

# Joint Load Scheduling and Voltage Regulation in Distribution System with Renewable Generators

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**Abstract**—By equipping with the advanced smart meters and two-way communications infrastructure, smart grids, as a key component of future smart cities, are able to improve the energy efficiency and reduce the energy cost through real-time monitoring and customer load scheduling. However, the high penetration of intermittent renewable energy such as solar power may cause frequent overvoltage and undervoltage problems at certain buses, making the load scheduling face new challenges on voltage regulation. In this paper, we investigate the impact of voltage constraints on load scheduling by power flow analysis in a power distribution system with renewable generators. A voltage regulator (VR) is introduced to regulate the voltage of buses in the distribution system and assist load scheduling. To jointly minimize the cost and stabilize the voltages of the distribution system, we propose a grid-customer coordinated load scheduling strategy, which simultaneously determines the tap changes of VR and scheduling of customer electricity loads in each time slot. Finally, we evaluate the performance of the proposed strategy based on realistic power demand and renewable energy generation datasets. Extensive numerical results demonstrate that the proposed strategy can remarkably reduce the energy cost and stabilize the voltage fluctuation of distribution systems.

**Index Terms**—Load scheduling, voltage regulation, distribution system, renewable generator, demand response.

## I. INTRODUCTION

TODAY, traditional power grids are under pressure to accommodate the increasing power demand and to provide a sustainable and stable supply of electricity. These significant challenges are motivating the evolution of smart grid technologies. By integrating wireless communication networks into power grids, smart grid is able to preform real-time monitoring and schedule the electrical loads of customers through smart meters and two-way communication infrastructures [1]. In this way, electricity consumption can be shifted from peak to off-peak periods depending on consumers' flexibility/preferences in operating their appliances, which is also referred to as demand response [2]. It is expected that load scheduling would be a promising and cost-effective alternative than adding generation capabilities to meet the ever-increasing electricity load. This advancement makes smart grid an important component

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## Frequently Used Notations

| Notation                       | Description   |
|--------------------------------|---|
| $N$                            | Set of electrical appliances.   |
| $\mathcal{G}$                  | Set of distributed renewable energy generators (DGs).                       |
| $\mathcal{N}$                  | Set of electrical appliances.   |
| $\mathcal{T}$                  | Set of time periods.  |
| $\xi_{g,t}$                    | Electricity generated by $g$ during the period $t$ .                        |
| $E_t$                          | Total electricity generated in a period $t \in \mathcal{T}$ .               |
| $\mathcal{N}_f, \mathcal{N}_s$ | Set of inelastic and elastic appliances, respectively.                      |
| $L_{f,t}, L_{s,t}$             | Loads of inelastic and elastic appliances, respectively.                    |
| $x_{i,t}$                      | Load of $i$ at time period $t$ .  |
| $x_t$                          | Load scheduling vector for all elastic appliances in $t$ .                  |
| $\mathcal{D}_i$                | Feasible working time periods for each $i \in \mathcal{N}_s$ .              |
| $\delta_{i,l}, \delta_{i,m}$   | Minimum and maximum power consumption of appliance $i$ at time period $t$ . |
| $\kappa_i$                     | Minimum energy for appliance $i$ to complete a given task.                  |
| $\mathcal{W}$                  | Set of buses.   |
| $\mathcal{BN}_i$               | Set of appliances connected to the bus $w_i$ .                              |
| $\mathcal{BG}_i$               | Set of DGs connected at $w_i$ .   |
| $V_i(t), V_j(t)$               | Voltages of two neighbouring buses in time period $t$ .                     |
| $P_{i,j}(t), Q_{i,j}(t)$       | Active and reactive power flow from $Bus_i$ to $Bus_j$ during period $t$ .  |
| $P_i(t), Q_i(t)$               | Active and reactive power injection at bus $w_i$ during period $t$ .        |
| $\mathcal{BG}_{i,t}$           | Power injection by the DGs at $w_i$ .                                       |
| $V_l, V_m$                     | Lower and upper voltage limits of the buses, respectively.                  |
| $V^*$                          | Nominal voltage of the distribution system.                                 |
| $\mathcal{U}$                  | Set of taps $\mathcal{U}$ in the LTC.                                       |
| $u \in \mathcal{U}$            | Corresponding voltage at $w_0$ .  |
| $u_t \in \mathcal{U}$          | LTC tap position in $t$ .   |
| $c_t$                          | Electricity price at time period $t$ .                                      |
| $C_e$                          | Total electricity cost.   |
| $\lambda, \mu$                 | Lagrange multiplier matrices.   |
| $\mathcal{P}, \mathcal{D}$     | Results of (PP) and (DP), respectively.                                     |

of future smart city to improve energy efficiency. Meanwhile, the global concern on reducing greenhouse gas emission motivates the increasing utilization of renewable energy sources (e.g., wind and solar) in the power chain. According to the forecast from International Energy Agency (IEA), the power generation from renewable energy sources will nearly triple from 2010 to 2035, reaching 31% of the global power generation. The wind and solar generation will provide 25% and

7.5% of the total renewable power generation, respectively [3], [4]. The proliferation of distributed generators (DGs) with high penetration of renewable energy sources and distributed energy storages has brought great opportunities to alleviate energy burden for smart grid. However, due to the intermittent electricity generation and penetration of renewable generators, load scheduling in smart grid also faces new challenges on voltage regulation.

Since renewable energy sources are stochastic and intermittent in nature, the electricity outputs of DG units usually vary from time to time. Consequently, overvoltage and undervoltage problems may frequently arise at certain buses (e.g., DG unit points and load connection points) [5], when performing load scheduling. An effective way to address this challenge is to monitor and predict the DG penetration level to keep the voltages at all buses within an acceptable range [6]. As a result, load scheduling has to face strict voltage constraints to meet the requirements of voltage regulation. In order to regulate the voltages of distribution lines more effectively, voltage regulators (VRs) are generally installed to accommodate the voltage constraints of buses by changing their tap positions, and hence to assist the load scheduling. For instance, the voltage of a feeder can be controlled from 10% boost to 10% buck by a widely used McGraw-Edison single-phase VR [6]. Since the tap changes of VR and load scheduling can both significantly impact voltage fluctuation in smart grid, voltage regulation strategy should be carefully determined and coordinated with the load scheduling strategy to optimize the overall objective of the power grid.

There have been a number of existing works focusing on load scheduling to reduce the *peak-to-average-ratio* (PAR) in load demand or economically minimize electricity expense. But few of existing works take account of voltage regulation into load scheduling to simultaneously minimize the energy cost and stabilize the stochastic voltage fluctuation, which is a critical issue for the power distribution system with DG integration. In this paper, we investigate the joint load scheduling and voltage regulation problem in a power distribution system, which is connected to the main grid and equipped with a number of distributed renewable generators. A grid-customer coordinated load scheduling strategy is proposed to minimize the electricity cost and stabilize the voltages of the distribution system. In summary, the main contributions of this paper are two-fold.

(i) We incorporate the voltage constraints of all buses into the load scheduling problem based on power flow analysis in a power distribution system with DGs. A VR is introduced to regulate the voltages of buses and assist the load scheduling. Based on the power flow analysis, we investigate the joint load scheduling and voltage regulation problem to determine a combined strategy, consisting of the tap position of VR and the scheduled load of each appliance in each time period. To minimize the total energy cost, we formulate this problem as a mixed-integer non-linear programming (MINLP) problem, and decouple it into two independent subproblems. The primal problem is finally addressed by solving the subproblems separately and efficiently.

(ii) According to the solution of the primal problem, we

propose both offline and online algorithms to schedule the electricity load of customers and determine the tap positions of the VR. By taking advantages of wireless communications, the online algorithm can adjust the scheduled load for each time period based on the real-time generation and environment parameters. Extensive numerical results based on realistic renewable energy generation and power consumption datasets demonstrate that the proposed load scheduling and voltage regulation strategy can significantly reduce the electricity cost and stabilize the voltages of the distribution system.

The reminder of the paper is organized as follows. We review the related work in Section II and introduce the system model in Section III. The joint load scheduling and voltage regulation problem is formulated in Section IV and solved in Section V, respectively. The grid-customer coordinated load scheduling strategy is proposed in Section VI. We evaluate the performance of the proposed strategy in Section VII. Finally, Section VIII concludes this paper and outlines our future work.

## II. RELATED WORK

With the ever-increasing power consumption, increasing attention has been drawn on load scheduling in recent years, to reduce the *peak-to-average-ratio* (PAR) in load demand or economically minimize electricity expenses. Existing works can be briefly divided into two categories, i.e., direct load control and indirect load control, according to the control mechanisms [7]–[9].

Direct load control refers to grid control center can remotely turn off or cycle customers' electrical appliances to achieve some specific objectives. This program is generally offered to low consumption customers (i.e., residential customers) by signing control contracts. Nikolaos et. al. [10] propose a residential load control strategy to minimize the electricity provider cost plus the total user dissatisfaction. This strategy can find near-optimal schedules even if the exchanged information between the utility company and customers is lost. Deng et. al. [11] address the residential load scheduling problem by a game-theory approach under the consideration of user interactions and temporally-coupled load demand constraints. Based on these solid research foundation, some researchers extend their focuses to demand response in smart grid. An adaptive electricity scheduling algorithm is proposed in [12] to minimize the grid operation cost and guarantee the residents' quality of service in electricity. Meanwhile, with the proliferation of renewable energy sources and electric vehicles (EVs), generated electricity prediction and flexible electricity storage scheduling are integrated into load scheduling in smart grid. Mosdaddek et. al. [13] present a centralized method to jointly schedule the load of home appliances and plug-in EVs to minimize the electricity expenses for customers. In [14], the impact of EVs is also investigated in a residential grid and a demand response strategy is proposed as a load shaping tool to tackle the problem of distribution transformer overloading.

On the other hand, indirect load control strategies aim to economically stimulate customers to schedule their loads by themselves, via dynamically adjusting the electricity price. Antonio et. al. [15] propose a real-time pricing scheme joint

with an optimization model to adjust the hourly customer load in response to hourly electricity prices. There are also some related works investigating the impacts that should be considered in electricity pricing determination, such as residential renewable generation [16], customer fairness [17], etc. A real-time electricity pricing strategy is designed in [16] to support high penetrations of renewable generation and flexibly achieve overall load control. Zahra et. al. [17] highlight fairness should be paid more attention in demand response according to customers' contribution on system optimization. A smart billing mechanism is then proposed to enforce both optimality and fairness in demand response. Maharjan *et al.* [18] establish a Stackelberg game between electricity providers and consumers, where providers behave as leaders maximizing their profit and consumers act as the followers maximizing their individual welfare, to optimize the load scheduling of the whole system.

In addition to the load scheduling solutions, there are also a number of existing works focusing on voltage regulation in smart grid through adjusting the tap position of voltage regulator. By taking consideration of the voltage changes caused by the increasing distributed generation, Senjyu et al. [19] optimize the distribution voltage and simultaneously reduce the power loss for the whole power system by adjusting the tap position of voltage regulator. Kryonidis et al. [20] propose a decentralized voltage regulation algorithm to minimize the active power losses and the total reactive power consumption in medium-voltage networks with radial topology. A cooperative on-load tap changer control is adopted to further reduce the network losses. Zakaria et al. [21] investigate the problem of the increasing distribution system losses caused by the voltage deviations and reverse power flow, under the high penetration of DGs. They propose a decision mechanism to achieve the optimal schedule for DGs, including battery energy storage system, controllable loads and tap changing transformers, to minimize the loss of the whole power system.

However, few of existing works jointly consider voltage regulation and load scheduling to optimize the energy cost of the grid, which is a critical issue for power distribution system with DG integration. Although many research efforts have been invested in voltage regulation from a power system point of view [5], [22], to the best of our knowledge, there is still no significant achievement to study coordinated voltage regulation and load scheduling. To this end, in this paper, we investigate the joint voltage regulation and load scheduling problem by adjusting the voltage regulator and scheduling the load of buses, to achieve simultaneous energy cost reduction and voltage stabilization.

### III. SYSTEM MODEL

We consider a power distribution system, consisting of a set of distributed renewable energy generators (DGs)  $\mathcal{G}$  ( $|\mathcal{G}| = G$ ), a control center, a step-down transformer with a VR and a set of electrical appliances  $\mathcal{N}$  ( $|\mathcal{N}| = N$ ). The control center is responsible for scheduling and controlling the electricity loads of appliances, as well as adjusting the VR, in the system. Each DG is equipped with a remote terminal unit (RTU)

to send information to the control center. The appliances of each consumer are wirelessly connected to a smart meter through bluetooth or Zigbee. The smart meters can exchange information with the control center and control the working periods of the connected appliances. We divide a day into a set of time periods  $\mathcal{T} = \{1, \dots, T\}$ . For each DG  $g \in \mathcal{G}$ , let  $\xi_{g,t}$  be the electricity generated by  $g$  during the period  $t$ . Then, the total electricity generated in a period  $t \in \mathcal{T}$  is  $E_t = \sum_{g \in \mathcal{G}} \xi_{g,t}$ . In our generator model, we consider that the DGs consist of wind and solar power generators. Although both types of renewable energy sources are stochastic in nature, a number of existing models can predict the short-term power generation of renewable energy generators with a high accuracy [23], [24].

#### A. Electricity Load Model

The electricity load can be roughly divided into two categories: *inelastic* and *elastic* load. We use  $\mathcal{N}_f$  and  $\mathcal{N}_s$  to denote the set of inelastic and elastic appliances, respectively. Thus, we have the set of total home appliances is  $\mathcal{N} = \mathcal{N}_f \cup \mathcal{N}_s$ . For each time period  $t$ , if  $L_{f,t}$  and  $L_{s,t}$  are the loads of inelastic and elastic appliances, respectively, the total load  $L_t$  of the system can be calculated as  $L_t = L_{f,t} + L_{s,t}$ .

For each appliance  $i \in \mathcal{N}$ , let  $x_{i,t}$  denote the load of  $i$  at time period  $t$ . Since the inelastic load cannot be shifted, we have  $x_{i,t}$  is fixed for each  $i \in \mathcal{N}_f$  and each  $t \in \mathcal{T}$ . On the other hand, the elastic loads of the appliances  $\mathcal{N}_s$  (e.g., electric vehicle and clothes dryer) can be scheduled by the system to meet the system objective. For these loads, users only care about whether the task can be completed or not before a certain deadline [11], [25]. If  $x_{i,t}$  is the scheduled load for each  $i \in \mathcal{N}_s$  in  $t$ , we use  $\mathbf{x}_t \triangleq [x_{i,t}]_{i \in \mathcal{N}_s}$  to denote the load scheduling vector for all elastic appliances in  $t$ , and  $\mathbf{X} \triangleq [\mathbf{x}_t]_{t \in \mathcal{T}}$  to denote the load scheduling matrix for all elastic appliances over the whole day.

In addition, we define  $\mathcal{D}_i \triangleq [a_i, b_i]$  as the feasible working time periods for each  $i \in \mathcal{N}_s$ , where  $a_i$  and  $b_i$  are the starting time and deadline constraints for  $i$ , respectively. For instance,  $\mathcal{D}_i$  can be the time duration when an electric vehicle is parked at home. Therefore, we have the following constraint on  $x_{i,t}$ ,

$$\begin{cases} \delta_{i,l} \leq x_{i,t} \leq \delta_{i,m}, & \forall t \in \mathcal{D}_i \\ x_{i,t} = 0, & \text{otherwise} \end{cases}, \quad (1)$$

where  $\delta_{i,l}$  and  $\delta_{i,m}$  are the minimum and maximum power consumption of appliance  $i$  at time period  $t$ , according to its power levels [26]. Meanwhile, we have another *time-coupled* constraint on  $x_{i,t}$  for the total power consumption over the working periods  $\mathcal{D}_i$ , that is,

$$\sum_{t=a_i}^{b_i} x_{i,t} \geq \kappa_i, \quad (2)$$

where  $\kappa_i$  is the minimum energy for appliance  $i$  to complete a given task. For example,  $\kappa_i$  should be 16 kWh for an electric vehicle to achieve a daily 40 miles driving range [25]. This constraint is to guarantee that the scheduled task can be

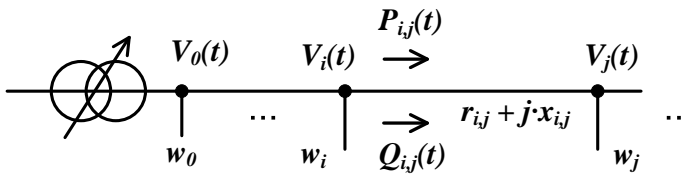


Fig. 1: Illustration of Power Flow between Two Adjacent Buses

finished by the deadline  $b_i$ . In summary, the total load of the system during  $\mathcal{T}$  can be calculated as

$$L_t = L_{f,t} + L_{s,t} = L_{f,t} + \sum_{i \in \mathcal{N}_s} x_{i,t}, \quad (3)$$

where  $L_{f,t}$  is constant.

### B. Voltage Regulation

We consider that the electricity transmission model of the distribution system can be abstracted as a one-line diagram with multiple buses [6], [27]. Let  $\mathcal{W} = \{w_0, w_1, \dots, w_W\}$  denote the set of buses. A transformer is equipped at the initial bus  $w_0$  to inject electricity from the main grid and distribute it to consumers (or reversely sell electricity to the main grid)<sup>1</sup>. For each bus  $w_i \in \mathcal{W} \setminus \{w_0\}$ , there is a set of appliances  $\mathcal{BN}_i$  connected to the bus. Thus, we have  $\bigcup_{i=1}^W \mathcal{BN}_i = \mathcal{N}$ . Since DGs are distributed at different buses, we refer to the buses with DGs as *generation buses* and the others as *load buses*. We use  $\mathcal{BG}_i$  to denote the set of DGs connected at  $w_i$ , and thus,  $\bigcup_{i=1}^W \mathcal{BG}_i = \mathcal{G}$ .

Given the certain line impedance, the voltage of each bus can be derived based on the active/reactive power injection via power flow analysis [6], [28]. As shown in Fig. 1, the relation between the voltages of two neighbouring buses in time period  $t$ , i.e.,  $V_i(t)$  and  $V_j(t)$ , is given by [27]

$$V_i(t) - V_j(t) = \frac{P_{i,j}(t) \cdot r_{i,j} + Q_{i,j}(t) \cdot y_{i,j}}{V_j(t)} \quad (4)$$

where  $P_{i,j}(t)$  and  $Q_{i,j}(t)$  are the active and reactive power flow from  $Bus_i$  to  $Bus_j$  during period  $t$ , respectively, and  $r_{i,j} + j \cdot y_{i,j}$  denotes the impedance of the feeder line  $i - j$ . In per unit, Eq. (4) can be approximately rewritten to [27]

$$V_i(t) - V_j(t) = P_{i,j}(t) \cdot r_{i,j} + Q_{i,j}(t) \cdot y_{i,j}. \quad (5)$$

If we use  $P_i(t)$  and  $Q_i(t)$  to denote the active and reactive power injection at bus  $w_i$  during period  $t$ , respectively, then we have

$$\begin{cases} P_{i-1,i}(t) = -\sum_{j=i+1}^W P_j(t) - P_i(t) = -\sum_{j=i}^W P_j(t) \\ Q_{i-1,i}(t) = -\sum_{j=i+1}^W Q_j(t) - Q_i(t) = -\sum_{j=i}^W Q_j(t) \end{cases} \quad (6)$$

Let  $BL_{i,t} = \sum_{j \in \mathcal{BN}_i} x_{i,t}$  be the electricity load at  $w_i$  in time period  $t$ . Since appliance load is fully supplied by the active power flow, we have

$$P_i(t) = BG_{i,t} - BL_{i,t}, \quad (7)$$

<sup>1</sup>The transformer of the power distribution system is a step-down transformer to reduce the voltage from a transmission level (above 110 kV) to a distribution level (below 50 kV).  $w_0$  refers to the low voltage side of the step-down transformer.

where  $BG_{i,t} = \sum_{j \in \mathcal{BG}_i} \xi_{g,t}$  is power injection by the DGs at  $w_i$ , and we have  $BG_{i,t} = 0$  if  $w_i$  is a load bus. Note that, if the power generation is larger than the load at  $w_i$  (i.e.,  $BG_{i,t} > BL_{i,t}$ ),  $P_i(t)$  is positive, which means that there is power injection at bus  $w_i$  and voltage rises; otherwise, it indicates that the voltage drops at  $w_i$ . Based on Eq. (5), we can derive the relation between the voltages of  $w_i$  ( $1 \leq i \leq W$ ) and  $w_0$  as

$$V_i(t) = V_0(t) - \sum_{j=1}^i (P_{j-1,j}(t)r_{j-1,j} + Q_{j-1,j}(t)y_{j-1,j}). \quad (8)$$

According to the requirements of voltage stability, the voltage of all the buses should stay with a certain range. Let  $V_l$  and  $V_m$  denote the lower and upper voltage limits of the buses, respectively. For example, if we use  $V^*$  to denote the nominal voltage of the distribution system, generally, we have  $V_l = 0.9 \cdot V^*$  and  $V_m = 1.1 \cdot V^*$ . Therefore, based on Eq. (8), (6) and (7), the voltage constraint of  $w_i$  in  $t$  is

$$V_l < V_0(t) + \sum_{j=1}^i \left\{ \sum_{k=j}^W (BG_{k,t} - BL_{k,t}) \cdot r_{j-1,j} + \sum_{k=j}^W Q_k(t) \cdot y_{j-1,j} \right\} < V_m. \quad (9)$$

Note that, in Eq. (9),  $r_{i,i+1}$  and  $y_{i,i+1}$  are fixed, and  $\sum_{k=j}^W Q_k(t)$  is the reactive power from  $w_j$  to  $w_{j+1}$  which can be monitored and predicted at each bus, and  $BG_{i,t}$  is the power generation of bus  $w_k$  that can also be predicted based on our generation model. Therefore, Eq. (9) is a constraint for the aggregated load of each bus.

To keep the stability of the distribution system, the total power generation capacity would never exceed the transformer capacity. Let  $PS_m$  be the maximum active power withdrawal capacity of the transformer, then the capacity limit of the distribution system can be written as  $\sum_{i=1}^W (BL_{i,t} - BG_{i,t}) \leq PS_m$ . According to Eq. (6) and (7), it is equivalent to

$$\sum_{i \in \mathcal{N}_s} x_{i,t} \leq \psi_t, \quad (10)$$

where  $\psi_t = PS_m + \sum_{g=1}^G \xi_{g,t}$ .

In addition, the equipped VR can change the voltage of the initial bus by adjusting the load tap changers (LTC) [29]. We assume that there is a set of taps  $\mathcal{U}$  in the LTC (e.g.,  $|\mathcal{U}| = 32$  for a 32-step LTC). Each tap  $u \in \mathcal{U}$  indicates the corresponding voltage at  $w_0$ :  $V_{0,u} \triangleq \Gamma(u)$ , where  $\Gamma(\cdot)$  is a linear function. If  $u_t \in \mathcal{U}$  is the LTC tap position in  $t$ , Eq. (9) can be rewritten as

$$V_l < \Gamma(u_t) + \sum_{j=1}^i \left\{ \sum_{k=j}^W (BG_{k,t} - BL_{k,t}) \cdot r_{j-1,j} + \sum_{k=j}^W Q_k(t) \cdot y_{j-1,j} \right\} < V_m. \quad (11)$$

### C. Electricity Pricing Model

We consider the distribution system and customers as a unity. If the generated electricity of DGs can satisfy the electricity load of the customers in  $t$ , i.e.,  $L_t \leq E_t$ , we say the unity is self-sustained at this time period. Once the load exceeds the generated electricity, the unity should buy

electricity from the main grid. We assume there is no storage device in the DGs, so the residual electricity, i.e.,  $E_t - L_t$ , will be sold to the main grid, if  $L_t < E_t$ . The electricity sell price is assumed to be equivalent to the buy price. Therefore, from the perspective of the utility, the electricity price of the main grid is the only price concerned to schedule the load, which changes during different time periods but can be known day-ahead, e.g., Hourly Ontario Energy Price (HOEP) in Ontario, Canada. If we use  $c_t$  to denote the electricity price at time period  $t$ , the total electricity cost  $C_e$  can be calculated as

$$C_e = \sum_{t \in \mathcal{T}} (c_t \cdot (L_t - E_t)). \quad (12)$$

Note that, if  $L_t < E_t$ ,  $C_e$  is negative, which means that the utility can make profits by selling electricity to the main grid; otherwise,  $C_e$  is the cost for the utility to buy electricity from the main grid to meet the power demand of customers.

#### IV. PROBLEM FORMULATION AND DECOMPOSITION

The objective is to minimize the electricity cost and stabilize the voltages of the distribution system. In this section, we mathematically formulate this joint load scheduling and voltage regulation problem as a constrained mix-integer non-linear programming (MINLP) problem. Since the MINLP is hard to be directly solved, we investigate its dual problem and decompose the dual problem into two subproblems that can be efficiently addressed.

##### A. Problem Formulation

The electricity cost  $C_e$  is the primary cost. It is determined by the deviation of the load and renewable generation, as well as the electricity price, during each time period  $t \in \mathcal{T}$ . On the other hand, the tap changing of VR will bring wear and tear to itself and shorten its lifetime. Thus, we have another cost on VR, caused by the difference between current and previous tap settings [6], [29]. The related cost function is defined as  $\Delta(\cdot) : \mathcal{U} \times \mathcal{U} \rightarrow \mathbb{R}$ , i.e., the cost associated with the wear and tear of VR during  $t \in \mathcal{T}$  is determined by  $\Delta(u_t, u_{t-1})$ , where  $\Delta(u_t, u_{t-1}) = 0$ , if  $u_t = u_{t-1}$ ; otherwise,  $\Delta(u_t, u_{t-1}) = c_u$ . Here, we set  $\Delta(u_1, u_0) = 0$ . If we use  $\mathbf{u} \triangleq [u_t]_{t \in \mathcal{T}}$  to denote the tap position vector of VR over the whole day, the total cost of VR is  $C_{vr} = \sum_{t \in \mathcal{T}} \Delta(u_t, u_{t-1})$ . Since the cost of VR tap changing  $C_{vr}$  is independent of the electricity cost  $C_e$ , the total cost can be calculated as

$$C(\mathbf{u}, \mathbf{X}) = \sum_{t \in \mathcal{T}} \left\{ c_t \cdot (L_{f,t} + \sum_{i \in \mathcal{N}_s} x_{i,t} - E_{g,t}) + \Delta(u_t, u_{t-1}) \right\}.$$

In summary, the joint load scheduling and voltage regulation problem can be formulated as determining  $\mathbf{u} = \{u_1, \dots, u_T\}$  and  $\mathbf{X} = \{x_{i,t} | \forall i \in \mathcal{N}_s, \forall t \in \mathcal{T}\}$  to

$$\begin{aligned} \text{(PP)} \quad & \text{minimize } C(\mathbf{u}, \mathbf{X}) \\ & \text{s.t. Eq.(1), (2), (10) and (11).} \end{aligned}$$

Obviously, (PP) is an MINLP problem. Due to the non-convex objective function (i.e.,  $\sum_{t \in \mathcal{T}} \Delta(u_t, u_{t-1})$ ), traditional solutions [27] can not be directly applied to address this problem. If we pay attention to the two decision variables  $\mathbf{u}$  and  $\mathbf{X}$ , their only relationship in (PP) is the constraint Eq. (11). Therefore, if we can remove this constraint by Lagrangian relaxation, this problem can be decomposed to two independent optimization problems for  $\mathbf{X}$  and  $\mathbf{u}$ , respectively.

##### B. Problem Decomposition

Since Eq. (11) is the only constraint coupling the optimization for  $\mathbf{u}$  and  $\mathbf{X}$ , we introduce two Lagrange multiplier matrices  $\boldsymbol{\lambda} = \{\lambda_{w,t} > 0 | 1 \leq w \leq W, 1 \leq t \leq T\}$  and  $\boldsymbol{\mu} = \{\mu_{w,t} > 0 | 1 \leq w \leq W, 1 \leq t \leq T\}$  to remove this constraint. Therefore, the Lagrangian  $\mathcal{L}(\cdot)$  associated with the primal problem (PP) is

$$\begin{aligned} \mathcal{L}(\mathbf{u}, \mathbf{X}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = & \sum_{t \in \mathcal{T}} \left\{ c(t) \sum_{i \in \mathcal{N}_s} x_{i,t} + \Delta(u_t, u_{t-1}) + z_t \right\} \\ & + \sum_{w=1}^W \sum_{t=1}^T \lambda_{w,t} \left( \Gamma(u_t) - \sum_{p=w}^W \sum_{q=p}^W \sum_{i \in \mathcal{W}_q} (r_{p-1,p} x_{i,t}) + s_{w,t} - V_m \right) \\ & + \sum_{w=1}^W \sum_{t=1}^T \mu_{w,t} \left( V_l - \Gamma(u_t) + \sum_{p=w}^W \sum_{q=p}^W \sum_{i \in \mathcal{W}_q} (r_{p-1,p} x_{i,t}) - s_{w,t} \right), \end{aligned}$$

where  $s_{w,t} = \sum_{p=w}^W \sum_{q=p}^W (r_{p-1,p} \xi_{q,t} + y_{p-1,p} Q_{q,t})$  and  $z_t = c(t) \cdot (L_f(t) - E(t))$  are both constants.

Denote  $\boldsymbol{\alpha}_t \triangleq [\alpha_{1,t}, \dots, \alpha_{|\mathcal{N}_s|,t}]^T$  and  $\mathbf{x}_t = [x_{i,t}, \dots, x_{|\mathcal{N}_s|,t}]^T$ . Then, we define

$$\begin{aligned} (\boldsymbol{\alpha}_t)^T \mathbf{x}_t & \triangleq \sum_{w=1}^W \left[ (\lambda_{w,t} - \mu_{w,t}) \sum_{p=w}^W \sum_{q=p}^W \sum_{i \in \mathcal{W}_q} (r_{p-1,p} x_{i,t}) \right], \\ \beta_t & \triangleq \sum_{w=1}^W (\lambda_{w,t} - \mu_{w,t}), \\ \theta & = \sum_{w=1}^W \sum_{t=1}^T [\lambda_{w,t} (s_{w,t} - V_m) + \mu_{w,t} (V_l - s_{w,t})] + \sum_{t=1}^T z_t. \end{aligned}$$

Therefore, the Lagrangian can be written as

$$\begin{aligned} \mathcal{L}(\mathbf{u}, \mathbf{X}, \boldsymbol{\lambda}, \boldsymbol{\mu}) = & \sum_{t \in \mathcal{T}} \left( \sum_{i \in \mathcal{N}_s} c(t) x_{i,t} - (\boldsymbol{\alpha}_t)^T \mathbf{x}_t \right) \\ & + \sum_{t \in \mathcal{T}} [\Delta(u_t, u_{t-1}) + \beta_t \cdot \Gamma(u_t)] + \theta, \end{aligned} \quad (13)$$

and the dual function is

$$\mathcal{D}(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \inf_{\mathbf{u}, \mathbf{X}} \mathcal{L}(\mathbf{u}, \mathbf{X}, \boldsymbol{\lambda}, \boldsymbol{\mu}). \quad (14)$$

In order to decouple  $\mathbf{u}$  and  $\mathbf{X}$  from the dual problem, we define the following subproblems,

$$\begin{aligned} \text{(SP1)} \quad \mathcal{S}_1(\boldsymbol{\lambda}, \boldsymbol{\mu}) & \triangleq \min_{\mathbf{X}} \sum_{t \in \mathcal{T}} \left( \sum_{i \in \mathcal{N}_s} c_t x_{i,t} - (\boldsymbol{\alpha}_t)^T \mathbf{x}_t \right) \\ & \text{s.t. Eq.(1), (2) and (10)} \end{aligned}$$

$$\begin{aligned} \text{(SP2)} \quad \mathcal{S}_2(\boldsymbol{\lambda}, \boldsymbol{\mu}) & \triangleq \min_{\mathbf{u}} \sum_{t \in \mathcal{T}} [\Delta(u_t, u_{t-1}) + \beta_t \cdot \Gamma(u_t)] \\ & \text{s.t. } u_t \in [1, |\mathcal{U}] \cap \mathbb{Z}, \forall t \in \mathcal{T}. \end{aligned}$$

Since  $\theta$  is independent of  $\mathbf{X}$  and  $\mathbf{u}$ , therefore, due to the separable property, the dual function can be rewritten as

$$\mathcal{D}(\boldsymbol{\lambda}, \boldsymbol{\mu}) = \mathcal{S}_1(\boldsymbol{\lambda}, \boldsymbol{\mu}) + \mathcal{S}_2(\boldsymbol{\lambda}, \boldsymbol{\mu}) + \theta. \quad (15)$$

Note that, the primal problem (PP) has been divided into two subproblems: (SP1) is to schedule the load of elastic appliances, and (SP2) is to determine the tap positions of VR.

Finally, the dual problem is to maximize the dual function over  $\lambda$  and  $\mu$ , i.e.,

$$\begin{aligned} \text{(DP)} \quad & \max_{\lambda, \mu} \mathcal{D}(\lambda, \mu) \\ \text{s.t.} \quad & \lambda_{w,t} \geq 0, \mu_{w,t} \geq 0, \forall w \in \mathcal{W}, t \in \mathcal{T} \end{aligned}$$

Due to the discrete variable  $u_t$  and non-convex function  $\sum_{t \in \mathcal{T}} \Delta(u_t, u_{t-1})$  in **(PP)**, only weak duality can be ensured by Lagrangian relaxation, and the duality gap exists [30]. Let  $\mathcal{P}$  and  $\mathcal{D}$  be the result of **(PP)** and **(DP)**, respectively. We have,  $\mathcal{D} < \mathcal{P}$  holds for all feasible solutions and  $\mathcal{D}$  actually becomes the lower bound of  $\mathcal{P}$  [31].

## V. SOLVING DUAL PROBLEM AND SUBPROBLEMS

In this section, we focus on solving the primal problem by addressing the dual problem and two subproblems.

### A. Subgradient Method for Dual Problem

For given  $\lambda$  and  $\mu$ , if we can address **(SP1)** and **(SP2)**, the dual problem **(DP)** can be iteratively calculated using a subgradient method [32]. Specifically, each Lagrangian multiplier can be updated in an opposite direction to the partial gradient of the Lagrangian dual function [30], [33], i.e.,

$$\begin{cases} \lambda_{w,t}(k+1) = [\lambda_{w,t}(k) + \gamma_\lambda \cdot f_{\lambda,w,t}(k)]^+ \\ \mu_{w,t}(k+1) = [\mu_{w,t}(k) + \gamma_\mu \cdot f_{\mu,w,t}(k)]^+ \end{cases} \quad (16)$$

where  $k \in \mathbb{N}^+$  is the iteration index,  $\gamma_\lambda > 0$  and  $\gamma_\mu > 0$  are the step sizes adjusting the convergence rate, and  $f_{\lambda,w,t}(k)$  and  $f_{\mu,w,t}(k)$  are subgradients of the dual function with respect to  $\lambda_{w,t}$  and  $\mu_{w,t}$ , respectively,

$$\begin{cases} f_{\lambda,w,t}(k) = \frac{\partial \mathcal{D}(\lambda, \mu)}{\partial \lambda} = s_{w,t} + \Gamma(u_t(k)) - V_m \\ \quad - \sum_{p=w}^W \sum_{q=p}^W \sum_{i \in \mathcal{W}_q} (r_{p-1,p} x_{i,t}(k)) \\ f_{\mu,w,t}(k) = \frac{\partial \mathcal{D}(\lambda, \mu)}{\partial \mu} = V_l - s_{w,t} - \Gamma(u_t(k)) \\ \quad + \sum_{p=w}^W \sum_{q=p}^W \sum_{i \in \mathcal{W}_q} (r_{p-1,p} x_{i,t}(k)) \end{cases} \quad (17)$$

where  $s_{w,t} = \sum_{p=w}^W \sum_{q=p}^W (r_{p-1,p} \xi_{q,t} + y_{p-1,p} Q_{q,t})$ , and  $x_{i,t}(k)$  and  $u_t(k)$  can be obtained by solving **(SP1)** and **(SP2)**. Since the concavity of **(DP)** always holds, the optimal Lagrangian multipliers can be achieved through this subgradient method.

### B. Solving Subproblems

In this subsection, we discuss the solutions to the subproblems **(SP1)** and **(SP2)**. Due to the iteration process in addressing the dual problem, subproblems should be solved efficiently to guarantee the efficiency of our approach. Specifically, the load scheduling problem, i.e., **(SP1)**, is a linear optimization problem, which can be directly solved by traditional linear programming techniques [34]. Meanwhile, the VR adjusting problem, i.e., **(SP2)**, is an integer optimization problem, which is generally hard to be addressed efficiently. Therefore, we focus on finding a polynomial solution for **(SP2)** in the following.

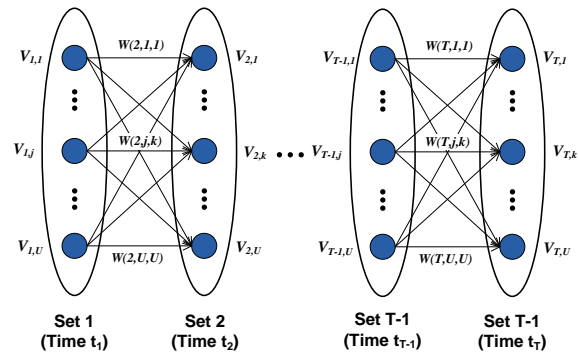


Fig. 2: The directed graph that is defined to solve **(SP2)**.

1) *Solution to (SP1)*: Given  $\lambda$  and  $\mu$ ,  $\alpha_t$  can be fixed for each period  $t \in \mathcal{T}$ . If we set  $\varphi_{i,t} \triangleq c_t - \alpha_{i,t}$ , the objective function of **(SP1)** can be rewritten as

$$S_1(\lambda, \mu) = \min_{\mathbf{X}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{N}} (\varphi_{i,t} \cdot x_{i,t}),$$

and the constraints can be rewritten as

$$\begin{cases} \gamma_i^{\min} \leq x_{i,t} \leq \gamma_i^{\max}, \forall i \in \mathcal{N}_s, \forall t \in \mathcal{T}, \\ \sum_{t \in \mathcal{T}} x_{i,t} \geq \kappa_i, \\ \sum_{i \in \mathcal{N}_s} x_{i,t} \leq \psi_t, \forall t \in \mathcal{T}, \end{cases}$$

where  $\gamma_i^{\min} = \gamma_i^{\max} = 0$ ,  $\forall t \notin [a_i, b_i]$ . Therefore, **(SP1)** becomes a classic linear programming problem. Karmarkar's algorithm [34] is a feasible solution for addressing this problem efficiently, which uses a projective method for linear programming and guarantees a polynomial bound.

2) *Solution to (SP2)*: Given  $\lambda$  and  $\mu$ , we have a fixed  $\beta_t$  ( $\beta_t \in \mathbb{R}$ ) for each period  $t \in \mathcal{T}$ . Since the objective function of **(SP2)** is dependent on all the deviation of  $u_t$  and  $u_{t-1}$ , the global optimal solution can not be derived by local optimization. In the following, we are trying to convert this problem into deriving the shortest path in a directed graph.

We define a directed graph  $\mathcal{G}$  as shown in Fig. 2, consisting of  $T$  independent vertex sets. In each set  $t$  ( $1 \leq t \leq T$ ), there are  $U$  nodes. We denote the node  $j$  ( $1 \leq j \leq U$ ) in set  $t$  as  $n_{t,j}$ . For  $2 \leq t \leq T$ , each node  $j$  of set  $t-1$  has an edge connected to all the nodes of set  $t$ . The weight of the edge from node  $j$  of set  $t-1$  to node  $k$  of set  $t$  is  $w(t, j, k)$ .

Now, we change our focus to the objective function of **(SP2)**. If we define  $F(t-1, t) \triangleq \Delta(u_t, u_{t-1}) + \beta_t \cdot \Gamma(u_t)$  for each  $1 \leq t \leq T$ , the objective function can be rewritten as  $S_2(\lambda, \mu) = \sum_{t=1}^T F(t-1, t)$ . Recall that, our objective is to determine a tap position  $u_t$  from the tap set  $\mathcal{U}$  at each period  $t$  to obtain  $S_2(\lambda, \mu)$ .

If we set in each time  $t$ , each tap position  $j \in \mathcal{U}$  is a node  $n_{t,j}$  in set  $t$  of  $\mathcal{G}$ , then the weight  $w(t, j, k)$  ( $1 \leq j \leq U$ ,  $1 \leq k \leq U$ ) can be defined as

$$\begin{cases} w(2, j, k) = \Delta(j, k) + \beta_1 \Gamma(j) + \beta_2 \Gamma(k), \text{ if } t = 2 \\ w(t, j, k) = \Delta(j, k) + \beta_t \Gamma(k), \text{ if } 2 < t \leq T \end{cases} \quad (18)$$

Therefore, **(SP2)** is equivalently transformed to choose a node  $n_{t,j}$  from each set  $t$  of  $\mathcal{G}$ , to minimize the total weight of the

whole path. We refer to the nodes that are chosen to form the set connection path as path nodes.

In order to solve this problem, we make some further modifications on this graph. If all the path nodes from set 1 to set  $t - 1$  have been determined, we can easily choose the path node from set  $t$ . It means that, if the path node is fixed as  $j$  in set  $t - 1$ , the best choice of set  $t$  should be the node  $k$  with the minimum weight  $\min_{1 \leq k \leq U} (\Delta(j, k) + \beta_t \Gamma(k))$ . Therefore, we can regard the set  $t$  as a single node  $S$ , and then the weight of the edge from node  $j$  of set  $t - 1$  to this node is

$$w(T, j, S) = \min_{1 \leq k \leq U} w(t, j, k), \quad (19)$$

and the path node of set  $t$  is

$$S = \arg \min_{1 \leq k \leq U} w(t, j, k). \quad (20)$$

If we choose  $S$  as the source node and the node  $n_{1,j}$  of the first set as the destination node, and reverse all the edge directions of the graph  $\mathcal{G}$ , then the minimum weight of the whole path is the shortest path from  $S$  to  $n_{1,j}$ . Thus we can iteratively calculate the values of the shortest path from  $S$  to  $n_{1,j}$  ( $1 \leq j \leq U$ ), and the minimum one is the solution of (SP2). Note that since the weight of each edge in  $\mathcal{G}$  could be negative, we can calculate the shortest path based on Bellman-Ford algorithm. We describe the main idea of our solution to (SP2) in Algorithm 1.

The time complexity of Bellman-Ford algorithm is  $O(|V| \cdot |E|)$ , where  $|V| = T \cdot U$  is the number of vertices in  $\mathcal{G}$  and  $|E| = T \cdot U \cdot U$  is the number of edges in  $\mathcal{G}$ . Therefore, the time complexity of our solution to (SP2) is  $O(U^3 T^2)$ .

## VI. GRID-CUSTOMER COORDINATED LOAD SCHEDULING STRATEGY

The previous section has illustrated the main ideas of addressing the dual problem (DP). In this section, we summarize the steps of our solution and propose a grid-customer coordinated load scheduling strategy to minimize the cost and stabilize the voltages of the distribution system. Specifically, the proposed load scheduling strategy consists of two phases: firstly, before the day, the control center determines the scheduled load and tap position vector of LTC over the next day; secondly, during the day, the control center will adjust the real-time scheduled load and tap position at the beginning of each time period.

### A. Day-ahead Load Scheduling

We first focus on the primal problem (PP), which is the objective of the grid-customer coordinated load scheduling strategy. According to our system model, except the decision variables  $\mathbf{X}$  and  $\mathbf{u}$ , all the parameters of (PP) are known in advance or can be predicted before the next day, including the electricity price of main grid  $\mathbf{c} = \{c_t | \forall t \in \mathcal{T}\}$ , the power demand of elastic appliances  $\boldsymbol{\kappa} = \{\kappa_i | \forall i \in \mathcal{N}_s\}$ , the fixed load of customers  $\mathbf{L}_f = \{L_{f,t} | t \in \mathcal{T}\}$ , and the predicted renewable generation  $\mathbf{E} = \{E_t | \forall t \in \mathcal{T}\}$ . Therefore, we describe the main idea of day-ahead load scheduling in

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### Algorithm 1: Bellman-Ford based Solution to (SP2)

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**Input** : The graph  $\mathcal{G}$  and  $\beta = \{\beta_1, \dots, \beta_T\}$ ;  
**Output**: The optimal value of  $S_2(\boldsymbol{\lambda}, \boldsymbol{\mu})$  and the tap position vector  $\mathbf{u} = \{u_1, \dots, u_T\}$ ;

- 1 Initialize the vertex matrix  $n[T-1][U]$ , the source node  $D$  and the edge matrix  $w[T][U][U]$  according to Eq. (18) and (19);
- 2  $src \leftarrow D$ ,  $dist[D] \leftarrow 0$ ;
- 3 **for each** vertex  $n[t][j]$  **in the vertex matrix** **do**
- 4      $dist[t][j] \leftarrow \text{infinity}$ ;
- 5      $predecessor[t][j] \leftarrow \text{null}$ ;
- 6 **end**
- 7 **for each** vertex  $n[t][j]$  **in the vertex matrix** **do**
- 8     **for each** edge  $w[t+1][j][k]$  **in the edge matrix** **do**
- 9         **if**  $dist[t+1][k] + w[t+1][j][k] < dist[t][j]$  **then**
- 10              $dist[t][j] \leftarrow dist[t+1][k] + w[t+1][j][k]$ ;
- 11              $predecessor[t][j] \leftarrow n[t+1][k]$
- 12         **end**
- 13     **end**
- 14 **end**
- 15  $S_2(\boldsymbol{\lambda}, \boldsymbol{\mu}) \leftarrow \min_{1 \leq j \leq U} dist[1][j]$ ;
- 16  $d \leftarrow \arg \min_{1 \leq j \leq U} dist[1][j]$ ;
- 17 **for each**  $t$  **from** 1 **to**  $T$  **do**
- 18     **if**  $t = T$  **then**
- 19         Set  $\mu_t$  according to Eq. (20);
- 20     **else**
- 21          $\mu_t \leftarrow d$ ;
- 22          $d \leftarrow predecessor(d)$ ;
- 23     **end**
- 24 **end**
- 25 **return**  $S_2(\boldsymbol{\lambda}, \boldsymbol{\mu})$  and  $\boldsymbol{\mu}$ ;

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### Algorithm 2: Day-ahead Load Scheduling

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**Input** : The predicted generation vector  $\mathbf{E}$ , the electricity price vector  $\mathbf{c}$ , the fixed load vector  $\mathbf{L}_f$  and the required load vector  $\boldsymbol{\kappa}$ ;  
**Output**: The optimal scheduled load  $\mathbf{X}(k)$  and tap position vector  $\mathbf{u}(k)$ ;

- 1 Let  $k = 0$ ; Initialize Lagrangian multipliers  $\lambda_{w,t}(k)$  and  $\mu_{w,t}(k)$ ;
- 2 **repeat**
- 3     With  $\lambda_{w,t}(k)$  and  $\mu_{w,t}(k)$ , calculate the scheduled load matrix  $\mathbf{X}(k)$  by solving (SP1), and determines the tap position vector  $\mathbf{u}(k)$  by solving (SP2) according to Algorithm 1;
- 4     With  $x_{i,t}(k)$  and  $u_t(k)$ , update  $\lambda_{w,t}(k+1)$  and  $\mu_{w,t}(k+1)$  according to Eq. (16) and (17);
- 5 **until** (i)  $k$  exceeds the maximum iteration number; or (ii) the gap between  $\mathcal{D}$  and  $\mathcal{P}$  is small enough;
- 6 **return**  $\mathbf{X}(k)$  and  $\mathbf{u}(k)$ ;

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Algorithm 2, which returns the globally optimal scheduled load of elastic appliances and tap positions of LTC over the next day.



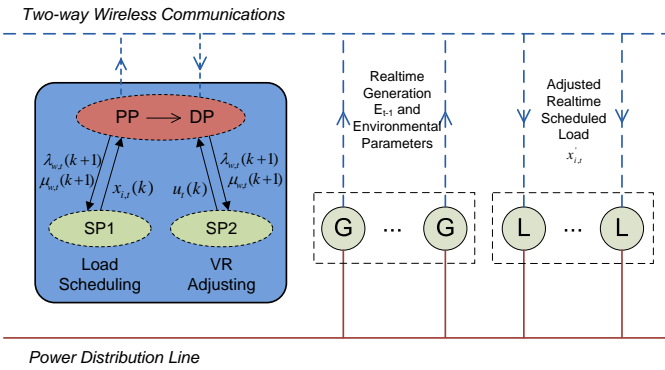


Fig. 3: Illustration of Intra-day Load Adjusting Strategy.

### B. Intra-day Load Adjusting

**Algorithm 2** provides an offline load scheduling strategy, which is run one day ahead based on the predicted renewable generation during the next day. Although existing generation prediction algorithms can achieve an acceptable accuracy, the prediction error still exists and may affect the system performance. In order to further mitigate the negative impact of generation prediction error, we propose an intra-day load scheduling strategy to update the real-time scheduled load  $x_t$  of elastic appliances and the tap position of LTC  $u_t$  for each time period  $t$  ( $2 \leq t \leq T$ ).

The operation of intra-day load adjusting is based on the day-ahead load scheduling, as shown in Fig. 3. That is, before the first time period, we use **Algorithm 2** to determine the scheduled load  $X$  and the tap position vector  $u$  of LTC, over the next  $T$ . After that, at the beginning of the  $t^{th}$  ( $2 \leq t \leq T$ ) time period, the control center will re-predict the electricity generation vector  $E' = \{E'_t, \dots, E'_T\}$ , based on the actual electricity generation during the last  $T - t$  time periods and the real-time environmental parameters (e.g., solar radiation and wind speed, etc.). Then, we can use  $E'$  as the input of **Algorithm 2** to determine the real-time scheduled load vector  $x_t$  and the tap position  $u_t$ .

## VII. PERFORMANCE EVALUATION

In this section, we evaluate the performance of the proposed load scheduling schemes based on the realistic power demand and renewable generation datasets. Simulations are performed based on the Kumamoto 15-bus system [35]. We add a VR between utility grid and bus 1, i.e., bus 0, a PV generator on bus 14 and a wind turbine generator on bus 9, as shown in Fig. 4.

### A. Simulation Configuration

The installed VR is the McGraw-Edison single-phase VR, with an adjustable range from 85% to 115% normal voltage [6]. The generated electricity data of PV generator and wind generator are adopted from the UQ solar dataset [36] and Belgium's wind-power capacity dataset [37], respectively. The real-time electricity price is adopted from the price for residents in Ameren Illinois [38], which dynamically changes at every hour. Fig. 5 shows the renewable generation and

### Algorithm 3: Intra-day Load Adjusting

**Input** : Same with **Algorithm 2**;  
**Output**: The adjusted scheduled load matrix  $X^*$  and tap position vector  $u^*$  for time period 2 to  $T$ ;

- 1 Schedule the load and adjust the LTC in the first time period, based on the results of **Algorithm 2**;
- 2 Adding  $x_1$  as the first column of  $X^*$ ,  $u_1$  as the first element of  $u^*$ ;
- 3 **repeat**
- 4     At the beginning of time period  $t$  ( $2 \leq t \leq T$ ), updating the predicted generation vector  $E' = \{E'_t, \dots, E'_T\}$ , the required load vector  $\kappa'$  with the scheduled load of the last  $T - t$  periods;
- 5     With the updated  $E'$  and  $\kappa'$ , running **Algorithm 2** to obtain the real-time scheduled load  $x_t$  and tap position vector  $u_t$  during time period  $t$ ;
- 6     Adding  $x_t$  as the  $t^{th}$  column of  $X^*$ ,  $u_t$  as the  $t^{th}$  element of  $u^*$ ;
- 7 **until**  $t = T$ ;
- 8 **return**  $X^*$  and  $u^*$ ;

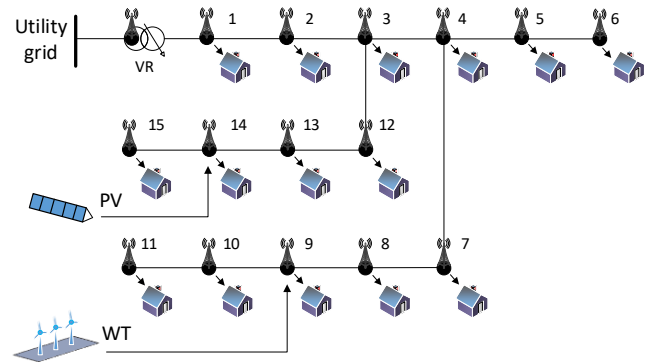


Fig. 4: System Architecture.

electricity price data. The statistics of demand for each customer at different times of a day is obtained from the smart meter readings of two residences subscribed to Waterloo North Hydro in the Laurelwood neighborhood of Waterloo Region in Canada [6]. The elastic load takes around 30% of the total load for each consumer [39], with specific scheduling constraints for each appliance (e.g., the electricity load of a charging vehicle can be scheduled between 6:00 pm and 8:00 am). We refer to the proposed day-ahead load scheduling algorithm as offline scheduling with VR, and the proposed intra-day adjusting algorithm as online scheduling with VR in our simulation figures. A whole day is set as a scheduling period, which is divided into 24 scheduling time slots (i.e., 1 hour is a scheduling time slot per day). The compared load scheduling algorithm is proposed by Deng et al. [40], which only focuses on optimally scheduling the loads without considering voltage regulation. The data of elastic loads and inelastic load used in the simulations are listed in Table I. The appliance whose type is 0 means that it produces inelastic



TABLE I: Appliance Usage Pattern

| Appliance          | Operating Power (KW) | Required Load (KWh) | Start Time | Type | Scheduling Constraints |
|--------------------|----------------------|---------------------|------------|------|------------------------|
| Dishwasher         | 0.467                | 1.342               | 8,12,17    | 1    | 12-18                  |
| Electrical Vehicle | 4.500                | 16                  | 18         | 1    | 18-8                   |
| Air Conditioner    | 2.550                | 5                   | 19         | 1    | 18-21                  |
| Cloth-dryer        | 4.115                | 4.110               | 11         | 1    | 11-22                  |
| Refrigerator       | 0.265                | 1.32                | 0          | 0    |                        |
| Lighting           | 0.1                  | 1.00                | 18         | 0    |                        |
| Heating            | 0.300                | 7.10                | 0          | 0    |                        |
| Compensation       | 2                    | 3                   | 9          | 0    |                        |

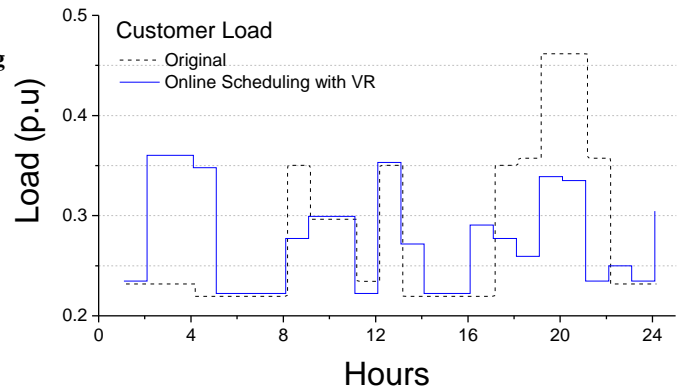


Fig. 6: Scheduled Customer Load.

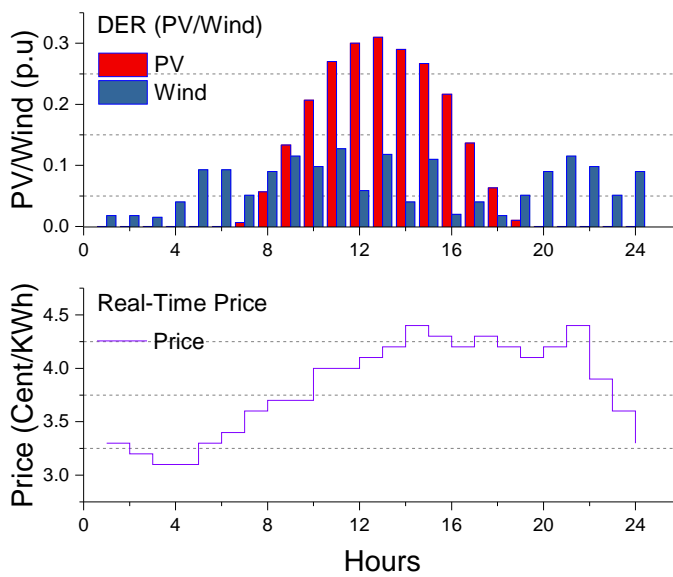


Fig. 5: Renewable Generation and Electricity Price.

loads. The simulations are based on Matlab R2012a using Matpower toolbox on Windows 10.

### B. Loading Scheduling and Electricity Price

Fig. 6 shows the comparison of original customer load and the scheduled customer load by the online scheduling algorithm. It can be seen from the figure that, by employing our algorithm, the customer load will be scheduled to reduce the peak load with high electricity price. Moreover, the online scheduling algorithm can leverage the generated electricity of DGs to meet the electricity demands of customers and sell the residual generated electricity at good price.

Fig. 7 compares the electricity costs of Deng's method and the online scheduling algorithm with VR. It can be seen that the online scheduling algorithm can significantly reduce the original electricity cost on different days. However, Deng's method can achieve lower electricity cost than the proposed algorithm. That is because Deng's method only focuses on achieving the optimal load scheduling without considering the

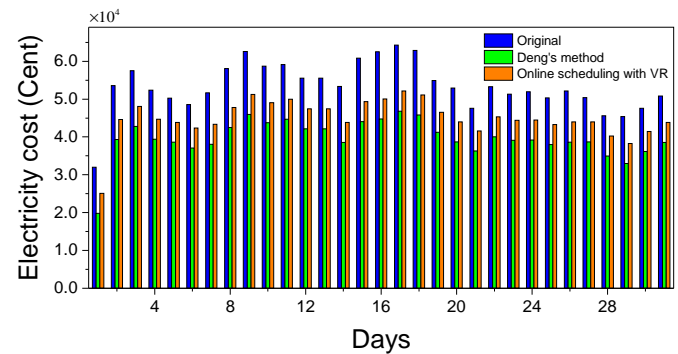


Fig. 7: Comparison of Electricity Cost.

constraint of voltage regulation. It can consequently achieve an improved electricity cost.

### C. Voltage Regulation

Fig. 8 presents the comparison of voltage violation probabilities, under Deng's method, the offline and online scheduling algorithms with VR. As shown in the figure, since we jointly consider load scheduling and voltage regulation, our proposed schemes can provide much lower voltage violation probabilities for the distribution system than Deng's method. Moreover, because the online scheduling can capture more accurate electricity generation and price information, the violation probability of online scheduling can be further reduced when compared to that of offline scheduling.

## VIII. CONCLUSION

In this paper, we have studied the joint load scheduling and voltage regulation problem in a power distribution system with renewable DGs. Based on the power flow analysis, the problem has been formulated as a constrained MINLP problem. We have addressed this problem by decomposing it into two independent subproblems that can be separately and efficiently solved. Furthermore, a grid-customer coordinated load scheduling strategy, including offline and online algorithms, has been proposed to simultaneously achieve improved load scheduling and voltage regulation. Finally, simulation

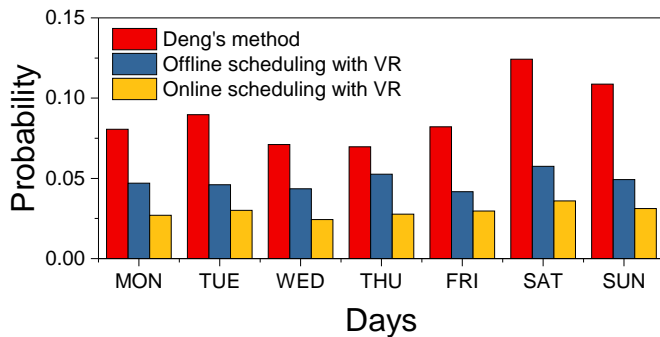


Fig. 8: Comparison of Voltage Violation Probability.

results based on realistic datasets have validated our theoretical analysis and demonstrated the high performance of our proposed strategy. In our future work, we will investigate the power distribution system where renewable generators are equipped with distributed electrical energy storages that can further assist the voltage regulation of the grid.

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