Low-Complexity Multiuser Detection in Millimeter-Wave Systems Based on Opportunistic Hybrid Beamforming

Quan Yu, Chao Han, Lin Bai, Jinho Choi,
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***Abstract*—In this paper, we study the multiuser detection in a millimeter-wave (mm-wave) wireless communication system using hybrid beamformers, where uplink transmissions are considered under limited scattering. For a base station, with the condition that the number of radio-frequency (RF) chains is relatively smaller than that of users, we consider opportunistic beamforming with analog beams for multiple-signal detection and derive a closed form expression for system sum rate. By applying the minimum mean square error (MMSE)-filter with the proposed opportunistic hybrid beamforming algorithm, the system performance can be further improved. Through simulation results, it is shown that the proposed MMSE-based multiuser detection approaches are attractive for hybrid mm-wave systems to support a large number of users with a small number of RF chains.**

***Index Terms*—Hybrid beamforming, millimeter-wave (mm-wave), opportunistic beamforming (OBF), multiuser detection, minimum mean square error (MMSE).**

I. Introduction

As the demand of the high-speed data transmission services grows, millimeter-wave (mm-wave) communications have attracted a lot of attention due to a large available bandwidth. Since the path loss of mm-wave is relatively high [1]–[3], it is expected to employ large-scale antennas for beamforming at a base station (BS).

Beamforming is widely employed to compensate for the high path loss and mitigate multiuser interferences. For mm-wave systems, fortunately, due to the high carrier frequency, a number of antenna elements can be deployed within a much small space which enables the application of large-scale antennas. However, considering high cost and significant power consumption, it would be impractical to implement full digital beamforming (DBF) even at BSs [2]. Thus, hybrid beamforming using an analog beamformer and a partial digital beamformer is proposed [2], [4], [5].

In [3]–[7], different hybrid beamforming algorithms have been studied. In [4], the proposed solution is not suitable to the single-antenna multiuser system where little computing power can be provided by users. The authors in [3], [5]–[7] proposed to find the optimal analog combiner ($\mathbf{W}_1$) and baseband combiner ($\mathbf{W}_2$) based on the optimization function (i.e., $\arg\min_{\mathbf{W}_1,\mathbf{W}_2}\|\mathbf{F}_{\mathrm{DBF}} - \mathbf{W}_1\mathbf{W}_2\|^2$, where $\mathbf{F}_{\mathrm{DBF}}$ is the optimal DBF solution). Although finite-resolution phase shifts are employed in the system [3], [6], the efficiency of the designed iterative matching methods for the function above is still limited. Therefore, to improve the efficiency of the hybrid beamforming (HBF) further, we start to study low-complexity methods for the mm-wave system with a number of users. In [8], a low-complexity multiuser detection system was designed where the BS is only equipped with analog beamformers (i.e., one radio-frequency (RF) chain is mounted), and the detection performance was effectively improved by searching the optimal analog beams exhaustively from a predetermined codebook. Since the exhaustive search method is time-consuming for the system with high-precision codebook, to find an acceptable trade-off between the performance and complexity, we consider the opportunistic beamforming (OBF) algorithm based on the approach in [9]. Assuming that the number the RF chains is much less than that of users to be served, this opportunistic approach can offer a near-optimal performance since a multiuser gain can be obtained.

In this paper, we develop the OBF strategy and propose two OHBF-based multiuser detection methods for the uplink transmission of mm-wave communication systems. Using a predetermined codebook with analog beams, the BS can randomly select a certain number of users for simultaneous transmissions. Based on this strategy, we employ multiple training slots and select the best user group using a signal-to-interference-plus-noise ratio (SINR)-based user selection method. After that, we combine the OHBF approach with a minimum mean square error (MMSE) filter, which can further improve the detection performance. In addition, since the signal detection performance is directly related to the equivalent channel matrix, then, under the circumstance where the communication quality is a priority, another low-complexity eigenvalue-based multiuser selection method is also designed for the system. From simulation results, we can find that the proposed approaches can achieve a reasonable trade-off between the performance and computational complexity.

The main contribution of this paper is the proposal of the algorithms which provide a near-optimal performance with low complexity for the uplink mm-wave system. Besides, to analyze the performance of the system, we derive an approximate closed form expression for the achievable rate of the system. With the closed form expression, we are able to analyze the system performance without using numerical simulations which involve significant time consumption.

Notations: Let $\mathbf{A}$ denote a matrix and $\mathbf{a}$ represent a vector. $\mathbf{A}^{\mathrm{T}}$ denotes the transpose of $\mathbf{A}$ while $\mathbf{A}^{\mathrm{H}}$ denotes the Hermitian transpose of $\mathbf{A}$. $\|\mathbf{A}\|$ represents the Frobenius norm of $\mathbf{A}$ while $|\mathbf{A}|$ denotes the determinant of $\mathbf{A}$. The notation $\mathrm{E}[\cdot]$ represents the mathematical

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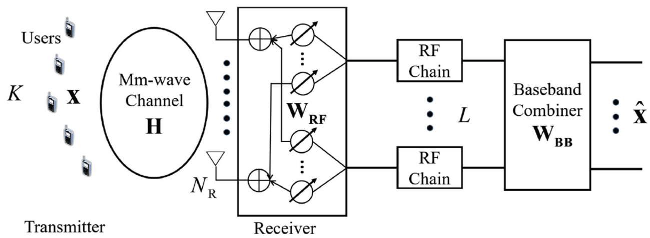

Fig. 1. An mm-wave system employing hybrid beamformers.

expectation and $\mathrm{Var}(\cdot)$ denotes the mathematics variance. $\mathbf{I}_N$ is the $N \times N$ identity matrix. The distribution of circularly symmetric complex Gaussian random vectors is denoted by $\mathcal{CN}(\mathbf{a}, \mathbf{R})$ where $\mathbf{a}$ is the mean and $\mathbf{R}$ is the covariance matrix. $\backslash$ denotes the set minus, vector vecd($\mathbf{A}$) is comprised of the diagonal entries of $\mathbf{A}$ where $\mathrm{diag}(a)$ is a diagonal matrix. For set $\mathcal{C}$, $|\mathcal{C}|$ represents the cardinality of $\mathcal{C}$.

II. System Model

In this paper, inspired by [8], a multiuser system that consists of a BS and K single-antenna users (i.e., no beamforming is involved at the user side) is considered which is illustrated in Fig. 1. At the BS, an $N_R \times 1$ dimensional antenna array is employed which consists of L RF chains, where $L \leq N_R$, for hybrid beamforming. Denote by $\mathbf{W}_{\mathrm{BB}} \in \mathbb{C}^{L \times K}$ the baseband combiner with L RF chains and by $\mathbf{W}_{\mathrm{RF}} \in \mathbb{C}^{N_R \times L}$ the RF combiner with the RF phase shifting network.

Let $\mathbf{x} = [x_1, \ldots, x_K]^{\mathrm{T}}$ be the transmitted signal vector from K users and $\mathbf{H} = [\mathbf{h}_1, \ldots, \mathbf{h}_K]$ be the uplink (narrow-band) channel matrix [2], where $\mathbf{h}_k \in \mathbb{C}^{N_R \times 1}$ is the channel vector from user k to the BS. Then, the received signal is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I})$ denotes the background noise. Throughout the paper, we assume that $x_k \in \mathcal{A}$ where $\mathcal{A}$ denotes the signal constellation. As in [2], the output signal of the hybrid beamformer is given by

$$\hat{\mathbf{x}} = [\hat{x}_1, \ldots, \hat{x}_K]^{\mathrm{T}} = \mathbf{W}_{\mathrm{BB}}^{\mathrm{H}} \mathbf{W}_{\mathrm{RF}}^{\mathrm{H}} \mathbf{H}\mathbf{x} + \mathbf{W}_{\mathrm{BB}}^{\mathrm{H}} \mathbf{W}_{\mathrm{RF}}^{\mathrm{H}} \mathbf{n}. \tag{2}$$

It is noteworthy that $\mathbf{W}_{\mathrm{RF}}$ and $\mathbf{W}_{\mathrm{BB}}$ can be obtained by using different matching pursuit approaches [2], [4], which indicates that a high computational complexity will be caused by simultaneous access requirements from different users. Therefore, it is desirable to design an efficient approach for the multiuser detection, which will be discussed in Section III.

Throughout the paper, a poor scattering environment is considered and the channel from the k-th user to the BS is given by [8]

$$\mathbf{h}_k = \sum_{p=1}^{P} \alpha_{k,p} \mathbf{a}(\theta_{k,p}) \tag{3}$$

where P is the number of paths. As in [8], under limited scattering environments, P is usually small ($\leq$3). In (3), $\alpha_{k,p}$ represents the channel coefficient while the angle of arrival (AoA) of the p-th path is denoted by $\theta_{k,p}$. For the system, the BS is equipped with a uniform linear array (ULA) and the array response vector (ARV) of the ULA is denoted by $\mathbf{a}(\theta_{k,p}) \in \mathbb{C}^{N_R \times 1}$ which is represented as [10]

$$\mathbf{a}(\theta) = \left[1, e^{-j\frac{2\pi}{\lambda} d \sin(\theta)}, \ldots, e^{-j\frac{2\pi}{\lambda}(N_R - 1) d \sin(\theta)}\right]. \tag{4}$$

Here, λ denotes the wavelength and d represents the antenna spacing in this paper.

In the uplink transmission of mm-wave systems, we need the channel state information (CSI) for detection, which can be estimated by uplink training. However, due to a large number of antennas in mm-wave system, the complexity of the channel estimation is high. Fortunately, since some low-complexity channel estimation methods are well-studied in the literature on mm-wave systems [8], [10], it is possible for the BS to estimate the CSI with low complexity. Here, throughout the paper, we suppose that $\mathbf{H}$ is perfectly known at the BS.

III. Signal Detection Based on the Proposed OHBF Algorithm

For mm-wave communication systems, the complexity in designing $\mathbf{W}_{\mathrm{RF}}$ and $\mathbf{W}_{\mathrm{BB}}$ is relatively high, especially when N_R is large. Thus, in this section, we propose two OBF-based methods where multiple training slots are employed and a certain user group can be selected with an SINR-based or a max-min eigenvalue (ME)-based user selection criterion. Then, an MMSE-based filter is considered to improve the detection performance.

A. Signal Detection With the OHBF Approach

Since DBF is carried out by adjusting the amplitudes and phases of the signals received at each antenna to form the desired beam, designing an exact combiner for a large scale antenna system is time-consuming. Thus, OBF, where random analog vectors from a predetermined analog codebook are chosen by analog beamformers at the BS during different training slots [9], was designed to offer a reasonable trade-off between the performance and complexity. Therefore, we apply the OBF strategy in this paper. The codebook is denoted by $\mathcal{C} = \{\mathbf{c}_1, \ldots, \mathbf{c}_{|\mathcal{C}|}\}$ where $|\mathcal{C}| = N$. Here, we assume $\mathbf{c}_n = \mathbf{a}(\theta_{(n)})$, where the $\sin(\theta_{(n)})$'s are uniformly distributed and the quantized AoA $\theta_{(n)}$ is the nth element in $\mathcal{G} = \{\theta_g, \theta_g \in [-\pi/2, \pi/2], g = 1, \ldots, G\}$ [10]. In this paper, we first consider the OHBF where the baseband combiner is merely a beam selector (i.e., $\mathbf{W}_{\mathrm{BB}} = \mathbf{I}$). During time slot S_{ts}, $\mathbf{W}_{\mathrm{BB}} = \mathbf{I}$, the output of the hybrid beamformer can be represented as

$$\mathbf{r}_{S_{ts}} = [r_{S_{ts},1}, \ldots, r_{S_{ts},L}]^{\mathrm{H}} = \mathbf{W}_{\mathrm{RF},S_{ts}}^{\mathrm{H}} \mathbf{y} \in \mathbb{C}^{L \times 1}, \tag{5}$$

where $i \leq T_{\mathrm{scan}}$ is the stage index of beam training time slots at the BS.

During different training slots, we assume that the BS can choose the optimal $\mathbf{W}_{\mathrm{RF}} \in \mathcal{C}$ which maximizes the following SINR-based criterion:

$$\xi(\mathbf{W}_{\mathrm{RF}}) = \prod_{\substack{\mathbf{w}_{\mathrm{RF},i} \in \mathbf{W}_{\mathrm{RF}} \\ i=1,\ldots,L}} \frac{\max_{1 \leq m \leq K} \left|\mathbf{w}_{\mathrm{RF},i}^{\mathrm{H}} \mathbf{h}_m\right|^2}{\sum_{n \neq m} \left|\mathbf{w}_{\mathrm{RF},i}^{\mathrm{H}} \mathbf{h}_n\right|^2 + N_R N_0}, \tag{6}$$

where $\mathbf{w}_{\mathrm{RF},i}$ is the i-th column of $\mathbf{W}_{\mathrm{RF}}$. First, the analog beamforming vector, which maximizes the criterion as $\tilde{\mathbf{W}}_{\mathrm{RF}} = \arg\max_{\mathbf{W}_{\mathrm{RF}} \in \mathcal{C}} \xi(\mathbf{W}_{\mathrm{RF}})$, will be chosen by the BS. Then, from the outputs of the hybrid beamformer, the detected signals transmitted from user m are found as

$$\tilde{x}_m = \mathrm{Dec}_{\mathcal{A}}\left(\tilde{\mathbf{w}}_{\mathrm{RF},i}^{\mathrm{H}} \mathbf{y}\right) \tag{7}$$

where $\mathrm{Dec}_{\mathcal{A}}(x) = \arg\min_{s \in \mathcal{A}} |x - s|^2$ and $\mathcal{A}$ is the signal constellation.

Algorithm 1: The Proposed OHBF Algorithm.

Input: L, K, T_{scan}, τ, η, $\mathbf{H}$, $\mathcal{C}$.
Output: The RF combiner $\tilde{\mathbf{W}}_{\text{RF}}$, indices of the detected users $\mathcal{M}$.
First stage: RF combiner selection.
Initialization: $maxSINR = 0$.
for $t = 1, \cdots, T_{\text{scan}}$ **do**
  $\hat{\xi} = \xi(\mathbf{W}_{\text{RF},t})$; $\mathbf{W}_{\text{RF},t} \in \mathcal{C}$.
  if $\hat{\xi} > maxSINR$ **then**
    $\tilde{\mathbf{W}}_{\text{RF}} = \mathbf{W}_{\text{RF},t}$; $maxSINR = \hat{\xi}$.
  end
end
Second stage: multiuser selection.
Initialization: $\mathcal{M} = \{1, \cdots, K\}$; $i = 1$.
for $l = 1, \cdots, L$ **do**
  $\{\mathcal{I}_l, \mathcal{U}_l\} = \text{sorting}(\text{vecd}((\tilde{\mathbf{w}}_{\text{RF},l}^{\text{H}}\mathbf{H})^{\text{H}}(\tilde{\mathbf{w}}_{\text{RF},l}^{\text{H}}\mathbf{H})))$.
end
$\{\hat{\mathcal{I}}, \hat{\mathcal{F}}\} = \text{sorting}([\mathcal{I}_1(1), \cdots, \mathcal{I}_L(1)])$.
while $i < L$ **do**
  for $j \in \{2, \cdots, |\mathcal{M}|\}$ **do**
    if $\frac{\mathcal{I}_{\hat{\mathcal{F}}(i)}(j)}{\mathcal{I}_{\hat{\mathcal{F}}(i)}(1)} > \tau$ **then**
      $\mathcal{M} = \mathcal{M} \setminus j$.
    end
  end
  $i = i + 1$; **Update:** $\hat{\mathcal{I}}$, $\hat{\mathcal{F}}$.
  if $\hat{\mathcal{I}}_{\hat{\mathcal{F}}(i)}(1) < \eta$ **then**
    $i = i + 1$.
  end
end

The proposed OHBF algorithm for multiuser detection is summarized in Algorithm 1. First, we choose a certain RF combiner $\mathbf{W}_{\text{RF},t} \in \mathcal{C}$ to maximize $\xi(\mathbf{W}_{\text{RF},t})$. Then, the users, which are in the directions of different analog beams at the BS, will be selected for the detection. Finally, the BS will communicate with the users and feedback them of the decision. As in [11], when the order of the signals to be detected is taken into consideration, the system performance will be further improved. Thus, in Algorithm 1, we consider the following definition.

Definition 1: Denote by $\text{sorting}(\mathcal{P})$ the function which reorders the elements in $\mathcal{P}$ in the descending order. For example, if $\{\hat{\mathcal{I}}, \hat{\mathcal{F}}\} = \text{sorting}(\mathcal{P})$, we have $\hat{\mathcal{I}} = \mathcal{P}(\hat{\mathcal{F}})$ and $\hat{\mathcal{I}}(1) \geq \hat{\mathcal{I}}(2) \cdots \geq \hat{\mathcal{I}}(|\mathcal{P}|)$, where $\mathcal{F}$ is the index set of $\mathcal{P}$.

Note that the correlation between the channel matrix $\mathbf{H}$ and the analog combiner $\mathbf{W}_{\text{RF}}$ is closely related to the system performance. To reduce the multiuser interference effectively, here, we propose to utilize thresholds τ and η to simply seek a suboptimal user group. For example, if $\hat{\mathcal{I}}_i(1) < \eta$, the i-th RF chain will not be considered for signal detection. Furthermore, we will not choose user j if $\frac{\mathcal{I}_{\hat{\mathcal{F}}(i)}(j)}{\mathcal{I}_{\hat{\mathcal{F}}(i)}(1)} > \tau$. In the second stage, when K is large enough, the optimal user group whose channel vectors are orthogonal to each other will be selected to communicate with the BS.

B. The OHBF-ME Method and MMSE Detection

Considering the circumstance where the communication quality is of high priority, we also propose another two-stage multiuser selection method to improve the system detection performance. In the first stage, let the set of the desirable users be

$$\mathcal{K} = \{k \in \mathbb{Z}^+ \mid \bar{\text{V}}(k) \geq \text{E}_{i \in \mathbb{Z}^+; i \in [1,K]}[\bar{\text{V}}(i)]\} \tag{8}$$

where $\mathbb{Z}^+$ denotes the positive integer domain, $\bar{\text{V}}(i) = \text{Var}(\text{diag}(\mathbf{h}_i^{\text{H}}\mathbf{W}_{\text{RF}})\mathbf{W}_{\text{RF}}^{\text{H}}\mathbf{h}_i)\}$. Then, in the second stage, we choose the optimal users for communication based on the ME multiuser selection criterion [11]. Note that $\lambda_0 = \sqrt{\lambda_{\min}(\mathbf{H}^{\text{H}}\mathbf{H})} \geq 5$ can guarantee the communication quality (pairwise error probability (PEP) $\leq 10^{-5}$). Here, the proposed method above is referred as the OHBF-ME approach.

For the OHBF and OHBF-ME methods, $\mathbf{W}_{\text{BB}} = \mathbf{I}$, here, we propose to utilize the MMSE filter to calculate $\mathbf{W}_{\text{BB}}$ according to the CSI and $\mathbf{W}_{\text{RF}}$.[1] The detail is omitted and in-depth description can be found in [14, Section II] or [11, Section 2.3].

IV. Performance Analysis of the Proposed OHBF Scheme

In this section, we focus on deriving a closed form expression to calculate the achievable system sum rate of the proposed OHBF algorithm. Since the exact probability density function (pdf) of the random variable (RV) $R_m = |\mathbf{w}_{\text{RF}}^{\text{H}}\mathbf{h}_m|^2$ is unknown, a close-formed expression for the pdf of R_m needs to be derived first. Then, we study the expression for approximately estimating the uplink sum rate. Moreover, the computational complexity of the proposed approach is also analyzed.

A. Sum Rate of the Uplink Communication

Inspired by [9], the exact pdf of R_m can be approximated through matching the first raw moments of R_m. Specifically, a tractable formula approximating $f_{R_m(x)}$ is given by [9]

$$f_{R_m}(x) \approx \frac{(u/2/\mathcal{R})^{\frac{u}{2}}}{\Gamma(u/2)} \left(x^{\frac{u}{2}-1} e^{-\frac{u}{2\mathcal{R}}x}\right)\left(v_2 x^2 + v_1 x + v_0\right), \tag{9}$$

where the distribution of R_m is a corrected version of the χ^2 distribution. Here, u is the degree of freedom, which is given by $u = 2\lfloor \mathcal{T} + \frac{1}{2} \rfloor$, and $\Gamma(x)$ denotes the Gamma function. The gain $\mathcal{R}$ and the fading figure $\mathcal{T}$ are defined as

$$\mathcal{R} = \text{E}[R_m] \text{ and } \mathcal{T} = \mathcal{R}^2 / \left(\text{E}[R_m^2] - \mathcal{R}^2\right), \tag{10}$$

respectively. This approximation has been proved to be reasonable (see [9, Appendix I]).

Lemma 1: Denote by $F_{R_m}(x)$ the cumulative distribution function (CDF) of R_m. Then, we have [12], [13]

$$F_{R_m}(x) = \frac{(u/2\mathcal{R})^{u/2}}{\text{C}_1 \Gamma(u/2)} \big[(-2\mathcal{R}/u)\big(v_2 x + (2\mathcal{R}/u)v_2 + v_1\big)$$
$$(x^{\frac{u}{2}} e^{\frac{-u}{2\mathcal{R}}x})(v_0 - (2\mathcal{R}/u)((2\mathcal{R}/u)v_2 + v_1))\text{I}_0(x)\big] \tag{11}$$

where C_1 denotes the normalization factor to ensure $F_{R_m}(\infty) = 1$. Moreover, $\text{C}_1 = v_2(\frac{u+2}{u})\mathcal{R}^2 + v_1\mathcal{R} + v_0$, $\text{I}_0(x) = \gamma(\frac{u}{2}, \frac{u}{2\mathcal{R}}x) / (\frac{u}{2\mathcal{R}})^{u/2}$ and $\gamma(\alpha, x)$ denotes the incomplete gamma function.

Proof: Since the pdf of R_m is given in (9), (11) can be derived based on the integral theorem. ■

For the channel vector in (3), we assume that one path is the line-of-sight (LoS) path while the others are non-LoS (NLoS) paths. In this paper, we set $\alpha_{k,1} = 1$ for the LoS path and $\alpha_{k,p} \sim \mathcal{CN}(0,1)$ for

[1] Note that the HBF methods become the OHBF-MMSE and OHBF-ME-MMSE methods.

NLoS paths. Applying the expectation operator in (10) and we have (the derivation is shown in Appendix A)

$$\mathcal{R} = 3N_R^2,\ \mathrm{E}\left[\left|\mathbf{w}_{\mathrm{RF}}^{\mathrm{H}}\mathbf{h}_m\right|^4\right] = 11N_R^4 + 12N_R^3, \tag{12}$$

$$\mathcal{T} = 9N_R/(2N_R + 12). \tag{13}$$

Then, the sum rate can be calculated as (the derivation is given in Appendix B)

$$RS \approx \hat{L}\log_2\left(1 + \sigma - N_0 N_R \sigma^2 \mathrm{W}\right) \tag{14}$$

where $\hat{L}$ denotes the number of the selected RF chains. In addition, we have $\sigma = \frac{\sqrt{2}}{\tau \hat{L}}$ and $\mathrm{W} = \int_0^\infty \frac{1}{x + \sigma N_0 N_R} f_{R_m}(x)dx$. From (14), we can learn that the average achievable rate of the BS using OHBF approach grows with $1/N_0$.

B. Complexity Analysis

For the signal detection, as in (6), a large number of inner products between $\mathbf{h}_m$ and $\tilde{\mathbf{w}}_{\mathrm{RF},i}$ are needed. Fortunately, according to [8], the computational complexity is only $\mathcal{O}(P)$ for each inner product when we consider analog beamforming. The computational complexity of the proposed algorithm is comprised of the following three parts:

1) The first one is from the computation of SINR, which contains $K + PN_R$ real additions and L real multiplications as the first stage of Algorithm 1 can be implemented in parallel.
2) The second one is caused by the sorting method. Its time complexity is about $\mathcal{O}(L\log_2(L))$.
3) The third one is from the user selection and the computational complexity is $\mathcal{O}(L)$.

Since the calculation of MMSE detector has a complexity of $\mathcal{O}(L^3)$, with $K = L$, the complexity of the OHBF-MMSE is less than $\mathcal{O}(PN_R T_{\mathrm{scan}} + L^3)$, the complexity of the OHBF-ME-MMSE becomes $\mathcal{O}(3L + PL^3)$ while the complexity becomes $\mathcal{O}(N_R K^2 + K^3)$ for the HBF algorithm [14] and $\mathcal{O}(N_R^2 K^2)$ for the OMP-based HBF algorithm presented in [15]. It is noteworthy that the complexity analysis of the latter two algorithms in [14] is based on a similar system model used in this paper. In general, since $N_R \gg L$, we can find that the computational complexity of the proposed algorithm is low.

V. Simulation Results

In this section, we evaluate the performance of the proposed OHBF algorithm and present simulation results with the channel vector in (3). We assume that $P = 3$. Moreover, we set $d = \lambda/2$, $N_R = 60$ and $L = 10$, and the AoAs are assumed to be independent uniform RV from $-\pi/3$ to $\pi/3$. For convenience, the transmission power of each user is the same. In this paper, the results are obtained by 20000 runs.

The BER performance for different values of signal-to-noise ratio (SNR) is shown in Fig. 2, where E_{b}/N_0 denotes the SNR. Here, E_{b} is the bit-energy. It is shown that the performance of the proposed algorithms can be improved as the SNR increases (similar trend when the normalized mean squared error (NMSE) of CSI is considered [8], $\mathrm{NMSE} = \mathrm{E}[|\hat{\mathbf{h}} - \mathbf{h}|^2]/\mathrm{E}[|\mathbf{h}|^2]$). The performance of the proposed algorithms is reasonable, especially when there are numerous users (e.g., $K = 80$). In Fig. 2, the HBF algorithm proposed in [14] utilizes the Gram-Schmidt method to generate $\mathbf{W}_{\mathrm{RF}}$ and its performance is better. However, its complexity is 5 times higher than that of Algorithm 1. In addition, when $\mathrm{NMSE} = 1$, we can also see that the detection performances are reasonable under high SNR. From the simulations, it can

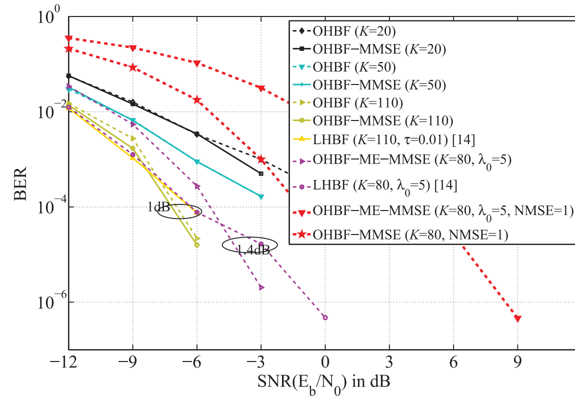

Fig. 2. BER versus SNR with $T_{\mathrm{scan}} = 1$.

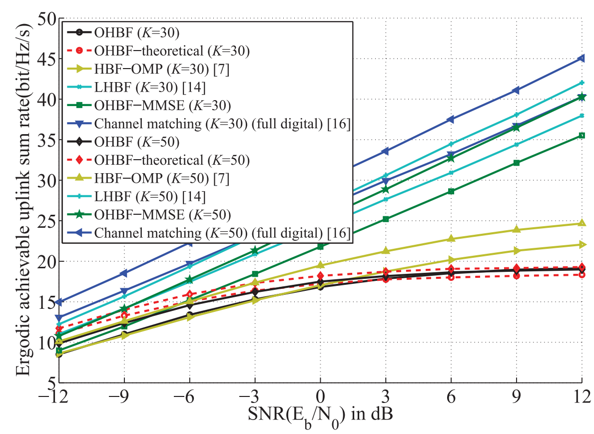

Fig. 3. The ergodic achievable uplink sum rate versus SNR with $T_{\mathrm{scan}} = 1$.

be learned that the proposed OHBF-ME-MMSE method can provide a high-quality detection performance.

Fig. 3 shows the achievable sum rate for the uplink mm-wave system. As expected, the sum rate increases with SNR and the derived approximate expression in (14) is also verified to be reasonable (e.g., the deviation $\leq$0.4 dB when $K = 30$). We can learn that the performance of the OHBF-MMSE is lower than that of "LHBF" since the Gram-Schmidt process is utilized to reduce most of the inter-user interference in analog domain [14]. For the proposed system, the performance of the method in [7] is not good since the orthogonality of the generated analog combiner can not be guaranteed. Moreover, although the number of RF chains in full DBF systems is 6 times larger than that in HBF systems, the rate achieved by the proposed OHBF-MMSE algorithm is about 90% of that achieved by the full DBF while the computational complexity of the OHBF-MMSE is less than that of the full DBF by a factor of $\frac{1}{10}$. Note that the method in [16] becomes pure DBF under the system setup of this paper.

The achievable sum rate performance for the number of training stages and RF chains is shown in Fig. 4. We can see that the sum rate increases with T_{scan}. When the SNR is low (e.g., -12 dB), we observe that the sum rate achieved by the OHBF algorithm is almost the same as that of the OHBF-MMSE algorithm. This results from that there might be some beams incorrectly selected under low SNR. When the SNR is high, the sum rate performance of the OHBF-MMSE method is better than that of the OHBF algorithm since the inter-user interference can be further reduced by using the MMSE filter. Besides,

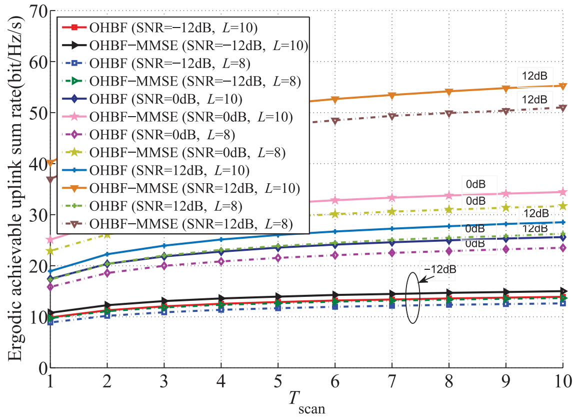

Fig. 4. The ergodic achievable uplink sum rate versus $T_{\rm scan}$.

the performance of the OHBF-MMSE algorithm can be improved effectively by increasing L.

VI. Conclusion

In this paper, we studied the multiuser detection methods for an uplink mm-wave transmission system where the number of users is large. Since the OBF is low-complexity and can provide a near-optimal performance, we proposed two OHBF-based algorithms. Besides, to improve the performance further, we applied the MMSE-based detector after deriving the received signals. From the simulation results, we can find that a reasonable trade-off between the performance and complexity can be achieved by the proposed approaches. To analyze the system performance efficiently, we also derived a closed-form approximation for the uplink sum rate in this paper, from which we can see that the OHBF-MMSE and OHBF-ME-MMSE algorithms can be used for hybrid beamformers with low computational complexity.

Appendix A
Derivation of (12)

According to the channel model, we have $\mathrm{E}(\alpha_2^2) = \mathrm{E}(\alpha_3^2) = 0$ and $\mathrm{E}(|\alpha_2|^2) = \mathrm{E}(|\alpha_3|^2) = 1$. Thus, it can be derived that

$$\mathrm{E}(R_m) = N_R \mathrm{E}\left[\left(N_R + N_R \sum_{p=2}^{3} |\alpha_p|^2\right)\right] = 3N_R^2. \tag{15}$$

Since $\mathrm{E}[\mathrm{Re}\{\mathbf{a}(\theta_p)^{\rm H}\mathbf{a}(\theta_q)\}] = 0$, $\mathrm{E}[|\mathbf{a}(\theta_p)^{\rm H}\mathbf{a}(\theta_q)|^2] = 3N_R$, $p \neq q$, $\mathrm{E}(R_m^2)$ is derived as

$$\begin{aligned}\mathrm{E}(R_m^2) &= N_R^2 \mathrm{E}\left[\left((\mathbf{a}(\theta_1) + \alpha_2\mathbf{a}(\theta_2) + \alpha_3\mathbf{a}(\theta_3))^{\rm H}\right.\right. \\ &\quad \left.\left. \times\ (\mathbf{a}(\theta_1) + \alpha_2\mathbf{a}(\theta_2) + \alpha_3\mathbf{a}(\theta_3))\right)^2\right] \\ &= 11N_R^4 + 12N_R^3. \end{aligned} \tag{16}$$

Appendix B
Derivation of (14)

For OHBF algorithm, since only few RF chains may utilize a high user diversity, we propose to use $f_{R_m}(x)$ to derive the approximate expressions of uplink sum rate. Thus, the sum rate of the system is given by

$$RS = \mathrm{E}\left[\sum_{l=1}^{\hat{L}} \log_2(1+\rho_l)\right] \overset{(a)}{\leq} \hat{L}\log_2\left(1+\mathrm{E}\left[\rho_l\right]\right), \tag{17}$$

where inequality (a) holds according to *Jensen inequality* and ρ_l is the SINR of the user l, which is represented by

$$\rho_l = \frac{|\mathbf{w}_{{\rm RF},l}^{\rm H}\mathbf{h}_m|^2}{\sum_{n\neq m}|\mathbf{w}_{{\rm RF},l}^{\rm H}\mathbf{h}_n|^2 + N_R N_0} = \frac{R_l}{I_l + N_R N_0}. \tag{18}$$

According to Algorithm 1, since τ is small (i.e., τR_m is less than 6) and the pdf of the multiuser interference is the tail function of the variable R_m, the distribution of I_l in (18) can be simplified as a triangle distribution [13]. Then,

$$I_l \approx \sqrt{2}\tau R_l \hat{L}/2. \tag{19}$$

Combining (17)–(19) with (9), we have

$$RS \approx \hat{L}\log_2(1+\mathrm{E}[R_l/(I_l + N_R N_0)]) \tag{20}$$

Then, we can have (14).

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