Control of Infinite-Dimensional Systems

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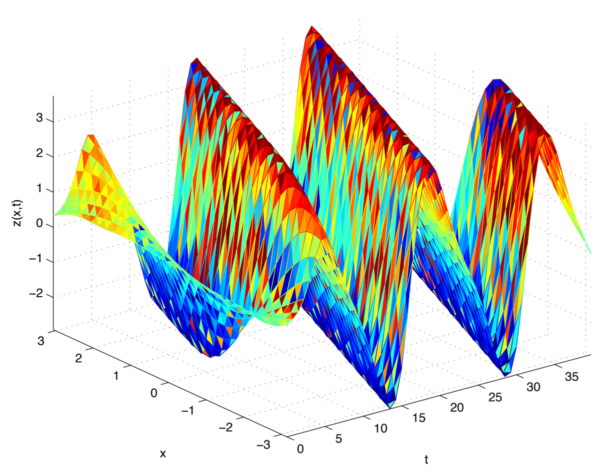
What is Control Theory?

Control theory is the introduction of an input into a dynamical system to steer the system to a desired objective. For example, a control objective can be steering the dynamics of a system from an unstable state to a stable state. Questions such as the choice, implementation, and robustness of the control arise. Control theory is a multidisciplinary research field with reaches in engineering systems, computer science, biological sciences and even economic models.

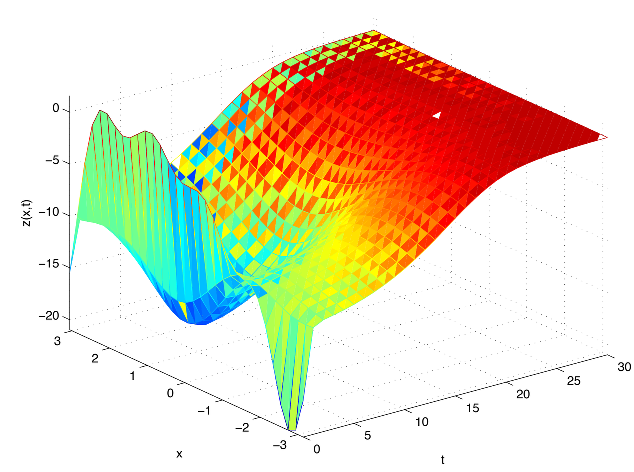
Stabilization of Kuramoto-Sivashinsky Equation

The Kuramoto-Sivashinsky (KS) equation is a nonlinear PDE which models reaction-diffusion systems and is related to various phenomena where turbulence and chaos appear. For instance, it models a thin liquid film falling down a vertical plane which may occur in both natural and industrial processes. For certain parameter values that depend on the physical model, such as density, viscosity or surface tension, this equation is unstable. It turns out that stabilizing the linearized KS equation implies local exponential stability of the nonlinear controlled system. This is used to develop a strategy for bounded controller design using a lumped approximation via input or output feedback control. These results indicate the system is stabilized and that spillover is avoided. In other words, a finite-dimensional controller stabilizes the full infinite-dimensional state. Furthermore, the KS equation can be stabilized so that the system is steered from one state to another.

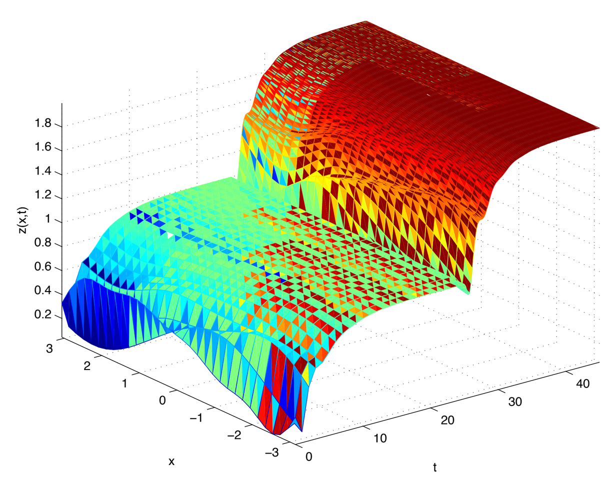
Simulations of the KS equation



Unstable system with no control

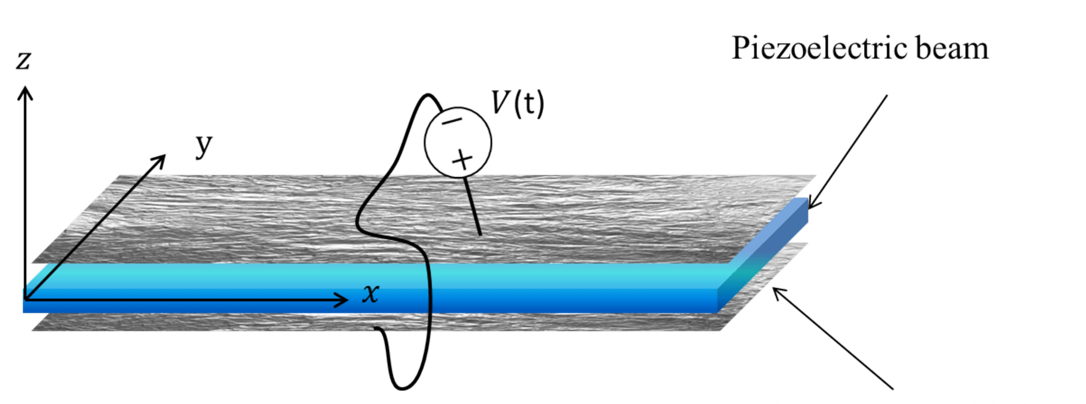


Control stabilizes the system



Control steers the system from one state to another

Modelling and Control of Piezo-electric Beams



Models for piezoelectric beams and structures with piezoelectric patches generally ignore magnetic effects because the magnetic energy has a relatively small effect on the overall dynamics. Hamilton's principle can be used to derive a model for a piezoelectric beam that includes magnetic effects. It turns out that magnetic effects have a strong effect on the stabilizability of the control system. Including magnetic effects leads to a model where (1) if voltage control is used, for almost all system parameters the beam is not exponentially stabilizable (2) if current control is used, the beam is never exponentially stabilizable. In both cases, strong stability can be achieved. This is quite different from the model without magnetic effects which can be exponentially stabilized with either current or voltage control.

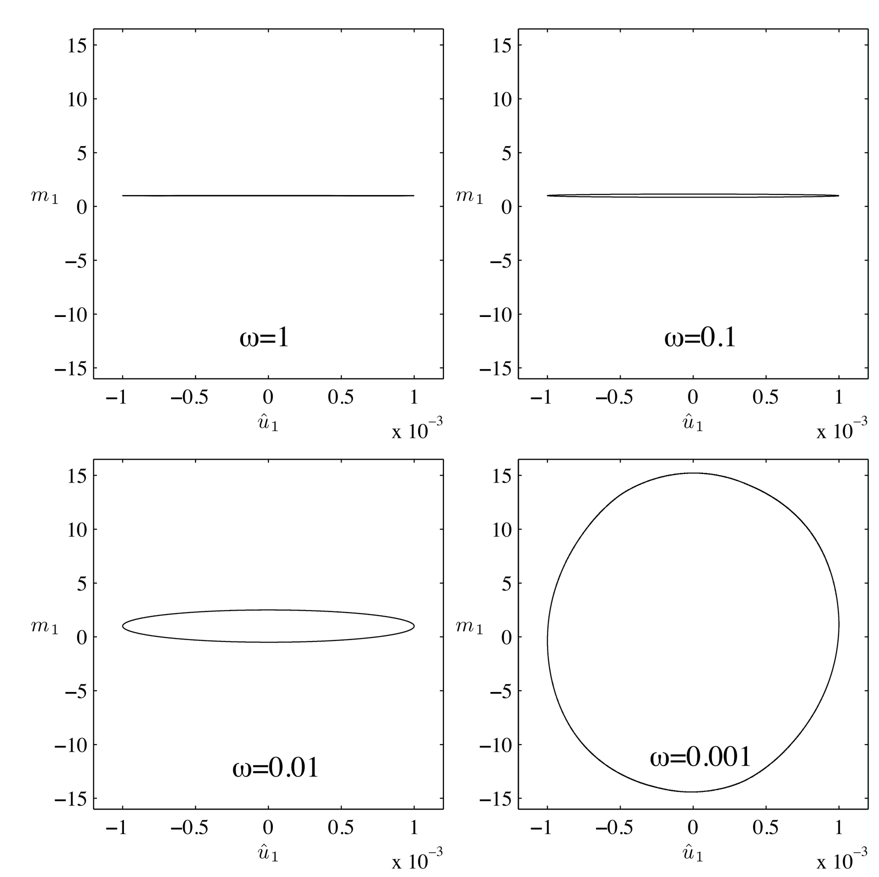
Challenges in Controlling Infinite-Dimensional Systems

Many systems of practical importance can only be described by infinite-dimensional systems. For example, vibrations and sound waves exhibit both time and space dependence and are, hence, modelled by partial differential equations (PDEs). This means that the state variables evolve on infinite-dimensional spaces. Consequently, analysis and simulation of these systems is often challenging. Research projects on the control of PDEs currently conducted at the University of Waterloo are presented.

Characterizing and Controlling Hysteresis

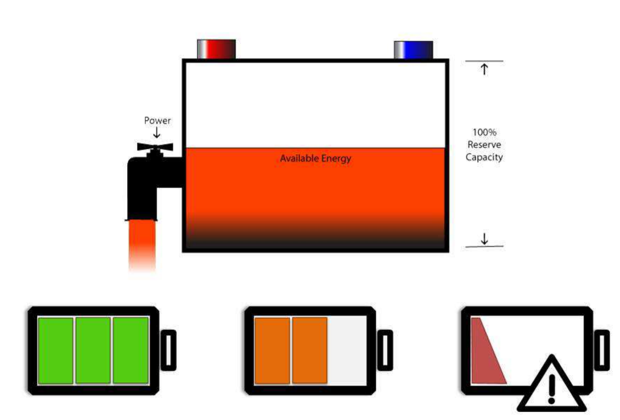
Hysteresis is a phenomenon that occurs in many processes. Examples include magnetization, smart materials, freezing and thawing processes and predator-prey relationships. A common theme in defining hysteresis is that of a looping behaviour displayed in the input-output map; however, the existence of a loop is not sufficient to identify hysteretic systems. Two definitions of hysteresis are considered: (1) systems that exhibit hysteresis have multiple stable equilibrium points and dynamics that are faster than the rate at which inputs are varied, (2) a system exhibits hysteresis if a nontrivial closed curve in the input-output map persists for a periodic input as the frequency component of the input signal approaches zero.

The Landau-Lifshitz equation is a nonlinear PDE that describes magnetization within ferromagnetic nanostructures. Magnetization governed by the Landau-Lifshitz equation exhibits hysteresis, which is demonstrated by the existence of persistent looping behaviour in the input-output maps as the frequency of the periodic input approaches zero.



A control that moves system dynamics from one stable equilibrium point to another stable equilibrium point essentially means the control of hysteresis. Such a controller design was applied to the Landau-Lifshitz equation, Hysteresis is absent in the controlled Landau-Lifshitz equation as looping behaviour is not observed in the input-output map.

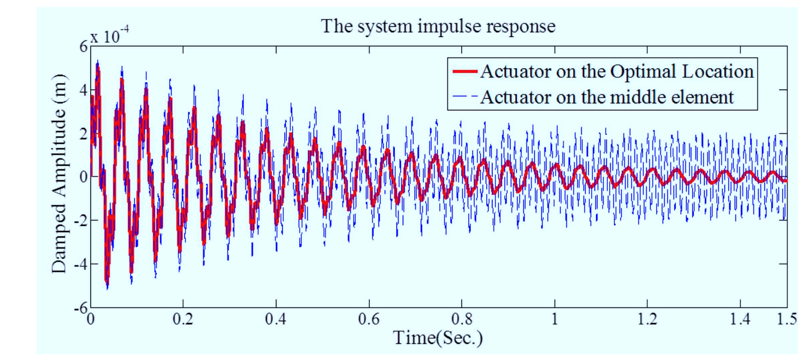
Estimation of Charge in Lithium-Ion Car Batteries



Determining the remaining charge in batteries is important for improving the safe time between charging. This cannot be measured directly and must be estimated. This is a difficult problem because the dynamics are governed by coupled nonlinear PDEs and hysteresis is a factor. An accurate but simple method that can be implemented in the processor available on an automobile is being sought.

The battery diagram is taken from
http://www.koldban.com/v/vspfiles/assets/images/images/kapower/Bat_Lo_Chg_Lg.gif
http://epg.eng.ox.ac.uk/content/electric-vehicles-using-physics-based-battery-models-improved-estimation-state-charge

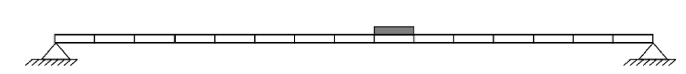
Optimal Actuator Location



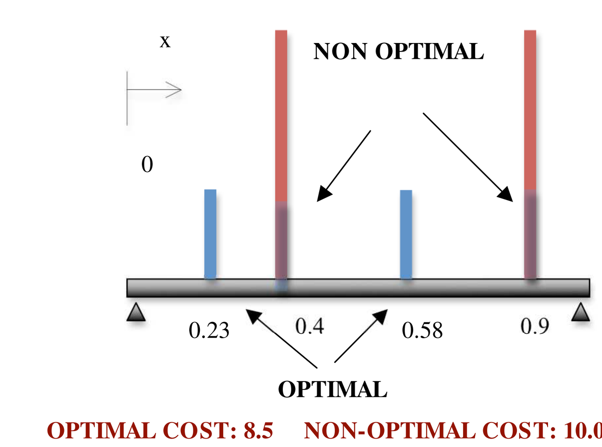
An actuator is a physical device which controls a given dynamical system. There is often freedom in choosing the location of actuators in systems governed by PDEs. The experimental data shown above for control of beam vibrations illustrates the fact the performance is strongly affected by actuator location. The actuator locations should be selected in order to optimize the performance criterion of interest. There are a number of theoretical and computational questions.

From a computational point of view, these optimization problems are generally nonconvex and the models for these systems often have a large number of degrees of freedom. Consequently, existing optimization schemes for optimal actuator placement may be inaccurate or computationally impractical. A subgradient-based optimization scheme for a linear quadratic cost, a popular design objective, was designed which leads to the global solution of the problem of finding optimal actuator locations. The optimization algorithm was applied to optimize the placement of piezoelectric actuators in vibration control of flexible structures. It is considerably faster and more accurate than the popular genetic algorithm. Experiments verified the efficacy of optimal actuator placement.

In many situations the control needs to attenuate the effect of disturbances. Both the controller and the actuator locations are chosen to minimize the effect of disturbances on the output of a full-information plant. For example, consider a beam of unit length fixed at both ends (depicted below) for which control of disturbances, such as vibrations, is desired. The actuator is represented by the grey block.



Where should the actuator(s) be placed in order to best reduce these disturbances? Often, the optimal location to place actuators do not agree with the intuitive location. For instance, if there are 2 locations of concentrated disturbances spread unsymmetrically (at 0.4 and 0.9) on a Kelvin-Voigt damped beam, an intuitive approach to solve the placement problem of 2 actuators is to collocate the actuators at the same location as the disturbances. However, our results demonstrate that the appropriate optimal locations are at 0.23 and 0.58. The error in the cost function when the actuators are placed at the disturbance locations is 15% with respect to the optimal location.



Current work is concerned with sensor location. The general problem is to determine the best locations for estimation despite noise and imperfect information. Since estimator design is mathematically dual to controller design, some of the results for actuator location can be used. A current project is to determine the best location to place a sensor that estimates the temperature of a large lake.

Future Research Opportunities

Opportunities for research in the analysis and control of infinite-dimensional systems (especially that of PDEs) are available. Please contact Professor Kirsten Morris for inquires (kmorris@uwaterloo.ca).

Acknowledgement

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Contributors

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