1/f$^{\alpha}$-charge-noise-robust voltage control of semiconductor quantum dot spin qubits using fractional calculus

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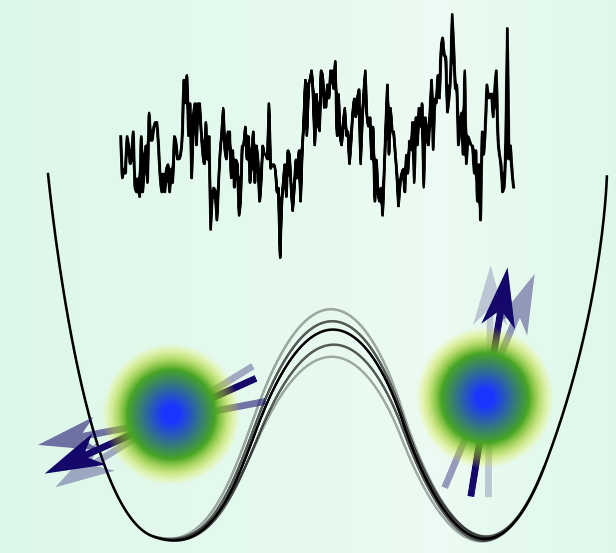
Long, low-amplitude, and **broad** exchange pulses, shaped according to the beta distribution function B(1−α/2,1−α/2), realize the SWAP^k gates least sensitive to stationary 1/f$^{\alpha}$ charge noise

➥ **strong exchange** interaction **may not be necessary** for the optimal performance of spin qubit processors

➥ this could **ease fabrication requirements**



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Motivation

Semiconductor quantum dot spin qubits:

✔ Precise voltage control of localized few-electron spin-orbital states

✘ Performance significantly hindered by low-frequency $1/f^{\alpha}$ charge noise with $0 < \alpha < 2$

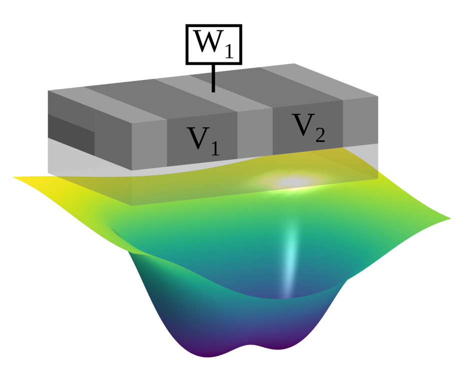


Image: Zach D. Merino

Goal

Develop a new method based on of fractional calculus [1] to design quantum gates least sensitive to $1/f^{\alpha}$ noise

Control problem

System

2 silicon quantum dot spin qubits with exchange interaction: $H(t) = \frac{J(t)}{4}\vec{\sigma}_1 \cdot \vec{\sigma}_2$

♦ charge noise is the dominant decoherence mechanism

Control

Nonlinear control of exchange with voltage (tunneling electrode / bias): $J(V) \approx J(V_0)\exp[\varkappa(V - V_0)]$

⇒ Exponential amplification of noise $\widetilde{v}(t)$ in voltage controls & environment: $J(V + \widetilde{v}) = J(V)\exp(\varkappa\widetilde{v})$

Quantum gate

2-qubit **SWAP^k**: $J(t) = \pi k \hbar S(t)/T$

for some normalized shape function $S(t)$:

$$\langle S(t)\rangle = \int_0^1 S(\tau)\,\mathrm{d}\tau = 1, \qquad \tau = t/T$$

Performance metric

Unitary infidelity: $\mathscr{F} = 1 - \frac{1}{4}\left|\mathrm{tr}\, U^{\dagger}[V]U[V+\widetilde{v}]\right|$

Average infidelity of a SWAP^k operation:

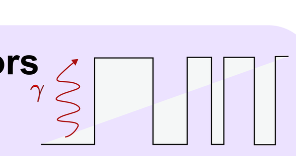
$$\langle \mathscr{F}\rangle \approx \frac{3\pi^2}{32}k^2\varkappa^2 \int_0^T \frac{\mathrm{d}t_1}{T}\int_0^T \frac{\mathrm{d}t_2}{T} S(t_1)S(t_2)R(t_1,t_2),$$

is determined by noise autocorrelation: $R(t_1,t_2) = \langle\widetilde{v}(t_1)\widetilde{v}(t_2)\rangle$

1/f$^{\alpha}$ noise models

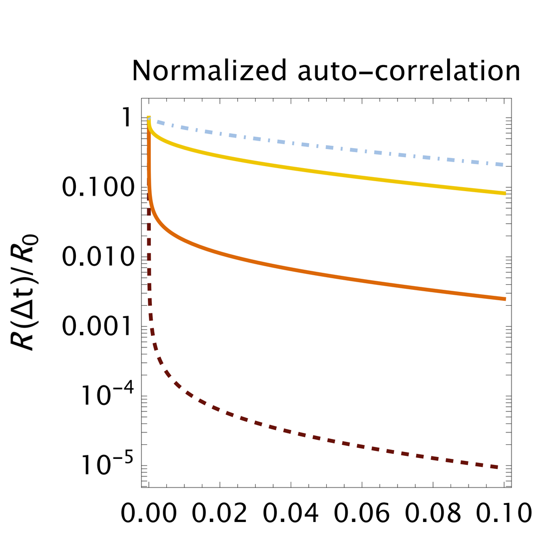
Stationary

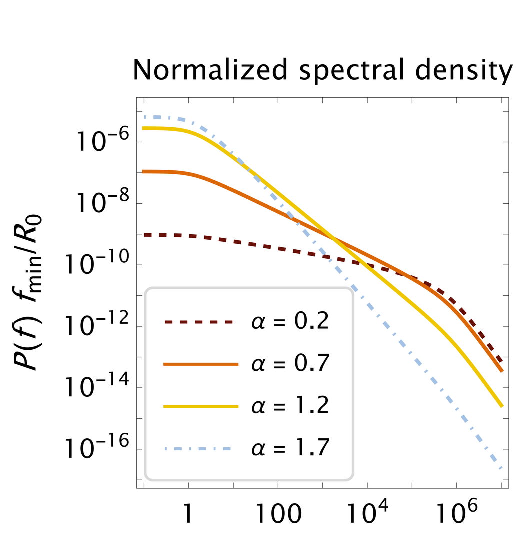
Ensemble of **two-level fluctuators** (TLFs): distributed charge traps



Power-law distribution of fluctuator energies per switching rate γ gives spectrum $\propto 1/f^{\alpha}$ in the bulk, with autocorrelation

$$R(t_1,t_2) \propto f_{\min}^{1-\alpha}E_{\alpha}(2\pi f_{\min}|t_1-t_2|) - f_{\max}^{1-\alpha}E_{\alpha}(2\pi f_{\max}|t_1-t_2|)$$





Nonstationary

Fractional Brownian motion (fBm): fractional charge transport due to disorder (high charge trap density)

$$R_{\mathrm{FBM}}(\tau_1,\tau_2) \propto \tau_1^{\alpha-1} + \tau_2^{\alpha-1} - |\tau_1-\tau_2|^{\alpha-1} \qquad (1 < \alpha < 2)$$

Methods

1 Fractional calculus

Generalization of integration/ differentiation to fractional orders

Motivation: $1/f^{\alpha}$ spectral behavior is intermediate between white noise $w(t)$ and Brownian motion $B(t)$: $B(t) = \int_0^t w(s)\,\mathrm{d}s$

Riemann-Liouville fractional integrals:

$$\left({}_aI^{\beta}\varphi\right)(x) = \frac{1}{\Gamma(\beta)}\int_a^x (x-s)^{\beta-1}\varphi(s)\,\mathrm{d}s\,, \quad \left(I_b^{\beta}\varphi\right)(x) = \frac{1}{\Gamma(\beta)}\int_x^b (s-x)^{\beta-1}\varphi(s)\,\mathrm{d}s$$

Riemann-Liouville fractional derivatives are inverse operators:

$${}_aD^{\beta}\varphi = (\mathrm{d}/\mathrm{d}x)^{[\beta]+1}\,{}_aI^{1-\{\beta\}}\varphi, \quad D_b^{\beta}\varphi = (-\mathrm{d}/\mathrm{d}x)^{[\beta]+1}I_b^{1-\{\beta\}}\varphi$$

Left- and right-sided operators are conjugate w.r.t. inner product:

$$\left(\varphi,\ {}_aI^{\beta}\psi\right) = \left(I_b^{\beta}\varphi,\ \psi\right), \qquad \left({}_aI^{\beta}\right)^* = I_b^{\beta}$$

2 Variational minimization

Lagrangian: average infidelity + normalization constraint on exchange pulse shape

$$\Lambda[S] = (S,\hat{R}S) - \lambda(1,S) \to \min_{S(t)}$$

Operator form of autocorrelation determines the existence and functional form of the minimizer!

Optimal shape: derivation for $0 < \alpha < 1$

For short SWAP^k pulses compared to the timescale of the experiment $\theta = 2\pi f_{\min}T \sim T/T_{\mathrm{exp}} \ll 1$:

$$R(\tau_1,\tau_2) \propto \frac{\theta^{\alpha-1}\Gamma(2-\alpha)}{|\tau_1-\tau_2|^{1-\alpha}} - 1, \quad \tau_{1,2} \in [0,1]$$

From [2], $|\tau_1-\tau_2|^{\alpha-1} = \frac{\pi\int_0^1 \mathrm{d}s K(\tau_1,s)K(\tau_2,s)\mathrm{d}s}{\Gamma(1-\alpha)\sin\frac{\pi\alpha}{2}} \leftrightarrow \frac{\pi}{\Gamma(1-\alpha)\sin\frac{\pi\alpha}{2}}\hat{K}\hat{K}^*$

$K(\tau,s) = \frac{\tau^{\alpha/2}}{\Gamma(\alpha/2)}(\tau-s)^{\alpha/2-1}\,s^{-\alpha/2}\,\mathbb{I}_{[0,\tau]}(s), \qquad \hat{K} = \tau^{\alpha/2}\,{}_0I^{\alpha/2}\tau^{-\alpha/2}$

This yields a quadratic Lagrangian for $M = \hat{K}^*S$:

$$\Lambda[M] \propto \left[(M,M) - \frac{\theta^{1-\alpha}\sin\frac{\pi\alpha}{2}}{\pi(1-\alpha)}\right] - \lambda(\hat{K}^{-1}1, M)$$

$\delta\Lambda = 0$ yields: $M_{\mathrm{opt}}(\tau) \propto \tau^{\alpha/2}\,{}_0D^{\alpha/2}\left(\tau^{-\alpha/2}\right) \propto \tau^{-\alpha/2}$

Optimal exchange pulse shape follows from $S = \tau^{-\alpha/2}D_1^{\alpha/2}\tau^{\alpha/2}M$ and the normalization $(1,S) = 1$:

Optimal exchange pulse shape

Stationary ensemble of TLFs

$$S_{\mathrm{opt}}(\tau) = \frac{\tau^{-\alpha/2}(1-\tau)^{-\alpha/2}}{B(1-\frac{\alpha}{2},1-\frac{\alpha}{2})}$$

Non-stationary FBM

$$S_{\mathrm{opt}}(\tau) = \delta(\tau - 0^+)$$

Results

Stationary noise

Optimal voltage pulse shape [Fig. (a)] is long, low-amplitude & broad:

$$V_{\mathrm{opt}}(t) = C(V_0,\alpha) - \frac{\alpha}{2\varkappa}\ln\left[\frac{t}{T}\left(1-\frac{t}{T}\right)\right]$$

$\langle\mathscr{F}\rangle(T)$: Average infidelity **decreases with pulse length** [Fig. (b)] for all noise colors $\alpha \in (0,2)$

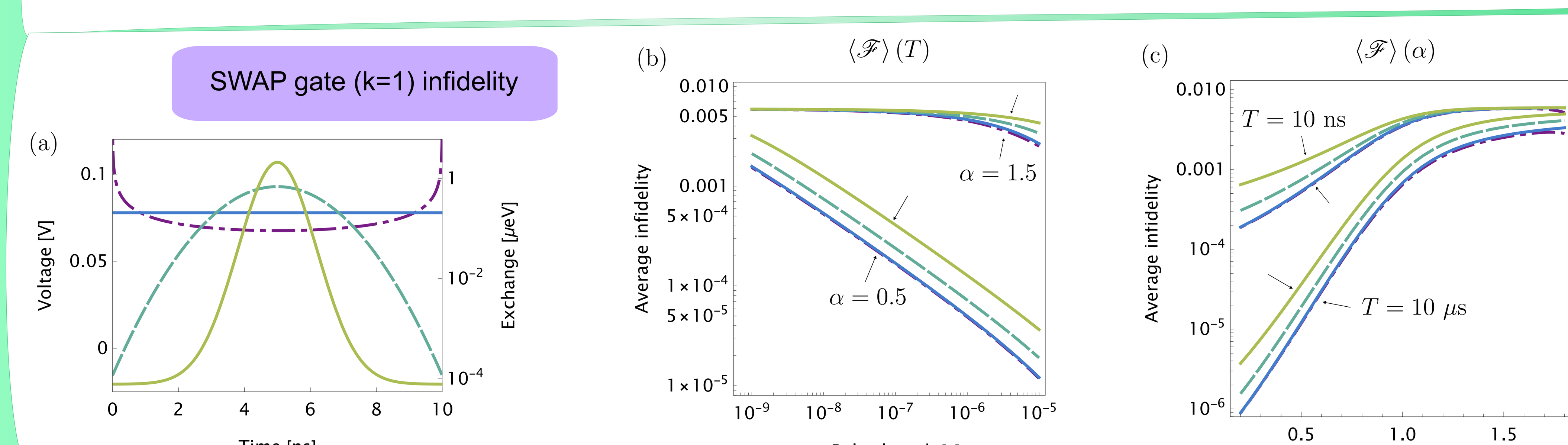
♦ Decrease is fastest in weakly-correlated noise environments (with small α values): -5.2 dB/dec for $\alpha = 0.5$

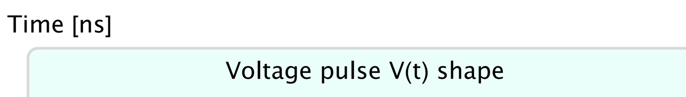
$\langle\mathscr{F}\rangle(\alpha)$: Noises with smaller α values [Fig. (c)] give:

✔ lower infidelity

✔biggest advantage of using optimal shape: ~4× drop in infidelity vs. a Gaussian voltage pulse at $\alpha = 0.2$

SWAP gate (k=1) infidelity





Device-specific parameters [3]: $V_0 = 0.04$ V, $J(V_0) = 0.01\ \mu$eV, $\varkappa = 80\ \mathrm{V}^{-1}$

Nonstationary fBm

♦ Optimal pulse: as short & localized as possible ♦ Slow growth of infidelity with pulse length: $\langle\mathscr{F}\rangle \propto T^{\alpha-1}$

Discussion: experimental implications

→ Strong exchange coupling regimes may not be necessary for high fidelity

→ This might ease requirements on fabrication by simplifying device layouts

→ With the presence of spin decoherence, we expect a **sweet spot** in pulse length T

→ Our strategy is anticipated to become **increasingly more useful** as T_2 values rise thanks to materials and fabrication improvements

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References

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[2] W. E. Williams, *A class of integral equations*, Mathematical Proceedings of the Cambridge Philosophical Society, Mathematical Proceedings of the Cambridge Philosophical Society 59, 589 (1963).

[3] B. Buonacorsi, M. Korkusinski, B. Khromets, and J. Baugh, Optimizing lateral quantum dot geometries for reduced exchange noise, arXiv:2012.10512 (2020).