Cosmology with One-Point Stats

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Good Old Days

**CMB**: one snapshot linear, almost Gaussian captured by 2-point statistics
**Good Old Days -> Future**

**CMB:** one snapshot
linear, almost Gaussian

**LSS:** motion picture
nonlinear, non-Gaussian

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Euclid
TRADITIONAL STATISTICS

Gaussian: 2-pt correlation

$$\xi(r) = \langle \delta(x)\delta(x + r) \rangle$$

nonlinear $\rightarrow$ non-Gaussian

Higher N-pt correlations

$N \geq 3$ hard to measure
EFFICIENT STATISTICS

My approach: 1-point PDF
recapture lost information
measure counts in cells
& plot histogram
if we only could observe dark matter
**Counts-in-Cells Idea**

Recapture lost information
smooth & plot histogram

$\rho(\rho)@z = 0.7, R_{\text{same}}\variance^2 = h^2 (x_i)$

Counts-in-Cells Idea
Counts-in-Cells Idea

matter density in symmetric cells

symmetry statistics $\leftrightarrow$ dynamics
Large-deviation statistics

large deviations exponentially unlikely

\[ \mathcal{P}_{r}^{\text{ini}}(\delta_L) \sim \exp \left[ -\frac{\delta_L^2}{2\sigma_L^2(r)} \right] \]
Counts-in-Cells Theory

Large-deviation statistics

most likely path dominates

\[ P_{R,z}(\rho) \sim \exp \left[ -\frac{\delta_L(\rho)^2}{2\sigma_L^2(z, r(R, \rho))} \frac{\sigma_L^2}{\sigma_{NL}^2} \right] \]

Bernardeau 94
CU++ 16

spherical collapse

linear variance & growth

nonlinear variance
**Spherical Collapse**

**mass conservation**

\[ r = \rho^{1/3} R \]

large density  \( \delta_L \)  \( r \)  \( \rho \)  \( R \)

small density  \( \delta_L \)  \( r \)  \( \rho \)  \( R \)

fixed final radius \( R \) mixes scales!
Spherical Collapse

density mapping \[ \delta_L(\rho) \approx \frac{21}{13} \left(1 - \rho^{-\frac{13}{21}}\right) \]

\sim \text{cosmology independent}
accurate PDF from first principles, not lognormal

\( P(\rho) \), \( R=10 \text{ Mpc}/h \)

sims: Quijote
Counts-in-Cells Cosmo

matter content $\sigma_8$ & $\Omega_m$: width & tilt

$\mathcal{P}_R(\rho)$

$\Omega_m = 0.26$: $w_0 = -1$

$\Omega_m = 0.21$: $w_0 = -1$

$\Omega_m = 0.31$: $w_0 = -1$

+ change of growth $D(z)$

sims: J. Shin
Counts-in-Cells Cosmo

dark energy $w$: growth

$\mathcal{P}_R(\rho)$

- $\Omega_m=0.26: w_0=-1$
- $\Omega_m=0.26: w_0=-1.5$
- $\Omega_m=0.26: w_0=-0.5$

more expansion

less expansion

Sims: J. Shin

Codis ++ (incl CU) 16
massive neutrinos $M_\nu$: partial clustering

fixed $\sigma_8$

\[ \log_{10} \mathcal{P}(\ln \rho), \, R=10 \, \text{Mpc}/h \]

also here!

sims: Quijote
Fisher for Complements

**Quijote simulations**

F. Villaescusa-Navarro ++ (incl **CU**) 19

15,000x fiducial cosmo

500x derivative cosmo

$\sigma_8, \Omega_m, \Omega_b, h, n_s, M_\nu, (w_0)$

>1 million PDFs
Fisher for Complements

Fisher matrix

$$F_{ij} = \sum_{\alpha, \beta} \left[ \frac{\partial S_\alpha}{\partial \theta_i} C_{\alpha \beta}^{-1} \frac{\partial S_\beta}{\partial \theta_j} \right]$$

fiducial covariance

summary stats: PDF bins

derivatives w.r.t. cosmo

marginalised errors \( \delta \theta_i \geq \sqrt{(F^{-1})_{ii}} \)
width: clustering amplitude \( \sigma_8 \)

\[
\mathcal{P}_{\sigma_8^+}(\ln \rho) - \mathcal{P}_{\sigma_8^-}(\ln \rho), z=0
\]
Fisher for Complements

tilt: matter density $\Omega_m$ & initial $n_s$

$P_+(\ln \rho) - P_-(\ln \rho)$

$\Delta \Omega_m$, $\Delta n_s$, $\Delta h$, $\Delta \Omega_b$

- Full pred.
- $\sigma_{NL}$ only
Fisher for Complements

environment-dependence: $M_v$

$$\mathcal{P}_{M_v}(\ln \rho) - \mathcal{P}_{\text{fidZA}}(\ln \rho)$$

$z=0, R=10 \text{ Mpc}/h$

$$z=0, R=20 \text{ Mpc}/h$$

$$(\ln \rho - <\ln \rho>_{\text{fidZA}})/\sigma_{\ln \rho, \text{fidZA}}$$
**Fisher for Complements**

**Covariance: PDF bin correlation**

determined by density-dependent clustering

Correlation matrix of density PDF bins

- **diagonal**
- **extra clustered**
- **corners**
  - **anti-clustered**
Fisher for Complements

power spectrum workhorse

PDF underdog
$z=0, 0.5, 1$

$V_{tot} = 6 \, (\text{Gpc}/h)^3$

$P(k), k_{\text{max}} = 0.2 h/\text{Mpc}$

PDF, $R=10,15 \, \text{Mpc}/h$

PDF disentangles $M_\nu$ from $\sigma_8$
Planck base_mnu_plikHM_TT_lowl_lowE
z=0,0.5,1, \( V_{\text{tot}} = 6 \text{(Gpc/}h)^3 \):
\( P(k), k_{\text{max}} = 0.2h/\text{Mpc} \)
PDF, \( R = 10, 15, 20 \text{ Mpc}/h \)
joint \( P(k) + \text{PDF} \)

PDF complements power spectrum & Planck
Cosmo with Counts-In-Cells

Powerful statistics
non-Gaussian, beyond PT
robust & accurate predictions

Cosmology & fundamental physics
\( \Omega_m, \sigma_8, M_\nu \)  
CU, Friedrich ++ 19  
\( f_{\text{NL}} \)  Friedrich, CU ++ 19

Reality: no 3D matter field
weak lensing: projected matter
galaxy clustering: bias & stochasticity
Weak Lensing in Cells

statistics of projected matter
Weak Lensing

source

(dark)
matter

convergence $\kappa$

& shear $(\gamma_1, \gamma_2)$

$|\kappa|, |\gamma| \ll 1$

<0

$\kappa$

size

$\gamma_1$

shape

$\gamma_2$

>0
Weak Lensing

many sources @ $z_s$

see different parts of large-scale structure along the line of sight

Euclid makes map of galaxy shapes
Weak Lensing

reconstruct convergence map from shapes
2D maps at source redshift $z_s$
Weak Lensing-in-Cells

weighted matter density in slices

$$\delta_{\text{disk}} < \theta D(z) w(z, z_s)$$

cylindrical collapse  lensing weight
Cylindrical Collapse

\[ r = \rho^{1/2} R \]

\[ \delta_L = \frac{7}{5} \left( 1 - \rho^{-\frac{5}{7}} \right) \]

\[ \delta_L = \rho \rightarrow \rho \]

large density

small density

\[ \rho_{\text{cyl}} \]

\[ \delta_L \]

\[ r \]

\[ R \]

non-linear

linear
Weak Lensing-in-Cells

weighted matter density in slices

cumulant generator

$$\phi_{\theta D(z)}^{\text{disk}} \left( \lambda w(z, z_s) \right)$$

cylindrical collapse

lensing weight
**Weak Lensing-in-Cells**

convergence: weight density slices

\[ \kappa_{<\theta} = \int_{0}^{\mathcal{D}(z_s)} d\mathcal{D}(z) \delta^{\text{disk}}_{<\theta \mathcal{D}(z)} w(z, z_s) \]

scale mixing
**Weak Lensing-in-Cells**

convergence: weight density slices

cumulant generator

\[
\phi^K_{\theta}(\lambda) = \int_0^\mathcal{D}(z_s) d\mathcal{D}(z) \phi^\text{disk}_{\theta \mathcal{D}}(z) (\lambda w(z, z_s))
\]

\[\rightarrow \text{reconstruct PDF} \quad \text{Bernardeau & Valageas 02}\]
Weak Lensing-in-Cells

Barthelemy, Codis, CU++ 19

$z_s = 1.5$

$P(\kappa)$

$\kappa$

- $\theta = 10$ arcmin
- $\theta = 20$ arcmin
- $\theta = 30$ arcmin
- $\theta = 40$ arcmin
- $\theta = 50$ arcmin

NO NULLING
Weak Lensing-in-Cells

Barthelemy, Codis, **CU++ 19**

\[ \frac{P_{\text{sim}}(\kappa)}{P_{\text{th}}(\kappa)} - 1 \]

\[ \kappa / \sigma \]

**NO NULLING**
Lensing Convergence PDF

single source redshift: $z_s=2$

simulations: DUSTGRAIN-pathfinder

Boyle, CU ++ in prep.
Lensing Convergence PDF

percent-level accuracy around the peak

Boyle, CU ++ in prep.
Lensing Convergence PDF

validated response to cosmo parameters

$\theta = 10.25'$

preliminary

simulations: DUSTGRAIN-cosmo

Boyle, CU ++ in prep.
$z_s = 2, V = 15000 \text{ deg}^2$

$\theta = 7.32', \theta = 10.25'$

Planck TT, TT, EE

lowl lowE

PDF 2 scales combined

2pcf $\theta_{min} = 5'$

complements

2pt correlation

& CMB

Euclid project

higher-order

WL statistics

Boyle, CU ++ in prep.
Cosmo with One-Point Stats

Powerful statistics
non-Gaussian, beyond PT
robust & accurate predictions
different density environments

Matter density PDF
\( \Omega_m, \sigma_8, M_v, w_0, f_{NL} \)  
CU, Friedrich ++ 19, Friedrich, CU ++ 19

Weak lensing convergence PDF
probes projected matter  
Barthelemy ++ (incl. CU) 20
\( \Omega_m, \sigma_8, w_0, M_v \)  
Boyle, CU ++ in prep