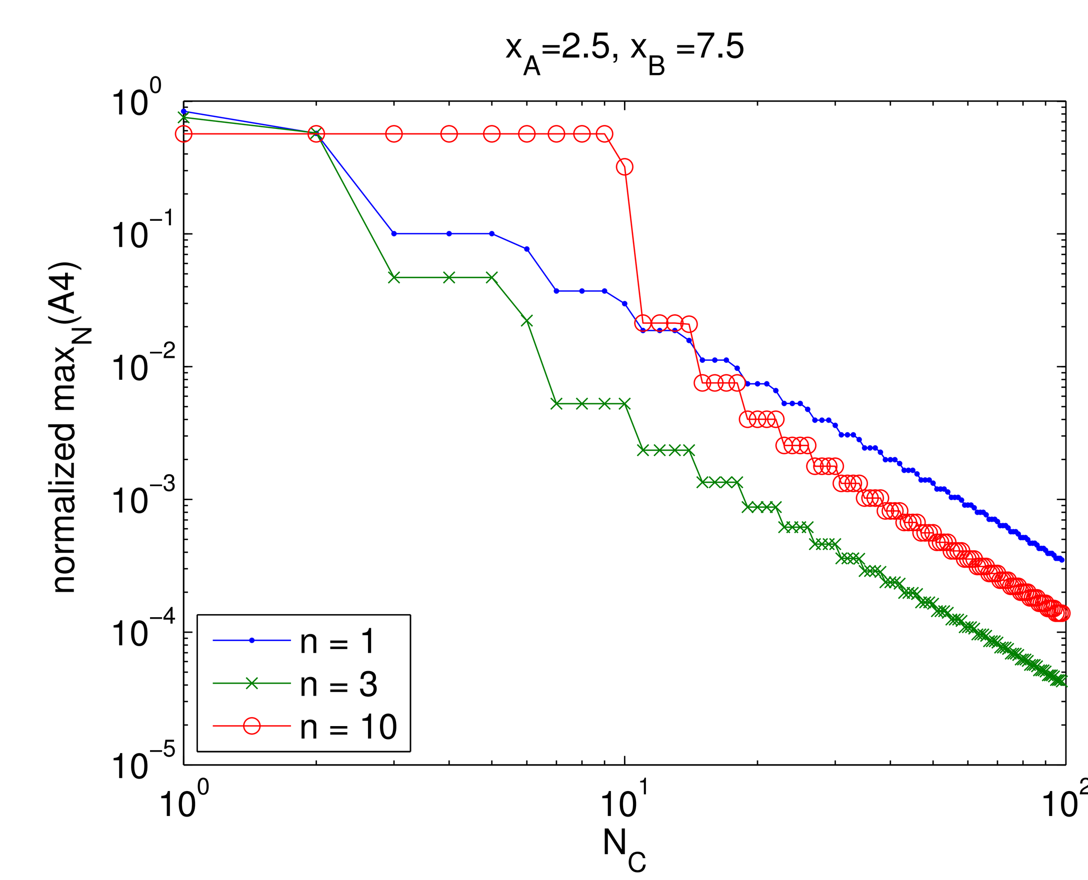
Lowest order contribution to the signalling term A for different cutoffs N_C. A is the probability for Bob to get excited by Alice starting out in her excited state. The dashed line indicates the light cone where $T = d/c$.

Non-causal Error Terms decay with Power Law in UV Cutoff

The error size in the signalling terms outside the light cone decays following a power law.

The power of the decay is universal in the sense that it is not dependent on the energy gap of the detectors, their position and initial state.

Hence, the UV cutoff in finite-mode approximations has to scale polynomially with the accuracy desired for the prediction.



Double logarithmic plot of the lowest order contribution to the signalling term A for $T = d/c$. The different plots are for detectors being resonant with the n-th field mode inside the cavity.

Reference

[1] RHJ, EMM, AK, *Quantum signalling in cavity QED*, arXiv:1306.4275 (2013)

Outlook

We want to investigate the quantum channel in relativistic and curved spacetime scenarios, close to causal and cosmological horizons.

Here the classical channel capacity

$$C \sim -\frac{1}{4}\left(|C| + |D|\right)^2 \log(P_2\lambda^2)\,\lambda^4 + \mathcal{O}(\lambda^6)$$

or, e.g., the maximum trace distance could be used to *'quant-ify'* the causal structure of spacetimes.