Image Inpainting: A Review of Early Work and Introduction Of A Multilevel Approach

Kenneth Webster

January 14, 2015

1 Abstract

There are instances where an image has data missing from it, so some form of approximation needs to be used to complete the image.In the literature, the approximating of missing data is referred to as image inpainting. A solution to this problem is to use a local process that when applied to the image, gives an approximation to the missing data. In this paper, a multilevel approach is used in conjunction with a single-level solution to the problem. This method has various advantages over other existing approaches due to the fact it utilizes multiple levels. This allows for a more global understanding of the problem since the fine and coarse level information are both considered. This multilevel approach has the same order of computational complexity as the single-level base method being used. Here, the coarse solution is used as an initial approximation to the solution of the finer level. The order of the complexity is not increased, rather, the size of the coarser levels decreases in such a way that the summation of those sizes only doubles the computational cost of the single-level problem.

2 Introduction

There is currently an explosive growth in the field of image processing. A new bre akthrough is discovered in area of digital computation and telecommunication almost every month. Noticeable developments came from the public as computers became affordable and new technologies such as smart phones and tablets now allowing the internet to be accessed anywhere. As a result of this there has been a wave of instant information available to many homes and businesses. This information can usually be found in four forms: text, graphics, pictures, and multimedia presentations. Digital images are pictures whose visual information has been transformed into a computer ready format that is made up of binary 1s and 0s. When referring to an image, it is understood that the image is a still picture that does not change over time unlike a video. Digital images are being used for storing, displaying, processing and transmitting information at an ever increasing frequency. So it is important that there be effectively engineered methods to allow for the efficient transmission, maintenance and improvement of the visual integrity of this digital information.

Image processing is an important topic of research due to its large amount of varied research and industrial applications. Many branches of science have sub disciplines using recording devices and sensors to capture image data from the world. The data is usually multidimensional and may be arranged in such a way to be viewable to us. Viewable datasets such as these may also be regarded as images and processed using methods that are currently in use in image processing, even if the datasets have not been obtained from visible light sources such as x-rays or MRI.

There are instances where an image has data missing from it. In the literature, the approximating of missing data is referred to as image inpainting. A way to correct the incomplete image is to use a local process that when applied to the image, gives an approximation to the missing data. In this paper, a multilevel approach is used in conjunction with another single-level solution to the problem. This method has various advantages over some other existing approaches because the multiple levels allow for a more global understanding of the problem since the fine and coarse level information are both being considered. In addition, this multilevel approach has the same order of computational complexity as the single-level method being used as a base. Here the coarse solution is used as an initial approximation to the solution of the finer level. The order of the computational complexity is not increased since the size of the coarser levels decrease, in such a way that the summation of those sizes only doubles the computational cost of the single-level problem. The approach allows for repeated iterations to get a better approximation.

The noise in the image affects a critical step in the original single level algorithm. The problem often affected the final result to such an extent to be noticeably 'wrong' to the human eye. A solution to the problem could be achieved in the following ways:

- Use an image restoration technique focusing on edge preservation
- Techniques that damp/remove noise

Neither approach can satisfactorily solve the issue. The edge preservation techniques are time costly, and ineffectively integrated into the framework. The damping/removing noise algorithms, while time-cost effective, suffers from requiring user defined values and loss of edge information on the local scale. This paper explores an approach which better solves the problem in a way that is less computationally intensive, and preserves the edges on the local scale. The solution is line-space or 2θ space.

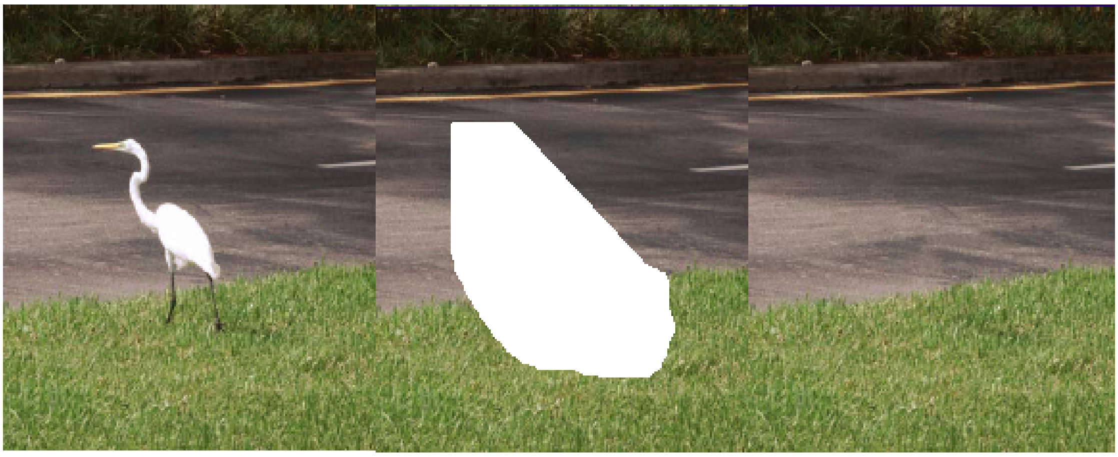


Figure 1: From left to right there is an object in the foreground of the image which is removed then filled in such a way as to appear natural. Image courtesy of [18]

3 Research Problem

Advances in technology allow a scene to be captured in the form of a picture stored in a digital format. This format allows images to be altered with the aid of a computer. Often when a picture is taken a part of the scene is judged to be undesirable and removed. The problem is how should this now missing information be completed without causing any other undesirable parts to appear within the scene? Figure 1 is the image of a bird at the edge of a road. The bird is deemed to be undesirable in the image so the user must manually remove the bird, creating a hole. This hole is called the mask in the literature. The mask is completed by using an approximation. Once the approximation is made, the image is said to be complete. One difficulty in completing an image is matching multiple textures. In image processing, a texture gives us information about the spatial arrangement of colour or intensities in parts of an image. In Figure 1 there are two distinct textures that border on the hole that was the bird. There is grass and there is road. Grass has many vertical lines that are relatively thin compared to the resolution of the image. This image contains low amplitude but high frequency changes in a local area. The road is smooth in a local area but has shadows cast on it that change the intensity over a global area. A good approximation for the missing data would match both of these textures in the mask. Mathematically this problem is said to be ill-defined because there does not exist a unique solution to the problem since being a 'correct' solution comes down to an individual preference for how the missing data is approximated. The preference for an image completion is to have the human brain perceive no artificially induced discrepancies, commonly referred to as wanting an image to 'look natural'. It is difficult to solve since the words 'look natural' are not well defined mathematically, so forming an equation to solve can be time consuming and it is quite difficult to know exactly what information our brain is using to determine if the image ‘looks natural’ or not. In the literature so far, it has proved important that the textures be replicated and continuing structures that extend into the missing data should be completed as well. Those two aspects should be maintained on a local and global scale simultaneously for the image to ‘look natural’. The main idea of the method being presented in this paper is to achieve very simple structure completion by propagating textures into the unknown region while maintaining a balance with respect to the global and local scale. Employing a given image completion method that works on a local scale within a suitable multilevel framework should achieve a completion that works globally and it should therefore improve the robustness of the local method substantially. The method will be used to complete synthetic and natural images.

4 Previous Work

Inpainting methods can generally be sorted into several categories. One approach is to view the image as the domain of a problem where the values in the grid are the discretized version of a PDE solution that is sought [3, 4, 11]. This approach utilizes numerical methods to solve diffusion equations by extending the image into the missing data region iteratively. In [7], convolution was used to propagate the diffusion to achieve a faster restoration compared to previous PDE methods. In [24] the algorithm approximated the missing data one pixel at a time by using the mean and variance of a block in a multi-resolution framework. It was proposed in [23] to use edge detection methods to approximate the structure of the data in the missing region. Once the structure is completed, only then is the rest of the missing data approximated. All of these methods produce very smooth results inside the missing data region which rarely matches the smoothness of the known parts of the image. These methods work well when the region of missing data is small in size compared to the entire domain. A second approach to the problem is to use texture synthesis to fill regions of missing data that are large compared to the domain, with texture sampled from the known data in the image. This approach was used by [9, 15, 19, 27] to approximate the missing data pixel by pixel. The value of the pixel was determined by sampling its local neighbourhood and using a best case match found in the known data of the image. One way to speed up this process involves copying a small region of known data to the missing data. The authors of [2, 15] define these regions as patches or blocks and the process of approximating the missing data as filling. From this, it is unclear in what order the mask should be filled. The order should create an image that minimizes seams between the patches. What constitutes a seam cannot be properly defined since it is a judgment of the human eye, but in practice the idea is to minimize any ‘noticeable’ pattern of discontinuities in the image that matches the pattern of the filling. [22] determines the shapes and which order to fill them in by using a graph cutting algorithm. The problem of determining the order to fill the missing data was shown to be relevant by Harrison in [19], where the priority of the next patch in the missing region to be filled was chosen by the entropy of its local neighbourhood. A combination of PDE and texture patching is used in [6].

Images were defined to be a combination of structure and texture, and completion of the mask is done on each component separately. Once the image is decomposed into two parts, a PDE method from [4] is used to fill the structured image and a texture synthesis method from [14] is used for the textures of the decomposed image. Once both parts are completed they are recombined to form the completion of the original image. This method captures both structure and textural data when filling in the image. However, since it uses a PDE diffusion model, it must be restricted to filling in small regions of missing data otherwise it suffers the same problem as the previous algorithms. In [10] the missing region was filled using a pyramid search in the known data, but the patch size was arbitrary, so there may have been better choices available. [25] included a form of guidance from the user where the user would extend the important structures into the missing region. The algorithm would then fill in the gaps using Belief Propagation [17] around the structures, and the remaining parts of the missing region would be filled using texture synthesis patch by patch. However, the major issue of this algorithm is that it requires supervision by the user, which may be time-consuming and inefficient.

The method presented in this paper is meant to compete with the method of Gazit [20]. Gazit uses multiple levels where each level has undergone edge-preserving smoothing of the previous level. Gazit uses the approach of Criminisi [12] to determine the order in which to fill the mask. A matching patch is selected by considering the Mean Square Error (MSE) on all levels. The matching patch provides and initial guess that is refined by moving from the smoothest level to the original image. Then a new patch in the mask to fill is chosen according to the Criminisi [12] algorithm and the whole process is repeated until the mask is completely filled in. One of the purposes of the algorithm presented in this paper is to lower the computational complexity necessary to match the single level algorithm of Criminisi [12] by using a multigrid framework, where the grids on the different levels are decreasing in size by a factor of two in each dimension. The Gazit [20] levels are all the same size. Using levels of the same size avoids problems encountered using decreasing grid sizes. They are described later in this paper.

The algorithm of this paper uses the method of [12] which is a computationally feasible method with complexity $O(mnlog(n))$ where m is the number of patches and n is the number of pixels defined by the resolution of the image. The algorithm uses exemplar-based synthesis to patch the missing region and the order of the patching is calculated by the direction and size of gradients that border the boundary of the missing region. The goal of Criminisi is to extend the linear structures located along the boundary of the missing region into the missing region with texture synthesis so that both structure and texture are filled in at the same time.

The method presented in this paper uses the single-level patching [12] as the smoother of a multigrid cycle for solving elliptic PDEs. Using the PDE solver framework ensures that the order of computational complexity is not increased from the single-level patching of [12], while improving the final results.

5 Notation

Let I denote a digital image represented by a matrix of real numbers on a set of pixels Ω. Let Ω be partitioned into two regions $\Omega = \Omega_k \cup \Omega_m$ where Ω_k is the subset of pixels where I is known and Ω_m is the subset of pixels where I is unknown. This unknown region is the missing data and in the literature is referred to as the mask, see figure below. An algorithm which fills in the missing data is referred to as a completion algorithm. It can be defined as a function C such that $J = C(I(\Omega))$ satisfies $J(\Omega_k) = I(\Omega_k)$, where C applied on an image with missing data returns an identical image except inside the missing region which now has been approximated. C approximates the missing data in the image I with the goal of the now-filled missing region not appearing 'unnatural' to the human eye, which is a subjective goal.

6 Bertalmio PDE method, [5]

6.1 Contribution

The algorithms devised for film restorations are not appropriate because they rely on the mask which is small in comparison with the image and they rely on the existence of information from several frames. Algorithms based on texture synthesis can fill large regions, but require the user to specify what texture to put where the technique the authors propose does not require any user intervention, once the region to be inpainted has been selected the algorithm is able to simultaneously fill regions surrounded by difference backgrounds without the user specifying "what to put where". No assumptions on the topology of the region to be inpainted or on the simplicity of the image are made. The algorithm is devised for inpainting in structured regions (e.g. regions crossing through boundaries) though it is not devised to reproduce large texture areas.

6.2 Algorithm

The algorithm is divided into two parts: Perona Malik Anisotropic diffusion, and the new part that is the isophote pushing part. An isophote is a region of sharp change in the pixel values that separate regions in a image. The Perona Malik part is to model the image as a discretized PDE of the form: The isophote pushing part is described by the authors of [5] as "estimate a variation of the smoothness, given by a discretization of the 2D Laplacian in our case, and project this variation into the isophotes direction".

6.3 Discussion

It is an early algorithm in the image inpainting field. The user only needs to define the mask. It connects simple structure in the image. The algorithm suffers from various limitations that render it difficult. It is complicated to implement. It uses only local information. The algorithm only works on small regions. It is heuristic with no guarantee of good result. There is no guarantee of convergence. It has no stopping criteria. It has many free parameters such as: time step, diffusion constant, epsilon to counter division by zeros, number of iterations of anisotropic diffusion, number of iterations of their method.

6.4 Conclusion

As early algorithm in image inpainting it is quite good. The main advantage is the automatic restoration of the mask. Unlike film restoration techniques, it does not use any other frames to complete the mask. The algorithm easily admits a parallelization. It fails to reproduce texture on large regions. The method is heuristic in nature with too many free parameters to be able to easily tune the algorithm for a good result. There is no stopping criteria.

7 Patch-based Inpainting

Many patch-based inpainting algorithms rely on the Mean Square Error (MSE) as a measure of the similarity of two images. The similarity of two images can be used to choose the best match. The benefit of using MSE is that it can be calculated efficiently using a Fast Fourier Transform (FFT). The FFT MSE calculation reduces linear calculations by orders of magnitude. The derivation below shows how to achieve the speedup.

7.1 FFT with analytical derivation

Let f and g be the images that are being matched with p and q as the shift in the horizontal and vertical directions respectively. h represents a binary choice with it being one when i and j are between 0 and $w-1$, zero otherwise. Using these functions allows an $O(n^4)$ calculation to be completed in $O(n^2 \log_2(n))$. a and b are the initial coordinates of the patch to be matched in g from the image f.

$$e[p,q] = \sum_{i=0}^{w-1} \sum_{j=0}^{w-1} (f[p+i, q+j] - g[i+a-p, j+b-p])^2, p, q = 0, 1, ..., n-1$$

$$e[p,q] = \sum_{i=0}^{w-1}\sum_{j=0}^{w-1}(f[p+i,q+j] - \tilde{g}[i,j])^2$$

$$e[p,q] = \sum_{i=0}^{w-1}\sum_{j=0}^{w-1}(\tilde{f}[p-i,q-j] - \tilde{g}[i,j])^2$$

$$e[p,q] = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}h[i,j](\tilde{f}[p-i,q-j] - \tilde{g}[i,j])^2$$

$$e[p,q] = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}h[i,j]\tilde{f}[p-i,q-j]^2 - 2h[i,j]\tilde{f}[p-i,q-j]\tilde{g}[i,j] + h[i,j]\tilde{g}[i,j]^2$$

$$e[p,q] = \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}h[i,j]\tilde{f}[p-i,q-j]^2 - 2\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}h[i,j]\tilde{f}[p-i,q-j]\tilde{g}[i,j] + \sum_{i=0}^{n-1}\sum_{j=0}^{n-1}h[i,j]\tilde{g}[i,j]^2$$

$$e[p,q] = \underbrace{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}h[i,j]\tilde{f}[p-i,q-j]^2}_{\text{convolve with FFT}} - 2\underbrace{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}h[i,j]\tilde{g}[i,j]\tilde{f}[p-i,q-j]}_{\text{convolve with FFT}} + \underbrace{\sum_{i=0}^{n-1}\sum_{j=0}^{n-1}h[i,j]\tilde{g}[i,j]^2}_{\text{constant for all } p,\ q}$$

8 Efros and Leung [16]

8.1 Approach

The algorithm grows the texture, pixel by pixel, outwards from an initial seed. A single pixel p is chosen as the unit of synthesis so that the model captures as much high frequency information as possible. All previously synthesized pixels in a square window around p (weighted to emphasize local structure) are used as the context. For each new context the sample image is queried and the distribution of p is constructed as a histogram of all possible values that occurred in the sample image. This non-parametric sampling technique, although simple, is very powerful at capturing statistical processes for which a good model has not been found.

8.2 Algorithm

It models texture as a Markov Random Field. It is assumed that the probability distribution of brightness values for a pixel given the brightness values of its spatial neighbourhood is independent of the rest of the image. The neighbourhood of a pixel is modeled as a square window around that pixel. The size of the window is a free parameter that specifies how stochastic the user believes this texture to be. If the texture is presumed to be mainly regular at high spatial frequencies and mainly stochastic at low spatial frequencies, the size of the window should be on the scale of the biggest regular feature.

8.3 Synthesizing 1 pixel

Take a window centered at the pixel of dimensions w by w. Use a distance metric to compare p with all possible pixels by using the metric to compare w(p) and the windows around all the other pixels. Copy the intensity value of the pixel with the lowest distance from w(p) to the missing pixel p. The metric itself is a normalized sum of square differences weighted with a Gaussian to give higher weights to the pixels closer to the unknown pixel itself. The success of the method is dependent on the assumption that there exists self similarity within the image.

8.4 Synthesizing Texture

Discussed previously is the synthesizing of a pixel when its neighbourhood is known. This (above) method cannot be used for inpainting because only some of its neighbourhood pixels will be known. The method is unsuitable for inpainting because not all of the neighbourhood pixel values are known. The correct solution would be to consider the joint probability of all pixels together but this is intractable for images of realistic size. To complete the image in a computationally realistic time, a heuristic is proposed where the texture is grown in layers outward from a 3 by 3 seed randomly taken from the sample image(in the case of hole filling, the synthesis proceeds from the edges of the hole). Now for any point p to be synthesized only some of the pixel values in w(p) are known (i.e. have already been synthesized). Thus the pixel synthesis algorithm must be modified to handle unknown neighbourhood pixel values. This can be easily done by only matching on the known values in w(p) and normalizing the error by the total number of known pixels when computing the conditional PDF for p. This heuristic does not guarantee that the PDF for p will stay valid as the rest of w(p) is filled in. However, it appears to be a good approximation in practice. One can also treat this as an initialization step for an iterative approach such as Gibbs sampling. However, the trials have shown that Gibbs sampling produced very little improvement for most textures. This lack of improvement indicates that the heuristic indeed provides a good approximation to the desired conditional PDF.

8.5 Discussion

This method can be parallelized. There is only a single free parameter which is the window size. Let n^2 be the number of pixels in the image, let m be the number of pixels in the mask. Then FFT means going from $O(mn^2w^2)$ to $O(mn^2log(n))$ operations. The constant in from is 3 and 8 respectively. The parallelization does not scale linearly with the number of processors. The structure inside the mask must be very simple. The algorithim may be slow if the size of the mask is $O(n^2)$ where the number of pixels in the image is $O(n^2)$. Manual selection of the window size can be problematic if the texture has many scales of textures. As with most texture synthesis procedures, only frontal parallel textures are handled. However it is possible to use Shape-from-Texture Techniques [1, 8, 21] to pre-warp an image into frontal-parallel position before synthesis and post-warp afterwards. One problem of the algorithm is its tendency for some textures to occasionally "slip" into a wrong part of the search space and start growing garbage or get locked onto one place in the sample image and produce identical copies of the original. These problems occur when the texture sample contains too many different types of texels (or the same texels but differently illuminated) making it hard to find close matches for the neighbourhood context window.

8.6 Conclusion

By performing in a pixel-by-pixel manner, the algorithm can capture and replicate high frequency textures either random if the picture is random or structure if the picture is structured. There is only a single free parameter The method admits a parallel version. If the size of the mask is in the order of the number of pixels of the image then the process takes $O(n^4w^2)$ or with FFT $O(n^4log(n))$ with constants 3 and 8 respectively. The growing of garbage is a fault but the authors claim it can be remedied by automatic backtracking when an accumulation of error is detected.

9 Criminisi [13]

The idea is to first fill regions of the mask that have linear structures penetrating the boundary of the mask. This is done in a way that allows the linear structures to be extended first, and then neighbouring regions are slowly filled in as well. Every pixel has its corresponding patch in the same way that every patch has a central pixel. Features of a pixel are computed using data within its patch. The pixels in the unknown region located on the boundary to the known region represent the set of patches that are to be filled in. Since each pixel has a patch, some patches overlap and share much of the same data. The patch is square and its size is defined by the user. Selection of the patch in the missing region is based on two components, the data term and the confidence term.

The data term $D(p)$ of the patch p is then defined as the absolute value of their dot product: $D(p) = |\nabla I^{\perp} \cdot \vec{n}|$ where n is the normal vector of the mask boundary. The linear structures of a patch are determined by using the maximum gradient inside the patch and then constructing a vector of equal magnitude that is perpendicular to the gradient. That vector is constructed from the colour data within the image and defined as $\nabla I^{\perp}$. The data term should then be normalized, which depends on the scale of the colour system used. For example, gray-scale may only have 256 distinct colours whereas RGB has 256*256*256 distinct colours.

The authors name the second component as the confidence of the patch, and in [12] it is defined as $C(p)$. It is a function of the relative size of the known region within the target patch; patches with more known data are preferred. Here, the target patch is the small region inside the missing data region that will be filled in by one patch. Pixels in the known region have a fixed confidence of one. The confidence of pixels inside the unknown region is calculated by the summation of all the confidences of the known pixels inside the target patch: $C(p) = \sum_{q \in p_k} C(q)$. It is then normalized over the total number of pixels in the target patch. This scales the confidences to be between zero and one.

The target patch p which has the greatest priority is the first patch to be filled. The product of the data term and the confidence term is called the priority of the patch: $P(p) = D(p)C(p)$.

Many algorithms are slow and benefit from a good initial guess for fast convergence. A way to do this is a multilevel approach. It has its origins in Elliptic PDEs where a method is called multigrid.

10 Multigrid

Before describing the method of this paper, a description of why multigrid is used to solve PDEs is useful. There is a discretized problem whose solution has been approximated. The exact error is unknown but it is known that the error is composed of high frequency components and lower frequency components. It is known that the method used to iterate towards the solution will smooth the high frequency error quickly but will not be effective at smoothing the lower frequency errors. The high frequency error will quickly disappear after a few iterations but the lower frequency errors require many iterations. The idea of multigrid is to smooth the high frequency components and then interpolate the error, now consisting of only the lower frequency components, from a fine grid to a coarse grid. The grid swap interpolates some of the lower frequency components to high frequency components since the number of points being used is smaller. Now the iterative method can effectively decrease the lower frequency components of the fine grid s high frequency error on the coarse grid. Multigrid is the framework that allows for the iterative method to be used on different grid sizes.

The method proposed in this paper consists of an iteration that is repeated until the image is as desired. The iteration is defined as a V-cycle because of its similarity to a multigrid V-cycle. The description of a V-cycle in the multigrid literature begins with a description of a two-level cycle which is extended recursively to achieve a V-cycle. Here, a two-level cycle first approximates the missing data with some form of single-level

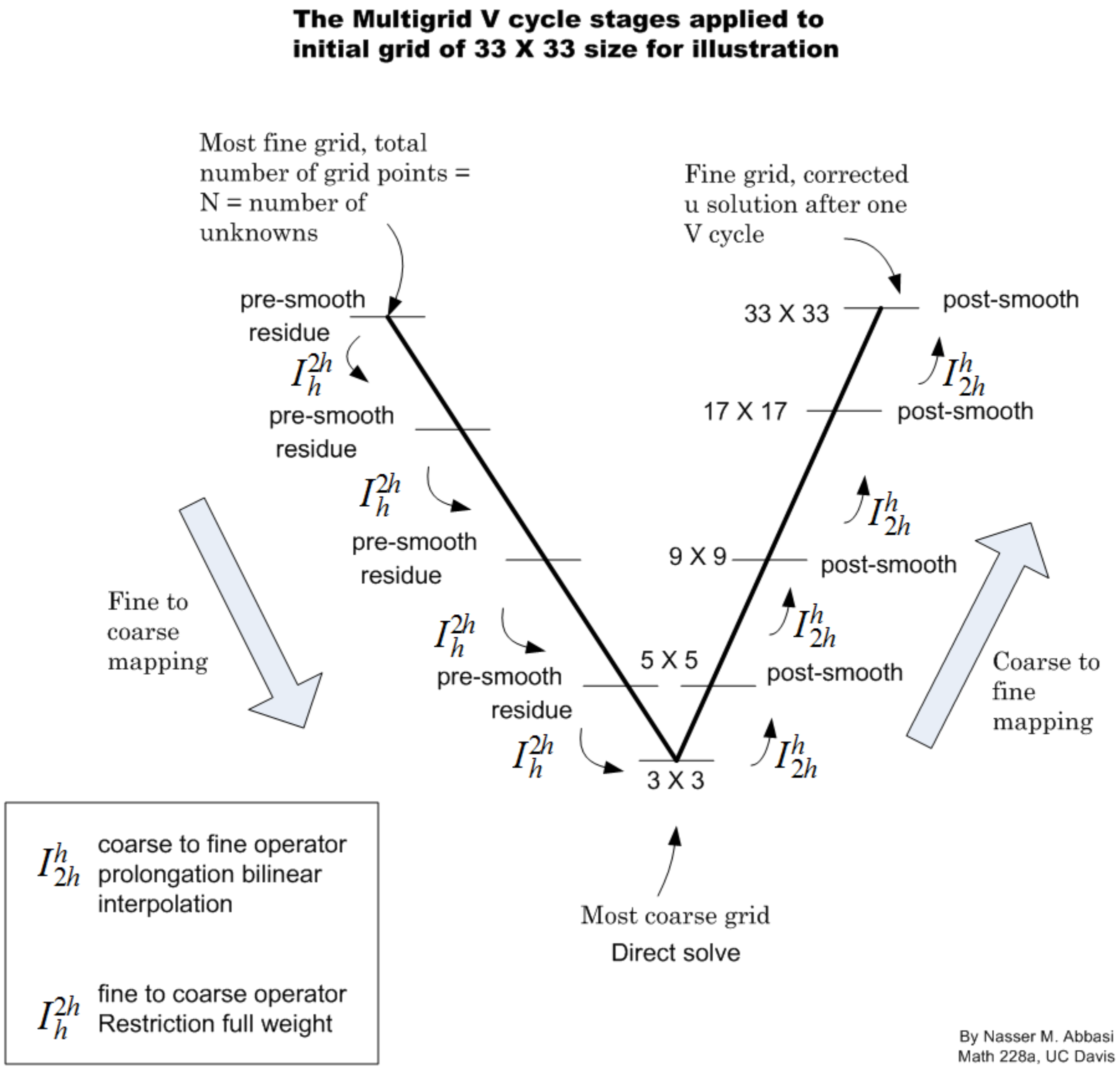


Figure 2: Image courtesy of Nasser M. Abbasi, UC Davis. In the context of this paper, the residue is the now-filled image.

completion [12] and then scales the image down by a factor of 2 in each dimension to get a smaller interpolated image. The single-level approximation algorithm is then used on this smaller image but altered to consider the approximated data now filling the missing region from the larger image. The missing region in the smaller image is expanded by a factor of 2 and copied to the original missing region of the large image. Finally the altered single-level method is used on the larger image to get a new approximation. Outlined above is a two level approach to solving the problem. A V-cycle is a recursive form of this two-level method. In the two level method there is a large image and a small image. The recursive part of the V-cycle is to call itself again with the small image as the new large image.

11 The Proposed Algorithm

The key idea of the method presented in this paper is to fill the missing data with an approximation and then iteratively improve the approximation.

Vcycle: input(*imLarge*) // where *imLarge* is an image

---

if size of *imlarge* is bigger than is allowed by algorithm [12] **then**
  return *imLarge*;
end if
$imV \leftarrow$ AlteredCriminisi($imLarge$)
$imSmall \leftarrow$ Coursen(imV) // Scale input by a factor of 0.5
$imW \leftarrow$ Vcycle($imSmall$) // This is the recursive step
$imX \leftarrow$ Interpolate(imW) // Scale input by a factor of 2
$imY \leftarrow imLarge$
$imY(\Omega_m) \leftarrow imX(\Omega_m)$
$imLarge \leftarrow$ AlteredCriminisi(imY)
return $imLarge$

The next step is determining if an iterated approximation is within a defined bound of an ideal solution. The stopping criterion for the iterations is user defined since an image itself does not satisfy many of necessary assumptions that PDEs require. A scene in nature does not necessarily have to be continuous at all points in the captured image. Nor will it have enough pixels to be locally smooth. One criteria is to stop iterating if nothing is different after an iteration, basically $x_{k+1} = x_k$ which should be included but this method is not guaranteed to become stable because in experiments it was observed that an image can oscillate between two images, i.e. $x_{k+2} = x_k$ but $x_{k+1} \neq x_k$. So the best way to ensure the method finishing is to stop after an arbitrarily specified number of iterations or when an iteration no longer changes the image, whichever happens first.

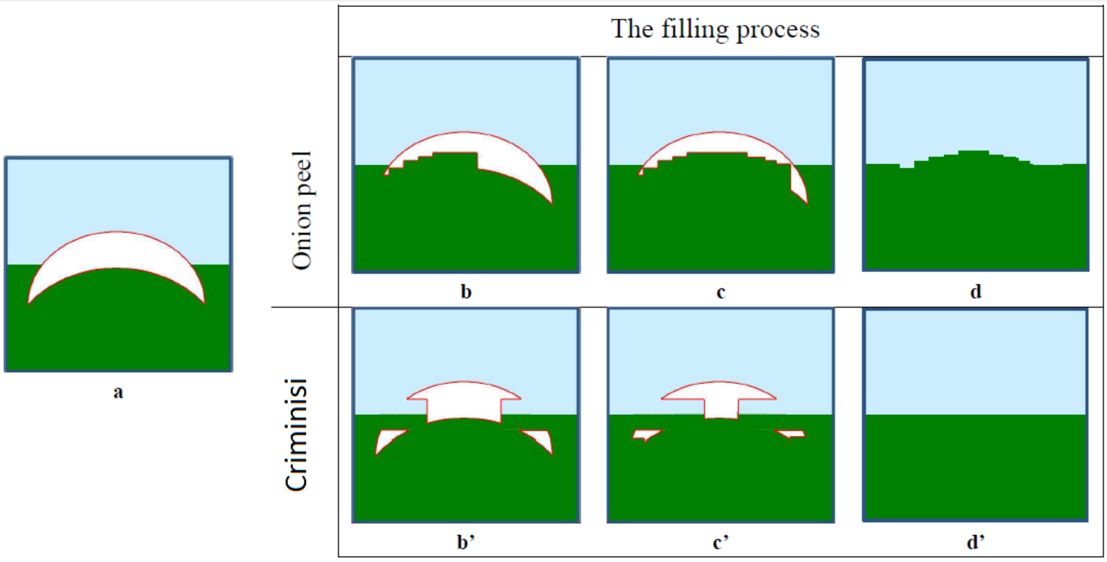


Figure 3: the importance of the filling order when dealing with concave target regions. (a) A diagram showing an image and a selected target (in white). The remainder of the image is the source. (b,c,d) Different stages in the concentric-layer filling of the target region. (d) The onion-peel approach produces artifacts in the synthesized horizontal structure (b', c', d') Filling the target region by an edge-driven filling order achieves the desired artifact-free reconstruction. (d') the final edge-driven reconstruction, where the boundary between the two background image regions has been reconstructed correctly. Image courtesy of [12]

A problem that was encountered in this multigrid setup was how the interpolation of the mask region from level to level was to be handled. Gazit [20] avoided this by using levels of the same size which meant the mask did not change between levels. However, in this framework, the size of the mask does not half in each dimension in relation to the grid dimensions, as the problem is moved to the coarser grid. Since the mask is unknown, the pixels that belong to it propagate more unknown regions downwards to the coarser level. This means that the number of levels in the V-cycle is restricted by the growth of the mask. Should the mask cover the vast majority of a level then patch-based inpainting will poorly fill the region, because of the lack of matching patches to choose from. This problem is countered by restricting the number of coarser levels in the V-cycle. Multigrid smoothing (the error is being smoothed) is not used because there is no correct solution to achieve. The completion algorithm $Criminisi()$ in its altered form takes the place of the smoothing methods in a V-cycle. The completion algorithm is slightly altered in two ways from [12]. The modifications are aimed to slightly improve one part of the algorithm and to allow for the inclusion of the data from previous iterations.

The completion method of [12] is characterized by the order in which to fill in the missing region and a similarity measure which determines what data will be used in the

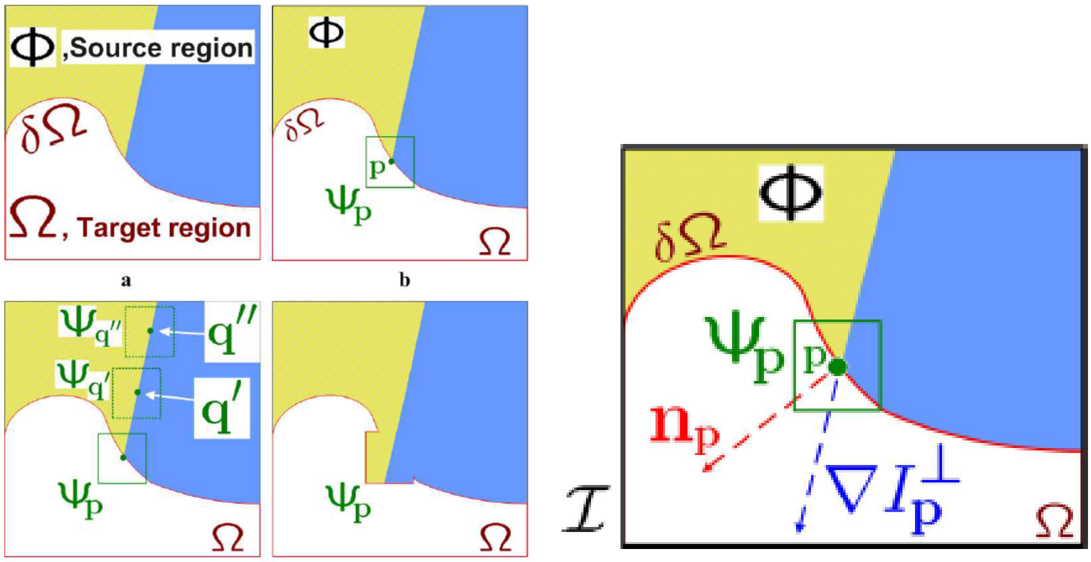


Figure 4: Notation diagram. Given the patch Ψ_p. n_p is the normal to the contour $\delta\Omega$ of the target region Ω and $\nabla I_p^{\perp}$ is the isophote (direction and intensity) at point p. the entire image is denoted with I. Image courtesy of [12]

patch. The authors of [12] showed that the order of completion affects how natural a completion may be. A good comparison can be found in Figure 3.

The two alterations to the algorithm in [12] are small in nature. The authors of [12] chose their data term by calculating the gradient of all pixels located within the target patch and then chose the gradient with the maximum magnitude to be representative of the central pixel. This means that multiple patches may have the same gradient since the same maximum magnitude gradient pixel may be located within several target patches. The modification is to multiply the gradient magnitude of the pixels by the inverse of their distance from the center of the target patch. This means that the target patch whose center is closest to the maximum gradient has a larger data term than its immediate neighbours. The second alteration is to the similarity measure, which is described first.

11.1 Similarity Measure

The completion algorithm requires that target patches are filled in with texture, in the form of patches, from the known data of the image. This can be done in several ways and the authors of [12] used one of the more common measures. The unknown pixels in the target patch are replaced by pixels from a source patch. The source patch is selected by it having the smallest deviation between itself and the target patch. The deviation is defined as the square of the difference in colour values of the known pixels in the target patch and the known pixels in the source patch. This is then normalized by the number of known pixels in the target patch. For a source patch $S(p)$ the deviation is defined as:

$$\frac{1}{|p|}\sum(p_k - S(p_k))^2$$

This is essentially the mean square error between the known pixels in the target patch and the pixels of the source patch in the same relative position within the patch as the target patch. The alteration is to have the mean square error over all the pixels in the target patch and source patch. This is because there is already an approximation to the solution in the missing data (But the very first approximation has no values in the missing region). Including the approximations allow the iterations to improve the image iteratively.

11.2 Computational Complexity

Although it may seem that using multiple images multiplies the cost of the entire completion, the implementation presented in this paper does not increase the complexity of the single-level completion algorithm [12]. This is because of the way in which the smaller images in the V-cycle are created. As in multigrid for PDEs, the next grid is smaller by a factor of two for every dimension of the domain, for images that means the next smaller image is one quarter the size. Each level, or smaller image, also uses the completion algorithm. Say that the completion algorithm has computational complexity of $O(mn \log n)$ where n is the number of pixels in the image and m is the number of patches needed to approximate the missing data. The $\log(n)$ comes from using Fast Fourier Transforms (FFTs) to calculate deviation between patches. Then summation of each level gives:

$$\begin{aligned} mn\log(n) + mn\log(n/4)/4 + mn\log(n/16)/16 + mn\log(n/64)/64 + \dots \\ <= mn\log(n)(1 + 1/4 + 1/16 + 1/64 + \dots) \\ = mn\log(n)(4/3) \end{aligned}$$

Therefore, the cost of a V-cycle will only be a constant multiplied by the computational cost of the algorithm of [12], and the algorithm presented in this paper is also $O(mn\log(n))$. Since stopping criteria is not guaranteed for the iterations, an arbitrary constant (k) should be used for the number of iterations then the order of the algorithm is $O(kmn\log(n))$. It should also be noted that the multiple levels are restricted to ones whose size is greater than the user defined size of a patch.

12 Results

The algorithm presented in this paper was tested on both synthetic and natural images that were not chosen because they had the best results. They were chosen to illustrate what the algorithm achieved over the single-level method [12] and what problems were encountered for both. All approximations in the V-cycle column were achieved with no more than 4 V-cycles.

13 Conclusion

This paper introduces a new concept to textured patching in image inpainting. The ability to repeatedly iterate an approximation while retaining the texture that is patched in. This was done by using a framework of multilevel approaches with its origins in solving elliptic PDEs. By using this multilevel framework in conjunction with a well devised texture patching method, it allows global structures to affect the local area around the missing data, instead of only using data from its local neighbours. Results of experiments vary but they appear to always be at least as good as the single-level method, and without incurring any additional computational complexity.

14 Future Research

There seem to be three areas where the method presented in this paper may be improved. When interpolating from a small image to a large image perhaps a transition which takes into account both images simultaneously would produce far less blur in the larger image on the damped update of a V-cycle.

Another area of future research would be to change the similarity measure because "mean-square error is a poor indicator of texture matching and also of gradient matching. Entropy or variance may be considered as alternatives" [26]. Changing this may lead to being able to remove the multilevel framework completely and only using the single-level method described in [12].

Another similarity measure [26] which has been shown to work very well is the Structural Similarity Index Measure (SSIM). It does however have a complexity cost at least one order of magnitude greater than MSE. To get around this the SSIM could be interspersed with MSE to reduce runtime but alternating between the two measures with any frequency will still increase the computational complexity of the original algorithm.

It could also be argued that the method in which the data-term is calculated may be improved by a more accurate reading of structure, because as can be seen in Figure 5 it focuses only on large magnitude changes instead of actual structure. In Figure 5 it can be seen that the line is a horizontal one and all the other lines are textures, but again this is quite difficult to describe to a computer. Altering the work done in [6] may lead

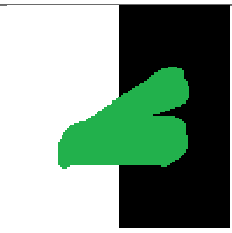
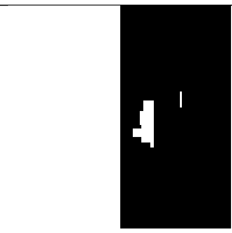
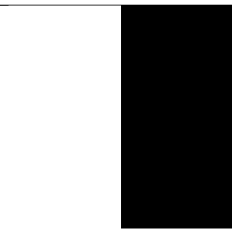
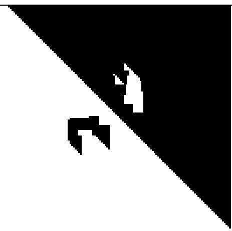
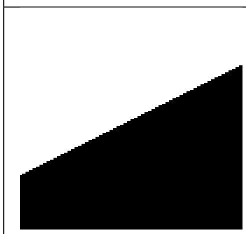
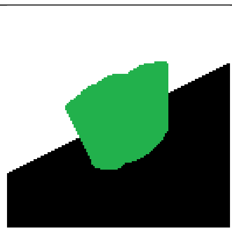
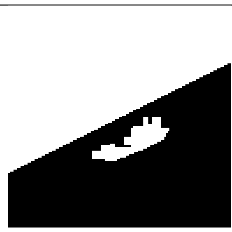
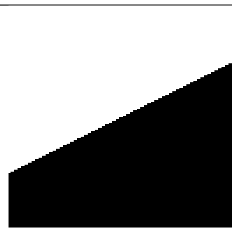
Original Image	Image with Mask	Criminisi Method	V-cycle
			
		the right side of the image has artifacts	
			
		On both sides of the diagonal there are noticeable artifacts	
			
		Below the diagonal there appears to be a large artifact	

Figure 5: Here is a comparison between the Criminisi method and the method proposed in this paper on several artificial images.

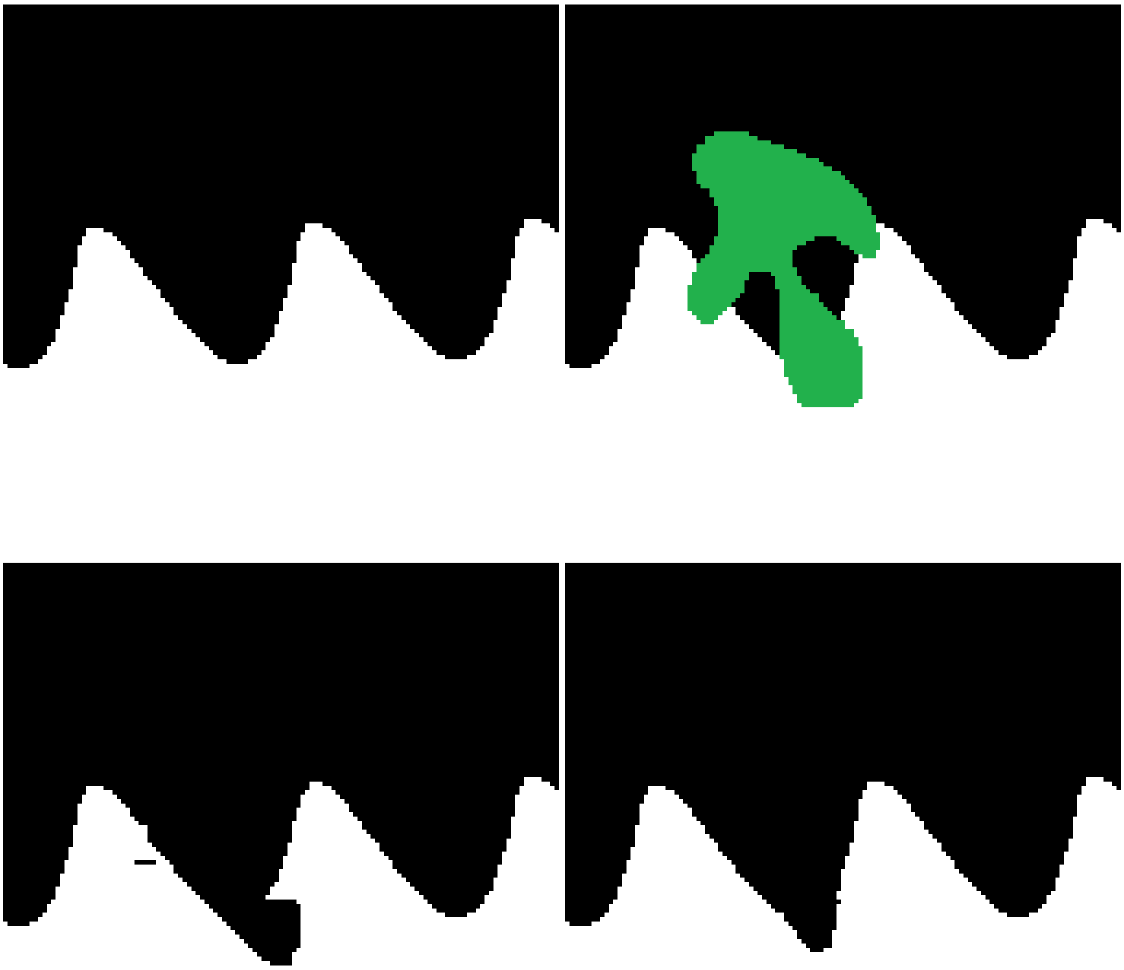


Figure 6: In the center of the image there appears to be a large area of black extending in an unnatural way from the wave. In the center of the image the bottom of the wave appears to be unnaturally sharp.

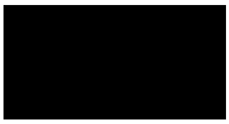
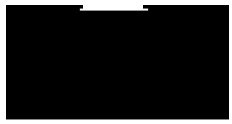
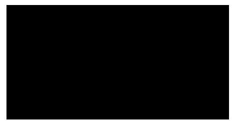
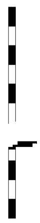
Original Image	Image with Mask	Criminisi Method	V-cycle
			
		the algorithm just missed filling in the region correctly but not significantly unnatural.	
			
	Here the pattern is obvious to the human brain but both methods fail to capture this aspect.	the horizontal extension from the bottom of the visible texture appears distinct from the rest of the image.	the center of the image was not approximated well enough to capture the aspects of the original image.
			
	Both methods fail to completely restore the image but they do not add any 'unnatural' artifacts.		

Figure 7: Experimental Results (part 2)

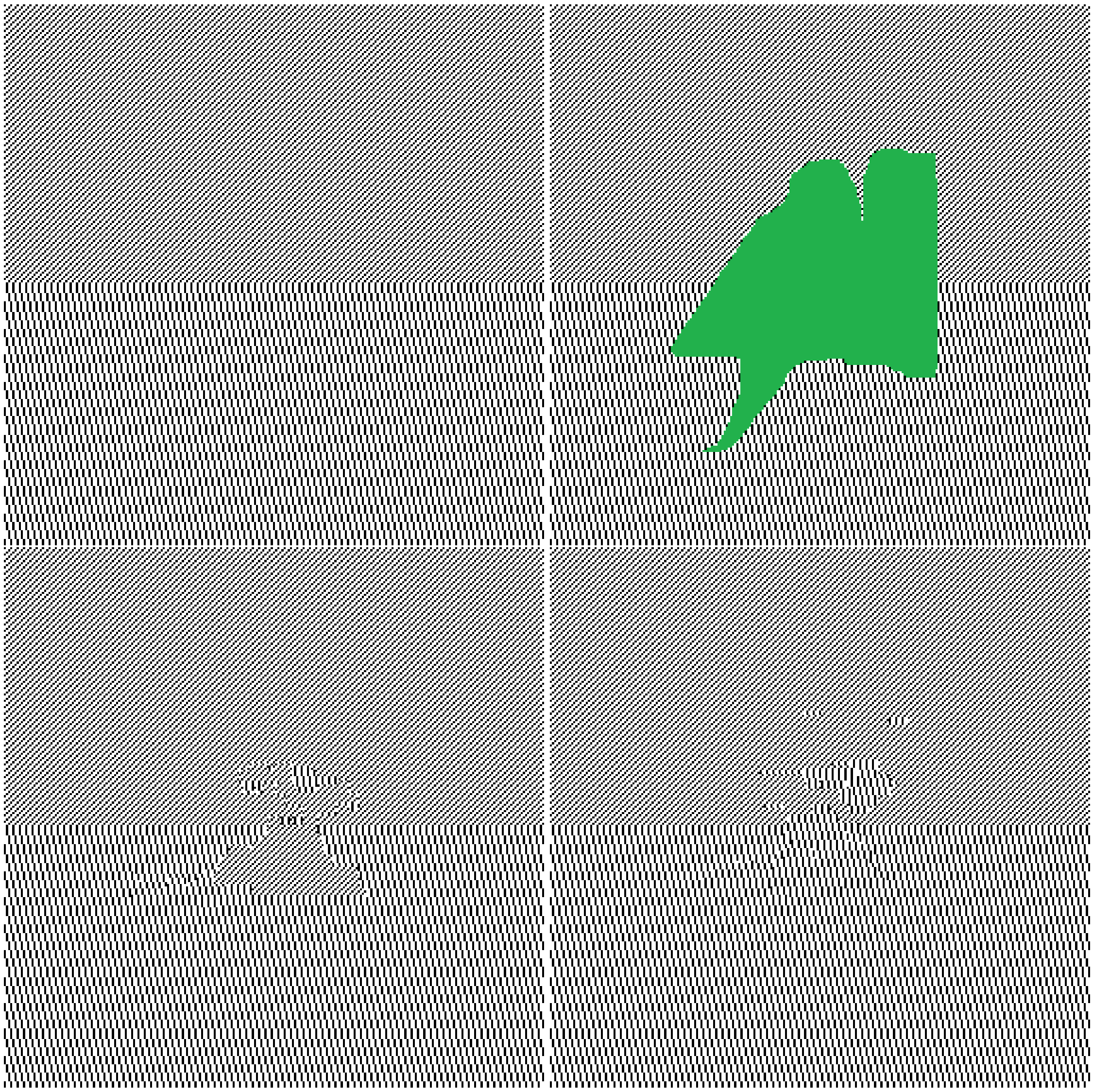


Figure 8: Top-Left: original image. Top-Right: Image with Mask. Bottom-Left: Criminisi Method. Bottom-Right: V-cycle. Both methods fail to distinguish texture from structure resulting in poor completions of the image.

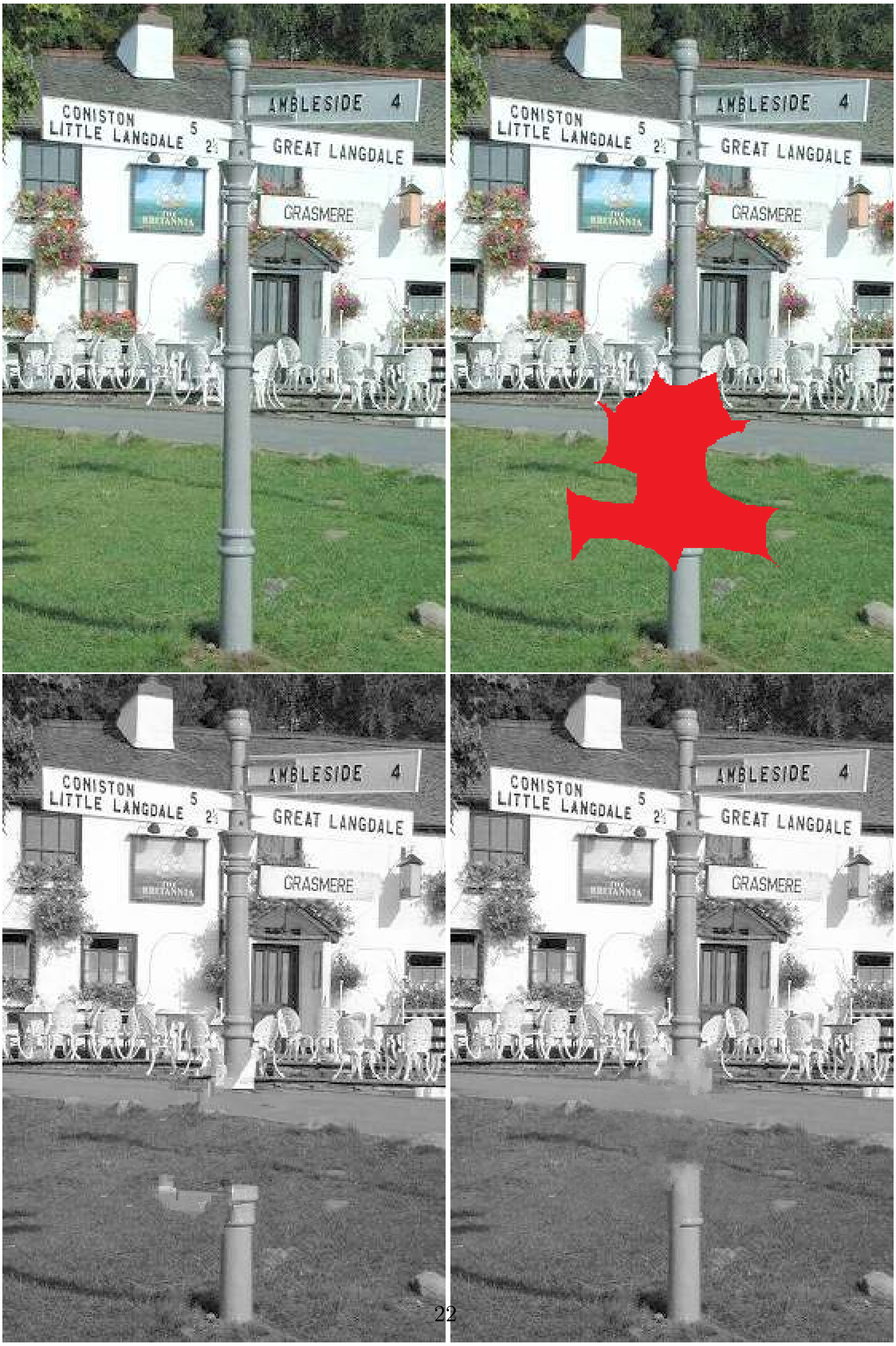


Figure 9: Top-Left: original image. Top-Right: Image with Mask. Bottom-Left: Criminisi Method. Bottom-Right: V-cycle. This image has two very distinct edges extending into the missing region. Looking at in a linear structure sense, there does not appear to be a preference if grass is over the top of the pole or the other way around. Noticeable artifacts have appeared in the missing region making the whole scene appear inconsistent. The pole had begun to extend upwards through the grass and the edge of the grass and the road was connected through the mask.

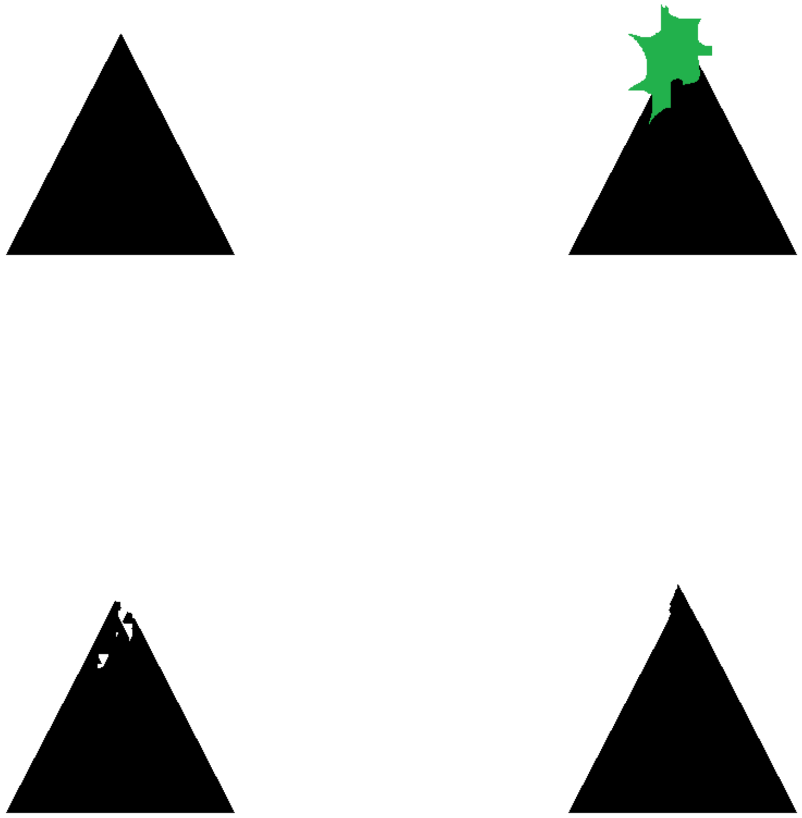


Figure 10: Top-Left: original image. Top-Right: Image with Mask. Bottom-Left: Criminisi Method. Bottom-Right: V-cycle. Parts inside the triangle were filled incorrectly. The top of the triangle is slightly lopsided.

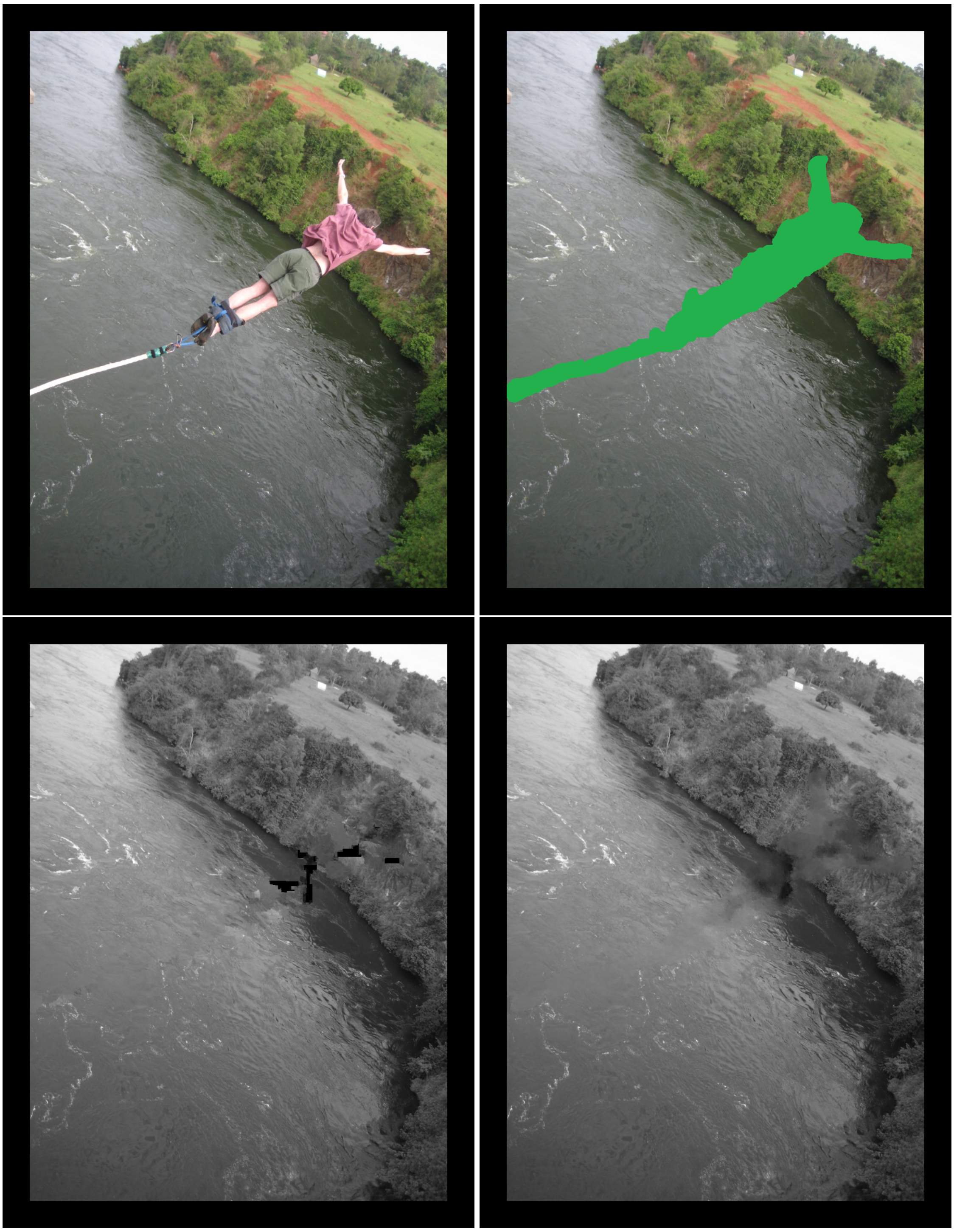


Figure 11: Top-Left: original image. Top-Right: Image with Mask. Bottom-Left: Criminisi Method. Bottom-Right: V-cycle. Parts from the border were included in the missing region. The approximation seems slightly smooth on close inspection.

Figure 12: Top-Left: original image. Top-Right: Image with Mask. Bottom-Left: Criminisi Method. Bottom-Right: V-cycle. Another eye was replicated in the Criminisi solution above and left of her right eye. In the V-cycle solution the approximation appears blurry

to better results for capturing the structures.

It should be said that all areas of the algorithm may benefit from further research, such as edge-preserving interpolation or the stopping criteria. Perhaps the use of Full Multigrid (FMG) would be preferable to V-cycles.

Another improvement would be to calculate the isophotes using a method which averages the directions within a moving window to provide a better estimate of the data-term in the Criminisi algorithm. Currently I have a method which I have labeled 2θ. It is a transform which allows the addition of vectors in a constructive manner when the vectors lie along similar lines. In 2θ lines become vectors in a simplistic sense. In Euclidean space, if a vector is added to a negative version of itself, it disappears which makes sense if it is worded as a vector minus itself. In 2θ-space these two concepts are different. In 2θ-space lines are the new vectors. An important distinction between lines and vectors is that lines have two opposite directions, so reversing the line returns itself. So adding a negative version of a line to itself should just yield the line back again. In this new space, if you add a perpendicular line to itself it results in nothing. This is the basic concept for 2θ-space. It is not rigorous in the sense of an actual space. It is more of an idea of how to perform basic operations on vectors that result in a more line friendly approach.

This 2θ-space is well suited to multiple colour channels as well. Let us set up the case where an image consists of 3 colour channels, so say red(R), green(G), blue(B). They correspond to the functions f, g, and h respectively. Each pixel in the image has a corresponding position in the domain, say $(x,\ y)$ so we have the image being $f(x, y), g(x, y), h(x, y)$. Now the setup is that we are performing a moving window average over an entire image for the purpose of extending structures linearly into a mask inside the image. However let us focus on just an average of a small window on the edge of the mask. As we see the image there are three clear isophotes in each of the colour channels. If the isophote of f and g are perpendicular and of equal magnitude then they would cancel each other out in 2θ-space, which is a benefit of this algorithm because the third function h has the isophote that decides the stalemate of the f and g interaction. Without loss of generality, if the isophote of h is strongly correlated to f then the isophote is 'probably' real and should be recorded as strong. If the isophote is not strongly correlated with the ones from either f and g then the structure is no longer linear and too complex to naturally extend into the mask. This type of action is strongly suited for the algorithm of Criminisi.

Another direction that this research could take is moving from still images to sequences of images which come under the heading of video processing. Adapting this algorithm for video can be done in a number of ways. We could treat each frame as a separate image and run the algorithm, but this does not take advantage of the information available in the image ahead and behind the current image. By using multiple images from the sequence, there is more data that can be used when inpainting the mask. I have constructed a multigrid framework for still images which shows that this approach does work. There needs to be a consistency between consecutive completed images so that the linear structures extend in to the missing region is consistent across all frames. This means more image processing techniques such as image registration and motion tracking may have to be investigated to be employed because although there is only a short interval in the time between frames, the scene will generally change.

A real world example of this is close-up time-lapse photography of a flower in bloom. Usually since flowers reproduce in some way with pollen it attracts various insects onto the petals for a short period of time. This means that there are insects appearing and disappearing from frame to frame as the video runs. These insects are not a desirable part of the video, if capturing the flower blooming is the purpose. Even in time-lapse the image of the flower does not change drastically from one frame to the next, but perhaps the insect that was captured in one frame has disappeared in the next. Using information from frames ahead and behind the current frame will help to effectively inpaint the mask better than using information from the surrounding area of the mask. In just this example of time-lapse photography there are many undesirable objects such as water droplets or shadows. Time-lapse photography is generally used to capture scenes that humans understand better when viewed over a shorter period of time. The idea is to capture events that are happening over a long period of time and not the events in a short time frame. Eliminating the short time events is desirable in the world of photography.

Another example of integrating this research into video processing would be to remove an object that appears in multiple frames entirely from the video. Perhaps there is something going on in the background that detracts from the visual appeal of video and the background event needs to be removed. As long as the background event is properly identified over the sequence of images, the mask can be inpainted properly and the algorithm would definitely benefit from having the information from multiple frames. This can be understood most easily when considering a motionless camera. By just averaging all the frames you cancel out moving objects because as the objects move they reveal the true information that was hidden behind them. By taking this approach it may be possible to correctly restore the video to match the physical scene it was capturing. Whereas in image processing the information behind the mask is not known nor can it be determined exactly from the rest of the image. Video processing may introduce another dimension into the problem but it also introduces new information with meaningful consequences.

Bibliography

[1] John Aloimonos. Shape from texture. *Biological cybernetics*, 58(5):345–360, 1988.

[2] M. Ashikhmin. Synthesizing natural textures. In *Proceedings of the 2001 Symposium on Interactive 3D Graphics*, pages 217–226, New York, NY, USA, 2001. ACM Press.

[3] Ziv Bar-joseph, Ran El-Yaniv, Dani Lischinski, and Michael Werman. Texture mixing and texture movie synthesis using statistical learning. *IEEE Transactions on Visualization and Computer Graphics*, 7:120–135, 2001.

[4] Marcelo Bertalmío, Guillermo Sapiro, Vicent Caselles, and Coloma Ballester. Image inpainting. In *SIGGRAPH*, pages 417–424, 2000.

[5] Marcelo Bertalmio, Guillermo Sapiro, Vincent Caselles, and Coloma Ballester. Image inpainting. In *Proceedings of the 27th annual conference on Computer graphics and interactive techniques*, pages 417–424. ACM Press/Addison-Wesley Publishing Co., 2000.

[6] Marcelo Bertalmio, Luminita Vese, Guillermo Sapiro, and Stanley Osher. Simultaneous structure and texture image inpainting. In *IEEE Transactions on Image Processing*, volume 12, pages 882–889, August 2003.

[7] Jose M Bioucas-Dias, Mario AT Figueiredo, and Joao P Oliveira. Total variation-based image deconvolution: a majorization-minimization approach. In *Acoustics, Speech and Signal Processing, 2006. ICASSP 2006 Proceedings. 2006 IEEE International Conference on*, volume 2, pages II–II. IEEE, 2006.

[8] Andrew Blake and Constantinos Marinos. Shape from texture: estimation, isotropy and moments. *Artificial Intelligence*, 45(3):323–380, 1990.

[9] Jeremy S. De Bonet. Multiresolution sampling procedure for analysis and synthesis of texture images. In *SIGGRAPH*, pages 361–368, 1997.

[10] Richard J Cant and Carol S Langensiepen. A multiscale method for automated inpainting. In *17th European Simulation Multiconference*, pages 148–153, 2003.

[11] T Chan and J Shen. Non-texture inpaintings by curvature-driven diffusions. *Journal of Visual Communication and Image Representation*, 12(4):436–449, 2001.

[12] Antonio Criminisi, Patrick Pérez, and Kentaro Toyama. Region filling and object removal by exemplar-based image inpainting. *IEEE Transactions on Image Processing*, 13(9):1200–1212, 2004.

[13] Antonio Criminisi, Patrick Pérez, and Kentaro Toyama. Region filling and object removal by exemplar-based image inpainting. *Image Processing, IEEE Transactions on*, 13(9):1200–1212, 2004.

[14] Alexei A. Efros and William T. Freeman. Image quilting for texture synthesis and transfer. In *SIGGRAPH*, pages 341–346, 2001.

[15] Alexei A. Efros and Thomas K. Leung. Texture synthesis by non-parametric sampling. In *ICCV*, pages 1033–1038, 1999.

[16] Alexei A Efros and Thomas K Leung. Texture synthesis by non-parametric sampling. In *Computer Vision, 1999. The Proceedings of the Seventh IEEE International Conference on*, volume 2, pages 1033–1038. IEEE, 1999.

[17] Pedro F Felzenszwalb and Daniel P Huttenlocher. Efficient belief propagation for early vision. *International journal of computer vision*, 70(1):41–54, 2006.

[18] Michal Holtzman Gazit. *Multi Level Methods for Data Completion.* PhD thesis, Israel Institute of Technology, 2010.

[19] Paul Harrison. A non-hierarchical procedure for re-synthesis of complex textures. In *WSCG*, pages 190–197, 2001.

[20] Michal Holtzman-Gatiz and Irad Yavneh. A scale-consistent approach to image completion. *International Journal for Multiscale Computational Engineering*, 6(6):617–628, 2008.

[21] Kenichi Kanatani. *Shape from Texture.* Springer, 1990.

[22] Vivek Kwatra, Arno Schödl, Irfan A. Essa, Greg Turk, and Aaron F. Bobick. Graphcut textures: image and video synthesis using graph cuts. *ACM Trans. Graph.*, 22(3):277–286, 2003.

[23] Andrei Rares, Marcel J. T. Reinders, and Jan Biemond. Edge-based image restoration. *IEEE Transactions on Image Processing*, 14(10):1454–1468, 2005.

[24] T.K. Shih, Liang-Chen Lu, Ying-Hong Wang, and Rong-Chi Chang. Multi-resolution image inpainting. In *2003 International Conference on Multimedia and Expo*, volume 1, pages I–485–8 vol.1, 2003.

[25] Jian Sun, Lu Yuan, Jiaya Jia, and Heung-Yeung Shum. Image completion with structure propagation. *ACM Trans. Graph.*, 24(3):861–868, 2005.

[26] Zhou Wang, Alan C Bovik, Hamid R Sheikh, and Eero P Simoncelli. Image quality assessment: from error visibility to structural similarity. *Image Processing, IEEE Transactions on*, 13(4):600–612, 2004.

[27] Li-Yi Wei and Marc Levoy. Fast texture synthesis using tree-structured vector quantization. In *SIGGRAPH*, pages 479–488, 2000.