

The Role of Dominant Cause in Variation Reduction through Robust Parameter Design

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Reducing variation in key product features is a very important goal in process improvement. Finding and trying to control the cause(s) of variation is one way to reduce variability, but is not cost effective or even possible in some situations. In such cases, Robust Parameter Design (RPD) is an alternative. The goal in RPD is to reduce variation by reducing the sensitivity of the process to the sources of variation, rather than controlling these sources directly. That is, the goal is to find levels of the control inputs that minimize the output variation imposed on the process via the noise variables (causes). In the literature, a variety of experimental plans have been proposed for RPD, including Robustness, Desensitization and Taguchi’s method. In this paper, the efficiency of the alternative plans is compared in the situation where the most important source of variation, called the “Dominant Cause”, is known. It is shown that desensitization is the most appropriate approach for applying the RPD method to an existing process.

1 INTRODUCTION

Reducing variation in critical outputs is a very important goal in process improvement. Reviewing many variation reduction algorithms including the Shainin System (Shainin, 1992, 1993), DMAIC or Six Sigma (Harry and Schroeder, 2000), Scholtes algorithm (1988) and Statistical Engineering (Steiner and MacKay, 2005), indicates diagnostic and remedial journeys (see Figure 2.1), described by Juran and Gryna (1980) and Juran (1988), as the common element of these algorithms. During the diagnostic phase, the problem of process is investigated by examining its symptoms in order to find the causes of the problem. In the second phase, the remedial journey, we search for a solution. The idea is that if we know the cause of the problem, we are more likely to find efficient and effective solutions.

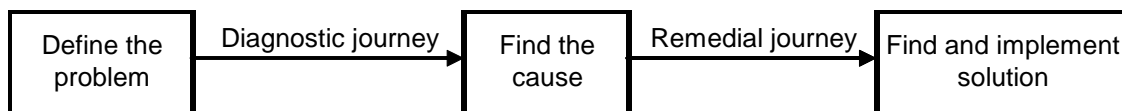


Figure 1.1: Common elements of well-known variation reduction algorithms

The inputs that operate on a system can be divided into two broad types (Wu and Hamada, 2000; Steiner and MacKay, 2005): varying inputs and fixed inputs. Varying inputs are process characteristics whose values change (unit to unit or time to time) in a process without deliberate intervention. Examples include: operators, pouring temperature, raw material characteristics and so forth. Fixed inputs, on the

other hand, are a process inputs/characteristics whose values can be adjusted, but remain fixed once they are chosen. These are parameters/factors that can be easily controlled and manipulate in a system's normal production. A cause of variation in process output is a varying input with the property that if all other inputs were held constant, then the output changes when the input changes. Note that although changing the level of a fixed input can be a solution for excessive variation in the output, a fixed input can not be a cause of variation in a process output (Steiner and MacKay, 2005). In the process improvement literature, varying and fixed inputs are also known as noise and control factors respectively (Wu and Hamada, 2000).

For any process there are a large number of causes, each with an effect. Applying the Pareto principle to the cause of variation, large effects can be attributable to only a few causes and these are called dominant causes (Steiner et al., 2007). A dominant cause(s) is varying input that has a large effect on the output with a relatively small change in its value. Juran and Gryna (1980, p. 105) define a dominant cause as "a major contributor to the existence of defects, and one which must be remedied before there can be an adequate solution". Consider the following simple model which describes relationship between a dominant cause (X) and an output (Y) (note that X and other varying inputs are assumed to be independent in this model). We can say the variable X is a dominant cause of output variation if standard deviation in the output due to X is large relative to the standard deviation due to rest of the causes (Steiner and MacKay, 2005).

$$Y = f(X) + \text{noise}$$

$$\text{sd}(Y) = \sqrt{\text{sd}(\text{due to } X)^2 + \text{sd}(\text{due to all other varying inputs})^2} \quad (1.1)$$

Throughout this paper we assume that a dominant cause(s) of variation in a process output exists and to simplify the language, we refer to a (single) dominant cause of variation, while recognizing that there may be more than one important cause.

Finding a dominant cause of variation in an output characteristic and trying to control and reduce its variation is one way to reduce variation. In some instances, however, the dominant cause may be difficult, expensive or even impossible to control in a system's normal production or usage condition. In these cases, finding some fixed input and identifying new settings for them which will make the process output less sensitive to changes in the dominant cause is a possible solution (Steiner and MacKay, 2005). This idea is known as Robust Parameter Design (RPD) or simply Parameter Design which was popularized and introduced in the United States in the 1980s by the Japanese engineer, Genichi Taguchi, (Taguchi, 1987; Ross, 1988; Taguchi and Wu, 1980; Kacker, 1985). The term parameter design comes from an engineering tradition of referring to product characteristics as product parameters (Taguchi and Wu, 1980). Parameter design works by identifying appropriate settings of some fixed inputs to exploit interactions between the fixed inputs and the dominant cause to reduce the variation in the output without the necessity of reducing the variation in dominant cause. Robust parameter design problems may arise in all three stages of the product development cycle: product design, process design and manufacturing. Despite Taguchi's suggestion that countermeasures against variation caused by environmental variables and product deterioration are best built into the product at the product design stage (Taguchi, 1987 and Kacker and Phadke, 1981), reviewing case studies given in ASI (1985 and 1986) reveals that Taguchi's method to RPD problems or a specific version of it, called "Robustness" in this paper, are mostly used in the manufacturing stage. In this paper, "Desensitization" is presented as an alternative to the robustness/Taguchi method and as the most appropriate approach to deal with RPD problems at the manufacturing stage in which, unlike product or process design stages, the main

source(s) of variation can in many cases be identified by observing the existing process. The efficiency of desensitization is examined and compared with the robustness and Taguchi's approaches to the RPD in the situation where a dominant cause of output variation exists and can be found.

2 EXPERIMENTAL PLANS FOR FINDING A ROBUST SOLUTION

The goal in robust parameter design is to find new levels for fixed inputs that reduce the output variation. Since the value of a fixed input doesn't normally change in the process, an experiment needs to be conducted in which we assign different levels to the selected fixed inputs and we examine the effect of those new settings on the output mean and variation. The goal of such an experiment is to find and exploit a favorable interaction between the selected fixed inputs (or candidates) and the dominant cause that makes process output less sensitive to uncontrollable changes in the dominant cause. In practice, process analysts have used at least three different types of experiments to find robust process settings. The first approach, called a desensitization experiment is useful within the Statistical Engineering algorithm as by Steiner and MacKay (2005). In the Statistical Engineering algorithm we first look for a dominant cause using observational studies and then run a desensitization experiment in which we also deliberately control the levels of the identified dominant cause. The second approach is to conduct a so called robustness experiment involving selected fixed inputs only. For the third option, an experiment is run with selected fixed inputs and a range of varying inputs that the experimenter believes are likely to be important causes. We call the third option a Taguchi experiment, although option #2 is also sometimes called a Taguchi experiment. Desensitization, robustness and Taguchi style experiments are described in the next sections as the three major experimental plans for finding a robust solution.

2.1 Robustness Experiment

In a robustness experiment a group of fixed inputs (called candidates) are selected based on engineering judgment and their effects on the output variation are examined. The experiment can be a full factorial or fractional factorial design. Once the candidates are identified, they will be systematically changed in the robustness experiment and a performance measure (usually the standard deviation of the output) will be recorded for each run. Then, the main effect and interaction plots are used to draw conclusions. A regression model can also be used to model log(s) as a function of the important effects and then the levels of candidates (fixed inputs) that minimize this function are suggested as the robust solution. Since knowledge of the dominant cause is not available, the length of experiment, the number of runs, the number of repeats in each run, and candidates are determined only based on engineering knowledge and the past experience of experimenters/analysts. A famous positive example of an application of a robustness experiment is a case study reported by Quinlan (1985) on speedometer cables. Shrinkage in the plastic casing material can sometimes make speedometer cables noisy. So a project was initiated to reduce variation in postextrusion shrinkage of the casing for the speedometer cable. When the team's efforts to find the cause of the shrinkage variation failed, they chose 15 fixed inputs and selected one new level for each. They then ran a two-level (one level of each candidate was the existing level) experiment with 16 runs (i.e. a 2_{III}^{15-11} fractional design). For each run 3000 feet of plastic casing were produced. Four samples were haphazardly cut out from each run and the percentage shrinkage measured on each specimen. Then, a performance measure (standard deviation of percentage shrinkage) was calculated (for each run) using the four sample values. Finally, the best combination of levels to reduce the variation was found. The new levels were confirmed and the process was improved.

As illustrated by the Speedometer Cable example, the robustness approach can be successful; however, there are some substantial drawbacks. To limit interference with regular production the robustness experiment is usually run over a short time (ASI, 1985; ASI, 1986). As a consequence there is a risk of running a high-cost experiment with no return, since if the dominant cause does not act with each run of the experiment and/or if the candidates (selected fixed inputs) do not include the one(s) that have interaction with dominant cause the robustness experiment will fail. We conclude that to have any hope of success in a robustness experiment the unknown dominant cause should act in the short-term family of variation (part-to-part for example). Otherwise the run lengths need to be very long to allow the dominant cause time to act during the experiment. If the dominant cause does not act within each run, it will not be possible to find a favorable cause/candidate interaction even if one exists.

2.2 Taguchi Method Experiment

We now consider the second experimental approach, a Taguchi experiment. Taguchi recommends a crossed array design for planning the experiment (Wu and Hamada, 2000). The *Inner-outer array* is a key concept in a crossed design or Taguchi’s approach to robust parameter design. In this approach a two-part experimental design is recommended. The Outer array (noise array) sets the levels of varying inputs while the inner array (control array) defines the treatments in terms of the levels of fixed inputs (Nair, 1992). Usually a 2^k or 2^{k-p} experiment is used for the inner array and a full factorial experiment is used for the outer array (Ross, 1988; Montgomery, 2001).

Each treatment combination in the control (inner) array is crossed with all level combinations in the noise (outer) array (Figure 2.1). Shoemaker et al. (1991) call this setup a product array since the outer array is run for every row in the control array.

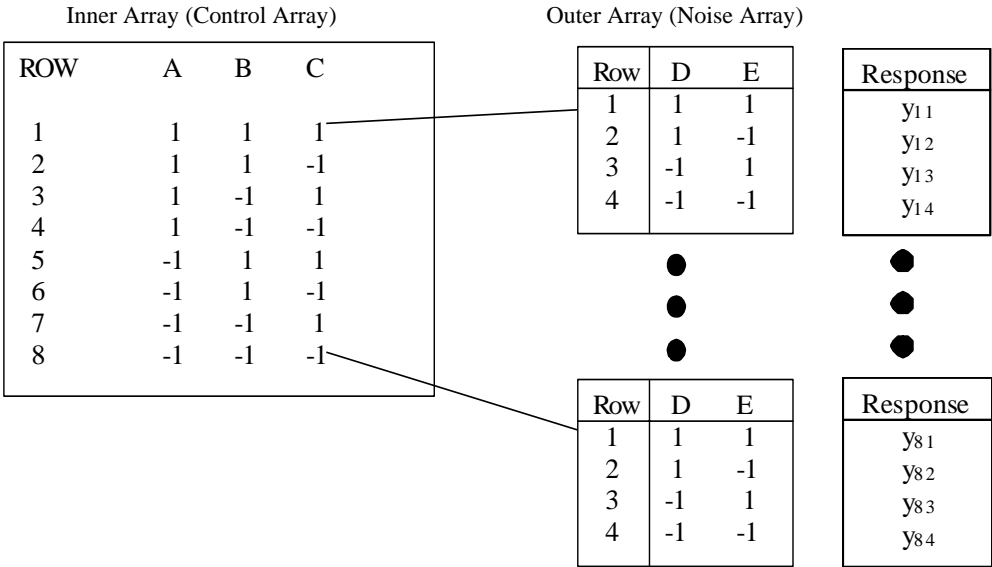


Figure 2.1: Product array in the Taguchi method for Robust Design

To define some notation, let y_{ij} be the observed response when the inner array is at its i th treatment combination and the outer array is at its j th treatment combination. Then, assuming there are “a” treatments in the inner array and “b” treatments in the outer array the typical data for Taguchi experiment with a product array design will appear as in Table 2.1.

Table 2.1: General arrangement for a Taguchi experiment – product array

		treatment combinations of outer array			
		1	2	...	b
treatment combinations of inner array	1	y_{11}	y_{12}		y_{1b}
	2	y_{21}	y_{22}		y_{2b}
	⋮				
	a	y_{a1}	y_{a2}		y_{ab}

For each inner array treatment then, \bar{y}_i and s_i can be defined as:

$$\bar{y}_i = \frac{\sum_{j=1}^b y_{ij}}{b} \quad i = 1, 2, \dots, a \quad (2.1)$$

$$s_i = \sqrt{\frac{\sum_{j=1}^b (y_{ij} - \bar{y})^2}{b-1}} \quad i = 1, 2, \dots, a \quad (2.2)$$

So, unlike robustness we now deliberately manipulate or control some noise factors. In this type of experiment once the noise factors (varying inputs) are selected, they should be systematically varied to reflect their variation in normal condition. So, the levels of noise factors are fixed during the experiment. Identifying optimal parameter settings in a Taguchi experiment requires specifying a criterion that is to be optimized. Taguchi suggests combining the mean and the variance, for each inner array treatment, into a single performance measure known as the signal-to-noise ratio (Kackar, 1985).

To derive conclusions, Taguchi recommends analyzing the mean response for each run in the inner array and also analyzing variation using an appropriate signal-to-noise ratio. Signal-to-noise ratios are derived from the quadratic loss function and the goal of quality improvement can be stated as attempting to maximize the signal-to-noise (S/N) ratio. Considering Table 2.1 the three of S/N ratios which are "standard" and widely applicable (Montgomery, 2001; Wu et al., 2000) is calculated for each i as follows:

1. Nominal is best: i.e. you ideally want all output values to be equal to a target value

$$S/N_T = 10 \log \left(\frac{\bar{y}^2}{s^2} \right)$$

where \bar{y} and s^2 are defined by Equations (2.1) and (2.2) respectively. This signal-to-noise ratio is applicable whenever there is a target value and a two side specification. For example, the size of piston

rings for an automobile engine must within the lower and upper limits and ideally close to a target to ensure product's high quality.

2. Larger the better: i.e. you want to maximize the output characteristics, e.g. breaking strength

$$S/N_L = -10 \log \left(\frac{1}{b} \sum_{j=1}^b \frac{1}{y_{ij}^2} \right)$$

where b is the number of observations at each treatment.

3. Smaller the better: i.e. you want to minimize the output characteristics, e.g. out of roundness

$$S/N_S = -10 \log \left(\frac{1}{b} \sum_{j=1}^b y_{ij}^2 \right)$$

Taguchi's methods of using the S/Ns in the analysis are detailed in Taguchi (1987b) and Wu & Hamada (2001). To illustrate, a case study, originally reported by Miller et al. (1993), is considered. We will use this example later to compare Taguchi, desensitization and robustness approaches in a simulation study. In automotive manufacturing, the drive pinion and gear "set" provides the transmission of power from the vehicle drive shaft to the rear axle. The parts are heat-treated to improve strength and wear characteristics. A quality problem arose from part distortion during heat-treatment, and a Taguchi style experiment was conducted in the attempt to find a way to improve the process. The five control factors (A-E) and three noise factors (F-H) are given in Table 2.2.

Table 2.2: The control and noise factors for the Gear experiment

Control Factors	Noise Factors
A carbon potential	F furnace track
B operating mode	G tooth size
C last zone temperature	H part position
D quench oil temperature	
E quench oil agitation	

The design matrix and response data are given in Table 2.3. The response is the dishing of the gear. Two levels were considered for each of the factors. A 2^{5-1} fractional factorial design was used for the inner (control) array and a 2^3 full factorial design was used for the outer (noise) array. There are $16 \times 8 = 128$ runs in total. The purpose of experiment was to find a way to run the process that has less gear dishing variation around a target value.

As the objective was to reduce the variation of response around a target value (nominal the best), S/N_T is used by experimenters. The last two columns of Table 2.3 contain \bar{y} and S/N_T values for each of the 16 inner-array runs.

Table 2.3: Design matrix and response data for the Gear experiment

														<i>Outer Array</i>								
														F	1	1	1	1	-1	-1	-1	-1
														G	1	1	-1	-1	1	1	-1	-1
														H	1	-1	1	-1	1	-1	1	-1
														<i>Inner Array</i>								
Run	A	B	C	D	E									Y bar	S/N _T							
1	1	1	1	1	1	7	12	6.5	14	3	14	4	16.5	9.625	5.4856							
2	1	1	1	-1	-1	13.5	14.5	5.5	17	-7.5	15	-4.5	12	8.1875	-1.2167							
3	1	1	-1	1	-1	3	11	5.5	18	3	19	1	21	10.188	1.9288							
4	1	1	-1	-1	1	10.5	14.5	6.5	17.5	3	14.5	9	24	12.438	5.4641							
5	1	-1	1	1	-1	10	23	3.5	23	4.5	25.5	10	21	15.063	4.4752							
6	1	-1	1	-1	1	6.5	22	14.5	23	5.5	18.5	8	21.5	14.938	6.1476							
7	1	-1	-1	1	1	5.5	28	7.5	28	4	27.5	10.5	30	17.625	3.5878							
8	1	-1	-1	-1	-1	4	14	6.5	23	9	25.5	9	24.5	14.438	4.4127							
9	-1	1	1	1	-1	-4	18.5	11.5	26	-0.5	13	0	16.5	10.125	-0.4057							
10	-1	1	1	-1	1	9	19	17.5	21	0.5	20	6.5	18	13.938	5.2955							
11	-1	1	-1	1	1	17.5	20	10	23	6.5	21.5	0	26	15.563	4.6716							
12	-1	1	-1	-1	-1	7	23.5	1	20	7	22.5	4	22.5	13.438	2.9881							
13	-1	-1	1	1	1	2.5	22	12	19.5	7	27.5	8.5	23.5	15.313	4.6048							
14	-1	-1	1	-1	-1	24	26	14.5	27.5	7	22.5	13	22	19.563	8.6539							
15	-1	-1	-1	1	-1	5.5	27	2.5	31	12.5	27	11.5	32.5	18.688	3.854							
16	-1	-1	-1	-1	1	11	21.5	12	27	16.5	29.5	16	28.5	20.25	8.708							

One approach to the analysis of this experiment is based on the “play the winners” rule. With this analysis we look for the treatment combination(s) that maximizes S/N_T. As can be seen in Table 2.3 the last treatment combination maximizes the signal-to-noise ratio and setting factors A, B, C, D to their low levels and E to its high level is the recommended solution based upon this rule. An alternative analysis involves using analysis of variance (Montgomery, 2001) or the half-normal and main effect plots (Wu and Hamada, 2000) to determine the main factors that influence the signal-to-noise ratio. For the Gear experiment, Table 2.4 and Figure 2.2 show that operating mode (B) and quench oil agitation (E) are marginally significant control factors.

Table 2.4: Estimated effects and coefficients for S/NT in the Gear experiment

Term	Effect	Coef	SE Coef	T	P
Constant		4.291	0.5230	8.20	0.000
A	-1.011	-0.505	0.5230	-0.97	0.357
B	-2.529	-1.265	0.5230	-2.42	0.036
C	-0.322	-0.161	0.5230	-0.31	0.765
D	-1.531	-0.766	0.5230	-1.46	0.174
E	2.409	1.205	0.5230	2.30	0.044

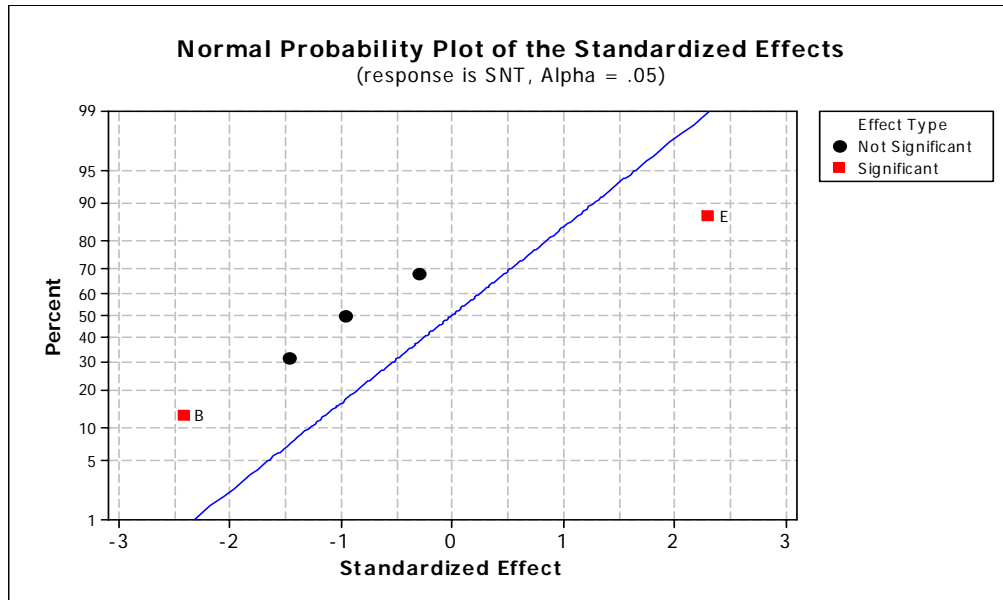


Figure 2.2: The normal probability plot of the effects for the Gear experiment

Once the significant control factors are determined, two different ways for deriving conclusions can be used. First, graphs of the main effects, called "marginal graphs" by Taguchi, are employed to find the robust solution. Figure 2.3 illustrates these graphs for the Gear example. The usual approach is to examine the graphs and "pick the winner" (Montgomery, 2001). In this case, factors B and E have larger effects than the others. As the objective is to maximize S/N_T , the low level of factor B and the high level of factor E are recommended as a robust solution.

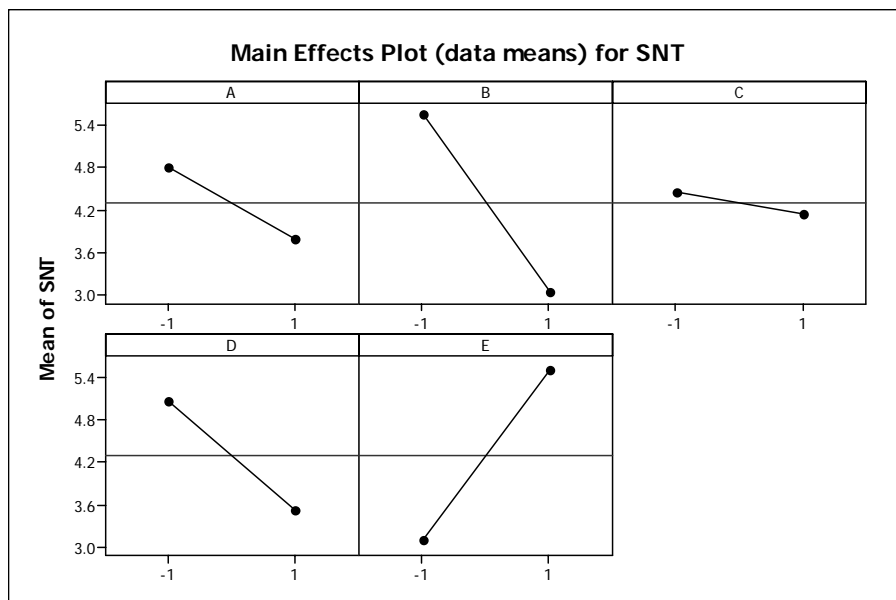


Figure 2.3: The main effect plot for the S/N_T in the Gear experiment

The graphical analysis can be supplemented with a regression model of the signal-to-noise ratio (Wu and Hamada, 2000). A regression model is used to model S/NT in terms of the significant control factors and the robust solution is obtained by maximizing the function. Based on Table 2.4 the corresponding signal-to-noise ratio model for the Gear experiment is:

$$S/N_T = 4.29 - 1.265 X_B + 1.205 X_E$$

To maximize this function we would select the low level of factor B and the high level of factor E which is the same conclusion as in the graphical approach. This kind of model building analysis of Taguchi experiments is called “loss-model analysis” in the literature.

Taguchi advocates claim that the use of the S/N ratio generally eliminates the need for examining specific interactions between the control and noise factors (Montgomery, 2001). However, we believe that examining control-noise interactions by either including noise terms in the response model or exploring the corresponding interaction plots can improve the efficiency of experiment and has the advantages of yielding additional information about the specific noise-control interactions that may allow reduction of output variability induced by varying (noise) inputs. Shoemaker et al. (1991) point out this drawback of the loss-model approach, but the role of the dominant cause in improving the efficiency of the experiment and the advantages of knowing the dominant cause in the planning stage of the experiment have not been given much attention.

Taguchi recommends using engineering judgment to select noise factors and assumes that the choice includes all important noise factors. Since we do not assume a known dominant cause(s), selecting noise factors and determining noise factor extreme levels is difficult in Taguchi experiment. This coupled with the difficulty of choosing appropriate fixed inputs usually leads to a large experiment. In the Gear experiment, for instance, 128 tests were run to try to find a robust solution. We show in Section 3.3 that only one of the three noise factors is a large cause and we could have gained this knowledge using inexpensive observational investigations before running the Taguchi experiment. Excluding two other noise factors from outer array (i.e. using desensitization experiment) can reduce the number of runs to $16 \times 2 = 32$ without reducing the efficiency of experiment. Some critics of Taguchi (e.g. Shoemaker et al., 1991 and Miller et al., 1993), recommend using a combined array instead of crossed array to reduce the number of runs, but we believe that a more critical issue is finding the dominant cause before proceeding with an experiment. This not only reduces the number of runs (by removing ineffective factors from outer and inner array) but is also, as shown later, more efficient.

2.3 Desensitization

In a desensitization experiment we choose a number of fixed inputs (candidates), based on knowledge of the dominant cause supplemented by engineering knowledge. We use an experimental plan to determine if these candidates and their new settings will make the process less sensitive to variation in the dominant cause.

Desensitization can be considered a version of the Taguchi method to RPD problem in which only the dominant cause is involved in outer array. Steiner and MacKay (2005) suggest using a full factorial design for the candidates, if there are three or fewer, and using a fractional design with resolution at least III otherwise. They also recommend selecting two levels for the dominant cause at the extremes of its normal range and using a crossed design where, for each treatment combination of candidates, there are runs for both levels of the dominant cause. Comparing desensitization and robust experiments, having knowledge of dominant cause reduces the size of the outer array and can lead better choices of

candidates for the inner array. Thus, desensitization experiments usually require fewer runs which reduces the cost and complexity of experiment. Also note that once a dominant cause is identified, in some instances, the remedy is obvious (dominant cause is controllable) and no further investigations are needed. Statistical Engineering methodology (Steiner and Mackay, 2005) and some other variation reduction approaches like Shainin System and Six Sigma (Steiner et al., 2007) present a diagnostic journey for finding the dominant cause using progressive search and observational investigations. Generally observational studies are cheaper than experimental investigations because changing process settings and interrupting normal operations of the process are not needed. The knowledge of the dominant cause also assist us in selecting appropriate levels of dominant cause which makes our experiment more effective.

Like the robustness and Taguchi method, analysis of a desensitization experiment can be carried out graphically or using a regression model. Drawing a plot of the output by each treatment is first step in the graphical analysis to look for promising treatment combinations. Then, all cause by candidate interaction effects plots are drawn and finally the levels of candidates that make the output less sensitive to variation in the dominant cause are determined by examining these plots.

To analyze the results of desensitization experiment using a statistical model, a regression model, known as “response model”, is employed to model the response (output) in terms of the control factors and the two term interactions of the control factors and the noise factor. A robust solution can be determined by minimizing the standard deviation of output based on the response model.

3 DESENSITIZATION VERSUS ROBUSTNESS AND THE TAGUCHI METHOD

3.1 Qualitative Comparison

Desensitization experiments have the following advantages over robustness and Taguchi style experiments:

- As mentioned in Section 3.2, in robustness experiments fixed inputs (candidates) are selected based only on engineering knowledge whereas in desensitization experiments engineering judgment is supplemented by knowledge of the dominant cause. Considering the dominant cause, the analyst tries to choose only fixed inputs that she/he feels are likely to have a favorable interaction with the dominant cause. This smart selection can improve the effectiveness of experiment. Generally, the more you know about the dominant cause of variation, the greater the chance you will select fixed inputs to change that will mitigate the variation in the dominant cause.
- Knowing the dominant cause in desensitization can also help experimenters reduce the size of outer array. Including only the dominant cause decreases the total number of experimental runs when comparing desensitization to a Taguchi style experiment. Fewer runs leads to an easier, cheaper and shorter experiment.
- Since noise factors or varying inputs are usually hard to control in the normal process operation, running a Taguchi experiment may be difficult, costly or sometimes impossible since you have a number of noise factors in the outer array and you need to fix the levels of these factors in each run of the experiment. This problem is mitigated somewhat in the desensitization approach that recommends an outer array defined only using the dominant cause.

- Having the dominant cause as a factor in the desensitization experiments, allows the analyst to model interactions between the dominant cause and the candidates directly whereas in the robustness experiments this interaction can not be assessed directly since dominant cause is not included as one of the experiment factors.
- As mentioned before, the desensitization approach recommends first finding the dominant cause of variation and then if the dominant cause is not controllable, running a desensitization experiment. In some situations, once a dominant cause is identified, the remedy is obvious and no further investigations are needed. In these cases the dominant cause is controllable and variation in the output can be reduced by reducing the variability of the dominant cause.
- Conducting baseline and observational investigations, as recommended by desensitization approach for finding the dominant cause, provides useful information about how the process operates under current conditions. This information can be used to specify the problem goal by stating how the baseline should be changed. Although experimenters who follow the Taguchi or robustness method may also conduct these kind of investigations before proceeding to experimental investigations. However, conducting observational studies before any experimental investigations is not explicitly mentioned in Taguchi or robustness literature. In desensitization approach, however, conducting observational experiments for finding the dominant cause is a requirement. So, the likelihood of limited information about the current process is high in the Taguchi or robustness methods and this is another drawback of these methods. Recall the examples presented in Sections 3.2 and 3.3; if none of the runs represent the current setup of the process, how can experimenters be sure that the new setting, recommended by experiment, improves the process? The recommended robust solution may be much better than other settings used in the experiment, but still worse than the existing setting.
- One of the most important requirements for a robustness experiment to be successful is that the unknown dominant cause acts in a short-term family of variation (Steiner and MacKay, 2005). This is important because the length of each run in a robustness experiment must be long enough to be sure that the dominant cause will vary over (close to) its full range within each run. Otherwise assessing the interaction between a dominant cause and the candidates is not possible (even indirectly) and we will not be able to see if any candidate settings make the process robust to the variation in the unknown dominant cause. If experimenters do not have any information about the time nature of the dominant cause they do not have any idea about the desired length of the experiment runs. If they know the unknown dominant cause acts in a time-to-time family, it will likely not be feasible to conduct a robustness experiment since the runs would need to be too long. In the desensitization experiment, however, the length of runs is not an important issue because we include the dominant cause in the experiment and we select two levels for the dominant cause at the extremes of its normal range which can reflect the full extent of output variation and this allows the experimenter to reasonably evaluate the effect of different settings of control factors and their interaction with the dominant cause on the output variation.
- As mentioned in the Section 3.3, Taguchi recommends using engineering judgment for selecting noise factors and assumes that the choice includes all the important noise factors. However, without substantial process knowledge and/or extensive preliminary investigations (as recommended in the desensitization approach) a poor choice of noise factors is possible.

We will consider this issue in the next chapter where it is shown that the effectiveness of a Taguchi method experiment depends critically on the choice of noise factors.

- In a desensitization experiment, the experimenter selects extreme levels of the dominant cause using information from preliminary investigations (conducted earlier when searching for the dominant cause). In Taguchi method, however, this information might not be available for experimenters since they are not required to conduct such preliminary investigations before conducting the experimental investigation; So, for Taguchi experiments we only on engineering judgment and past experience for selecting the levels of noise factors.
- Regarding model based analysis, using the response model in the desensitization approach is an advantage in comparison to the robustness and Taguchi approaches in which constructing a loss-model is recommended for the analysis. In the loss-model approach focus is on modeling the optimization criterion, signal-to-noise ratio in Taguchi experiments and usually log(s) in robustness experiments, which is a nonlinear, many-to-one transformation of response and It is shown by Shoemaker et al. (1991) that modeling the optimization criterion may hide some of the relationship between individual control and noise factors and it is less likely that the optimization criterion can be a low-order linear model. Shoemaker et al. (1991) give an elaborated comparison between the loss-model approach over response model approach in data analysis.

Considering all these qualitative reasons, we conclude desensitization experiments are more effective than robustness and Taguchi method experiments. This is shown quantitatively in the next section.

3.2 Quantitative Comparison

3.2.1 Modeling

To start, we consider the simplest situation where we have just one fixed input and only one dominant cause. Then, the idea of desensitization and robustness can be demonstrated by considering the following regression model:

$$Y = \beta_0 + \beta_1 X + \beta_2 z + \beta_3 Xz + R \quad (3.1)$$

where, Y represents a random variable that describes the possible values of output characteristic; X represents a random variable that describes the possible values of the dominant cause; z represents the levels of desensitizer (the fixed input that can desensitize the output to variation in the dominant cause) and R is a random variable that describes the effect of all other varying inputs on the response.

Equation (3.1) can be rewritten as:

$$Y = \beta_0 + (\beta_1 + \beta_3 z)X + \beta_2 z + R \quad (3.2)$$

If z_0 is the value of z in the current process, $\beta_1 + \beta_3 z_0$ is the slope of the relationship between the dominant cause (X) and the output (Y) with the current process settings (see Figure 3.1).

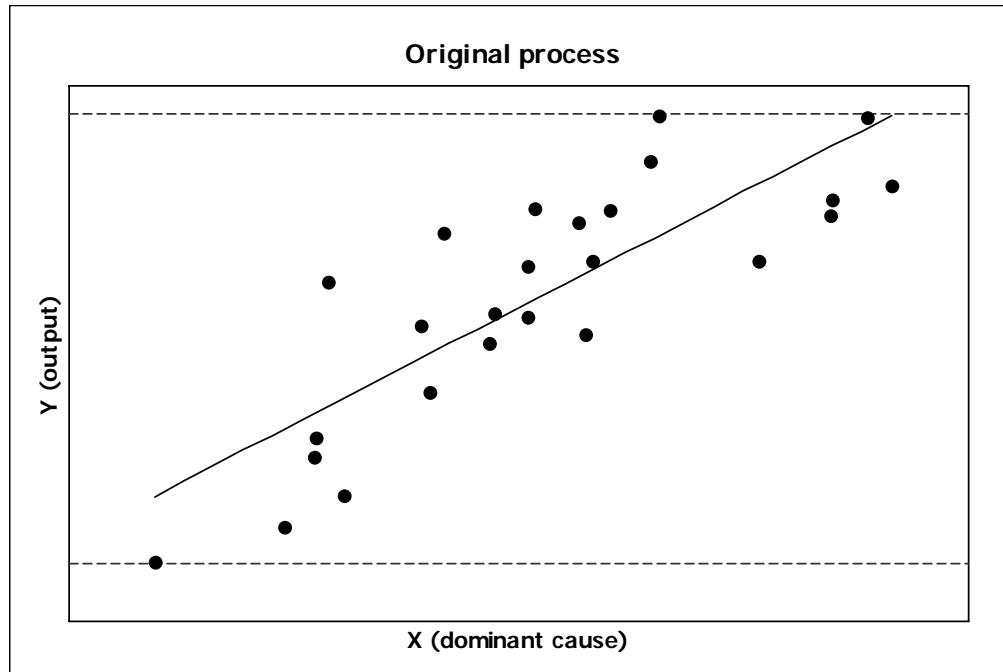


Figure 3.1: Range of output values in the current process ($z = z_0$)

Assuming the effect of all other causes, R , vary independently of the dominant cause, we can estimate the standard deviation of output using Equation (3.3).

$$\text{sd}(Y) = \sqrt{(\beta_1 + \beta_3 z)^2 \sigma_x^2 + \sigma_r^2} \quad (3.3)$$

where, σ_x^2 and σ_r^2 are the variances of the dominant cause and residuals respectively. The purpose of desensitization and robustness experiments is to find a new setting for z that flattens the relationship between output and dominant cause. This means we are looking for a new level of z , say z^* , where $\beta_1 + \beta_3 z^*$ is closer to zero than $\beta_1 + \beta_3 z_0$. With this change, while we continue to live with the variation of dominant cause (recall that we use these approaches when the dominant cause is hard to control or uncontrollable), we reduce the output variation (Figure 3.2) using the Xz interaction. We refer to this as a favorable interaction between X , a dominant cause, and z a (normally) fixed input.

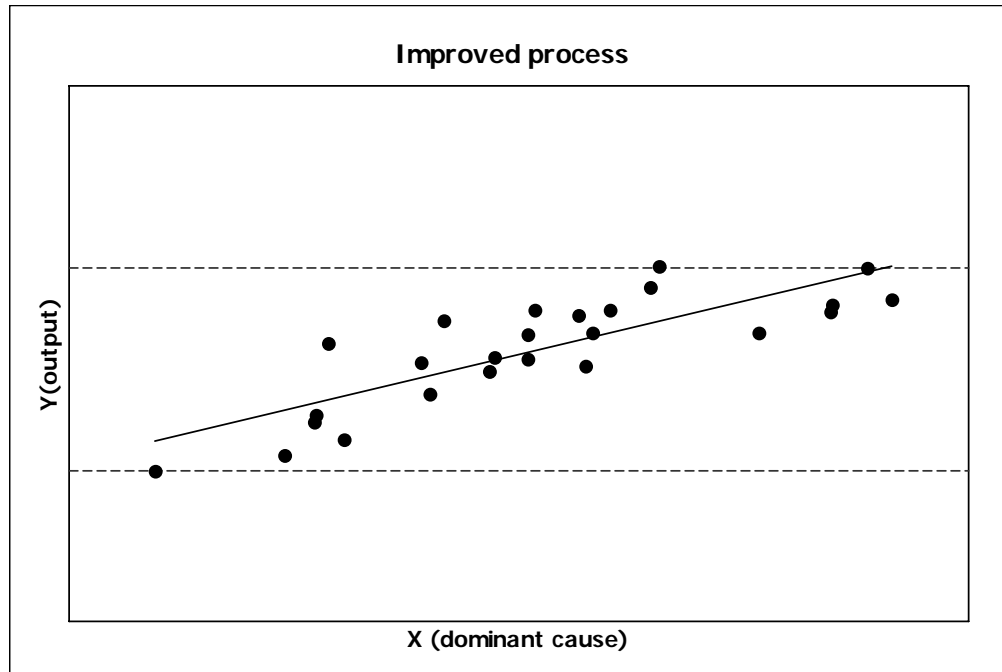


Figure 3.2: Range of output values with new setting ($z = z^*$)

The purpose of robustness and desensitization experiments is the same; however, in the robustness approach we assume that the dominant cause is not known and the experimenter tries to find the appropriate level of z without having the knowledge of a specific dominant cause.

3.2.2 Performance Measure

To compare the efficiency of desensitization and robustness experiments we need a performance measure. The method that provides a better prediction of output variation will be better at determining the best choice of the levels of the candidates. One way to define “good” prediction is to require the method have a reasonably consistent variance of the estimated response at points of interest (at specific levels of control factors used in the experiment). Consistent variance can be interpreted by smaller variation in estimated variance of output in either approach. So, we introduce the standard deviation of estimated response variance as a measure of efficiency or performance index, denoted by $\text{Std}(P)$ in this thesis. Next, we formulate $\text{Std}(P)$ for each method and then we compare each method using these formulated performance measures. The smaller the performance index the better.

In the case of desensitization, we first look for the dominant cause using observational investigations and a process of elimination (Shainin, 1993b; Steiner and MacKay, 2005), called the progressive search method. As such, to start we assume the standard deviation of dominant cause (σ_x), the slope of the relationship between the dominant cause and the output (i.e. $\beta_1 + \beta_3 z_0$), and the standard deviation of residuals (σ_r), are known from our prior investigations. In Section 3.2.4 we relax this assumption. The elimination method is detailed in Steiner and MacKay (2005) and we will describe it briefly later.

The model parameters are determined from our baseline investigation, an “input-output” investigation, and other preliminary enquiries for finding and verifying the dominant cause. Assuming σ_x , σ_r , and $\beta_1 + \beta_3 z_0$ are known and the current value of z (i.e. z_0) is equal to zero, the standard deviation of the

output can be estimated with a desensitization experiment by estimating β_3 (denote the corresponding estimator as $\tilde{\beta}_3$). Thus, if we define P_{des} as

$$P_{des} = \text{Var}(Y | z = z_1) = (\beta_1 + \tilde{\beta}_3 z_1)^2 \sigma_x^2 + \sigma_r^2 \quad (3.4)$$

The performance index in the case of desensitization is the standard deviation of P_{des} (i.e. $\text{Std}(P_{des})$).

In the robustness method, on the other hand, we estimate the standard deviation of output directly based on the experiment results. This means that P_{rob} is defined as:

$$P_{rob} = \text{Var}(Y | z = z_1) = s^2 \quad (3.5)$$

where s^2 is the sample variance of robustness experiment results when $z=z_1$.

Thus, the performance index in this case can be presented as the standard deviation of P_{rob} (i.e. $\text{Std}(P_{rob})$). Now, we derive $\text{Std}(P_{des})$ and $\text{Std}(P_{rob})$. For the case that was modeled and describe early, we have z as the fixed input or control factor and X as the dominant cause in the desensitization experiment; each at two levels (say $\pm a$ for z , where “a” is a constant value, and $\mu_x \pm 2\sigma_x$ for x which are extreme levels of x). Using a crossed design, there are runs for both levels of the dominant cause for each treatment (each level of z). For the robustness experiment we have only a fixed input or z with the same levels in desensitization experiment (i.e. $\pm a$). To be fair we compare desensitization and robustness experiments with the same number of runs. This means that if we have k replicates in the desensitization experiment, the number of replicates will be equal to $2k$ in the robustness experiment. The desensitization and robustness experiment plans for a simple case ($k=2$) are given in Tables 3.1.a and 3.1.b respectively.

Table 3.1: Design matrix for desensitization and robustness experiment ($k=2$ here)

Treatment	Run	z	x	Y
1	1	+a	$\mu_x + 2\sigma_x$	y_1
2	2	+a	$\mu_x - 2\sigma_x$	y_2
3	3	-a	$\mu_x + 2\sigma_x$	y_3
4	4	-a	$\mu_x - 2\sigma_x$	y_4
1	5	+a	$\mu_x + 2\sigma_x$	y_5
2	6	+a	$\mu_x - 2\sigma_x$	y_6
3	7	-a	$\mu_x + 2\sigma_x$	y_7
4	8	-a	$\mu_x - 2\sigma_x$	y_8

first replicate

second replicate

Treatment	Run	z	Y
1	1	+a	y_1
2	2	-a	y_2
1	3	+a	y_3
2	4	-a	y_4
1	5	+a	y_5
2	6	-a	y_6
1	7	+a	y_7
2	8	-a	y_8

1st replicate

2nd replicate

3rd replicate

4th replicate

a. Desensitization experiment with k replicates **b.** Robustness experiment with $2k$ replicates

Note that the only random variable in Equation (3.4) is $\tilde{\beta}_3$ and before formulating the $\text{Std}(P_{des})$ we need to determine variance of $\tilde{\beta}_3$. This variance can be determined using a regression model that we fit based on the desensitization experiment’s results. Note that with $z_0 = 0$, knowing $\beta_1 + \beta_3 z_0$ we know β_1 .

The regression model is presented as:

$$Y_i = \beta_0 + \beta_1 x_i + \beta_2 z_i + \beta_3 x_i z_i + R_i \quad i = 1, 2, \dots, 4k \quad \text{and} \quad R_i \sim N(\mu_r, \sigma_r^2)$$

or

$$Y_i - \beta_1 x_i = \beta_0 + \beta_2 z_i + \beta_3 x_i z_i + R_i ;$$

This model may be written in matrix notation as:

$$Z = X\tilde{\beta}$$

where

$$Z = Y_i - \beta_1 x_i \quad \tilde{\beta} = \begin{bmatrix} \beta_0 \\ \beta_2 \\ \beta_3 \end{bmatrix} \quad \text{and} \quad X = \begin{bmatrix} 1 & z_1 & x_1 z_1 \\ 1 & z_2 & x_2 z_2 \\ \vdots & \vdots & \vdots \\ 1 & z_{4k} & x_{4k} z_{4k} \end{bmatrix}$$

Using standard regression results (Montgomery, 2001) the variance of $\tilde{\beta}$ is expressed in covariance matrix:

$$\text{COV}(\tilde{\beta}) = \sigma_r^2 (X^T X)^{-1}$$

a symmetric matrix whose diagonal entries give the variance of the individual regression coefficient $\tilde{\beta}$. Thus, $\text{VAR}(\tilde{\beta}_3)$ is equal to $\sigma_r^2 (X^T X)^{-1}_{33}$ where $(X^T X)^{-1}_{33}$ is the 3th main diagonal element of the matrix $(X^T X)^{-1}$. Considering the design matrix of desensitization experiment the $(X^T X)^{-1}$ matrix can be calculated. By calculating $(X^T X)^{-1}$ we can see that its 3th diagonal element is $\frac{1}{16ka^2\sigma_x^2}$ where “k” is the number of replicates in the desensitization experiment and “a” is the absolute value of the levels of z. Accordingly $\text{VAR}(\tilde{\beta}_3)$ is equal to $\frac{\sigma_r^2}{16ka^2\sigma_x^2}$ (i.e. $\tilde{\beta}_3 \sim N(\beta_3, \frac{\sigma_r^2}{16ka^2\sigma_x^2})$).

To find the Std (P_{des}), we denote $\beta_1 + \tilde{\beta}_3 z$ as “A” in Equation (3.4) and rewrite the equation as:

$$P_{\text{des}} = A^2 \sigma_x^2 + \sigma_x^2, \quad A \sim N(\mu_A, \sigma_A^2) \quad (3.6)$$

In above equation, “A” is a random variable and σ_x^2 & σ_r^2 are constants, so

$$\text{VAR}(P_{\text{des}}) = \sigma_x^4 \text{VAR}(A^2) \quad (3.7)$$

where

$$A = \beta_1 + \tilde{\beta}_3 z ; E(A) = \mu_A = \beta_1 + \tilde{\beta}_3 z \quad \text{and}$$

$$\text{VAR}(A | z = a) = a^2 \text{VAR}(\tilde{\beta}_3) = a^2 \frac{\sigma_r^2}{16ka^2\sigma_x^2} = \frac{\sigma_r^2}{16k\sigma_x^2}$$

Based on the definition of noncentral chi-square distribution (Abramowitz and Stegun, 1972) we know

$$\text{that: } \left(\frac{A}{\sigma_A} \right)^2 \sim \chi_1^2(\lambda) \quad \text{with } \lambda = \left(\frac{\mu_A}{\sigma_A} \right)^2$$

Thus, $A^2 \sim \sigma_A^2 \chi_1^2(\lambda)$ and

$$\text{VAR}(A^2) = \sigma_A^4 \text{VAR}(\chi_1^2(\lambda)) = \sigma_A^4 2(1 + 2\lambda) = 2\sigma_A^2 (\sigma_A^2 + 2\mu_A^2) \quad (3.8)$$

Substituting Equation (3.8) into Equation (3.7), we get

$$\text{VAR}(P_{\text{des}}) = \frac{\sigma_r^4 + 2\sigma_r^2(16k\mu_A^2\sigma_x^2)}{8(4k)^2}$$

Thus, the performance index in the case of a desensitization experiment ($\text{Std}(P_{\text{des}})$) is the square root of above expression, namely:

$$\text{Std}(P_{\text{des}}) = \sqrt{\frac{\sigma_r^4 + 2\sigma_r^2(16k\mu_A^2\sigma_x^2)}{8(4k)^2}} \quad (3.9)$$

Next, we need to find $\text{Std}(P_{\text{rob}})$ when P_{rob} is defined by Equation (3.5) (i.e. s^2).

The sampling distribution of the sample variance is a scaled chi-square (Abramowitz and Stegun, 1972):

$$s^2 \sim \frac{\sigma_y^2}{n-1} \chi_{n-1}^2 \Rightarrow \text{VAR}(s^2) = \text{Var}(P_{\text{rob}}) = \left(\frac{\sigma_y^2}{n-1}\right)^2 2(n-1) = \frac{2\sigma_y^4}{n-1}$$

where n (# of data points used in the calculation of s^2) is equal to $2k$. So:

$$\text{Var}(P_{\text{rob}}) = \frac{2\sigma_y^4}{2k-1}$$

Thus, the performance index in the case of robustness ($\text{Std}(P_{\text{rob}})$) can be formulated as square root of above expression:

$$\text{Std}(P_{\text{rob}}) = \sqrt{\frac{2\sigma_y^4}{2k-1}} \quad (3.10)$$

The performance measures, Equation (3.9) and Equation (3.10), were also validated by a simulation.

3.2.3 Comparing Performance Measures

As mentioned before, the smaller the performance index the higher the effectiveness. So, to quantitatively prove our claim that a desensitization experiment is more effective than a robustness experiment we should show that Equation (3.9) is always less than Equation (3.10) or

$$\begin{aligned} \frac{2\sigma_y^4}{2k-1} &> \frac{\sigma_r^4 + 2\sigma_r^2(16k\mu_A^2\sigma_x^2)}{8(4k)^2} \quad \text{or} \\ 256k^2\sigma_y^4 &> (2k-1)(\sigma_r^4 + 32k\sigma_r^2\mu_A^2\sigma_x^2) \end{aligned}$$

Substituting $\sigma_y^2 = (\mu_A^2\sigma_x^2 + \sigma_r^2)$ into above expression and rearranging we obtain

$$\begin{aligned} 256k^2\mu_A^4\sigma_x^4 + 256k^2\sigma_r^4 + 512k^2\mu_A^2\sigma_x^2\sigma_r^2 \\ > \\ (2k-1)\sigma_r^4 + 64k^2\mu_A^2\sigma_x^2\sigma_r^2 - 32k\sigma_r^2\mu_A^2\sigma_x^2 \end{aligned} \quad (3.11)$$

Since $256k^2\mu_A^4\sigma_x^4$ is positive¹, $256k^2\sigma_r^4$ is greater than $(2k-1)\sigma_r^4$, and $512k^2\mu_A^2\sigma_x^2\sigma_r^2$ is also greater than $64k^2\mu_A^2\sigma_x^2\sigma_r^2$ we can conclude that inequality (3.11) is true and consequently conclude that $\text{Std}(P_{\text{des}})$ is always less than $\text{Std}(P_{\text{rob}})$. This conclusion indicates that the desensitization approach is always more efficient than the robustness approach (given our assumptions).

To generalize this conclusion we need to first consider cases in which there are more than one fixed input and one dominant cause and show that $\text{Std}(P_{\text{des}})$ is also less than $\text{Std}(P_{\text{rob}})$ in these situations. Second, we should challenge the assumption that we took in the desensitization case (i.e. σ_r^2 , σ_x^2 , $\beta_1 + \beta_3 z_0$ are known) and think about situations where one or all of these components are not known and we need to estimate them using either the desensitization experiment results and/or preliminary investigations. The next section shows how we generalized this comparison result.

3.2.4 Generalizing the Result

In Appendix, using a similar argument as given here, we show that the performance index of desensitization is less than the performance index of robustness even where we have “m” noise factors and “n” control factors. As mentioned early, the desensitization approach recommends using the method of elimination to find the dominant cause(s). This method concentrates on ruling out possibilities rather than looking directly for the dominant cause (Steiner and MacKay, 2005). Using elimination, the set of all causes is divided into families and then an observational investigation is conducted to rule out all but one family. This exercise is repeated on the remaining family until a single dominant cause or a small number of suspects cause(s) remain. At this point, when the family of remaining suspects is small, an “input-output” relationship investigation is used to isolate the dominant cause. In an “input-output” investigation a time frame is selected based on the full extent of output variation and a sample of 30 or more parts, spread across the time frame, is chosen. Then, for each part, the interested output characteristic and all remain suspects are measured. By plotting the output versus each one of the suspects any strong linear relationship can be found and the dominant cause can be identified. Steiner and MacKay (2005) not only recommend the method of elimination and a series of simple observational investigations to isolate a dominant cause but also recommend conducting a verification experiment to be sure that the suspected cause is dominant. Following these steps for finding the dominant cause before conducting the desensitization experiment it is reasonable to assume $\beta_1 + \beta_3 z_0$, σ_x and σ_r are already known (or well estimated) since these components can be estimated using the observational studies needed to find and verify the dominant cause. However, we shall also consider the situations where $\beta_1 + \beta_3 z_0$, σ_x and σ_r are not known and they are estimated using only the desensitization experiment results or using the desensitization results and a preliminary “input-output” investigation. For this reason a simulation study was employed. In the simulation study the model presented by Equation (3.2) is considered and without loss of generality we set:

$$\begin{aligned} \beta_0 &= 0 & \sigma_r^2 &= 1 \\ \beta_1 &= 1 & \mu_r &= 0 \\ \beta_2 &= 0 & \mu_x &= 0 \\ \beta_3 &= 1 & z_0 &= 0 \end{aligned}$$

¹ Note that k, the number of replicates in experiment is positive

With this setup, the levels of z in the desensitization and robustness experiments are used to quantify the size of the dominant cause and the potential to reduce process sensitivity to variation in the dominant cause. For fixed z the variance of output is :

$$\text{Var}(Y) = (\beta_1 + \beta_3 z)^2 \sigma_x^2 + \sigma_r^2 = (1 + 1z)^2 \sigma_x^2 + 1$$

Then X is a dominant cause if $(1 + 1z)^2 \sigma_x^2 > 1$. So in the current process where $z = z_0 = 0$, X is a dominant cause if $\sigma_x^2 > 1$. Note that with $z_0 = 0$ knowing $\beta_1 + \beta_3 z_0$ we know β_1 .

Then, four possible situations are considered:

1. Assume the relationship between x and y (i.e. β_1), σ_x and σ_r are known and then estimate the standard deviation of y at high and low levels of z by estimating β_3 and using Equation (3.3).
2. Assume the relationship between x and y (i.e. β_1) and the residual variation (i.e. σ_r) are not known, however σ_x is known. In this situation β_1 and σ_r are estimated using only the desensitization experiment results and then the standard deviation of y at high and low levels of z are estimated using Equation (3.3). This situation corresponds to a case where we know X is a dominant cause and know the distribution of X values (i.e. σ_x). If we know X is a dominant cause we would also have some knowledge of β_1 . So this situation is not overly realistic but is included for the sake of comparison.
3. The same situation as option 2, but we use a preliminary input-output investigation (sample size = 30) to help estimation of β_1 and σ_r .
4. Assume nothing is known, we only suspect that X is dominant cause and use the preliminary input-output investigation (with the same sample size as option 3) to estimate σ_x and use both input-output investigation and the desensitization experiment to estimate β_1 and σ_r .

As in the theoretical comparison, the levels of z for each run of the desensitization experiment are the same levels of z used in the corresponding robustness run and the level of X in desensitization runs are chosen to be extreme (i.e. $\mu_x \pm 2\sigma_x$). Simulation results are given by Figures 3.3 to 3.6.

In the figures we show contour plots of the performance ratio, which is $\text{Std}(P_{\text{rob}})$ divided by $\text{Std}(P_{\text{des}})$. Values greater than one suggests desensitization is more effective than robustness. The simulation estimates the standard deviation of the output using 1000 trials of each of the desensitization and robustness experiments. Each of earlier listed four options is considered. Figures 3.3 to 3.6 show the performance ratio for option 1 through 4, respectively. These figures present the results for high levels of z , where X is a dominant cause. The number of replicates in all options is equal to 2 and the number of observations in the preliminary input-output investigations for options 3 and 4 is equal to 30.

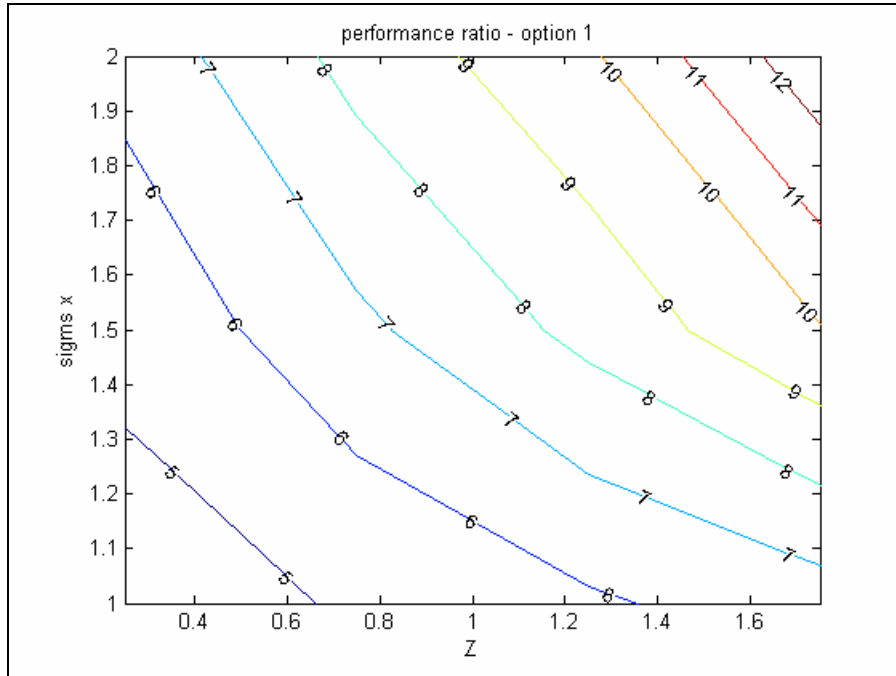


Figure 3.3: Performance ratio in situation 1

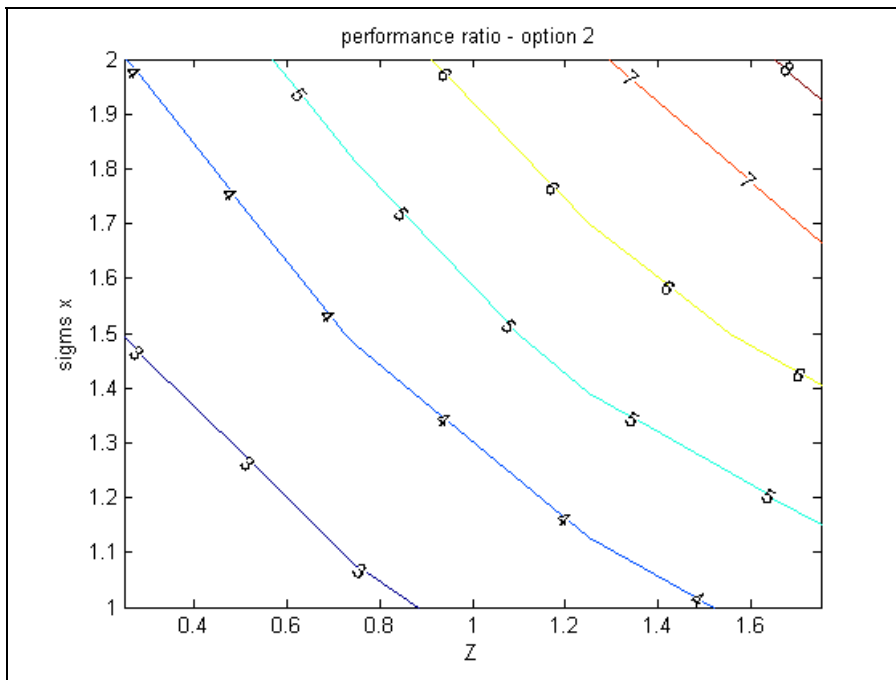


Figure 3.4: Performance ratio in situation 2

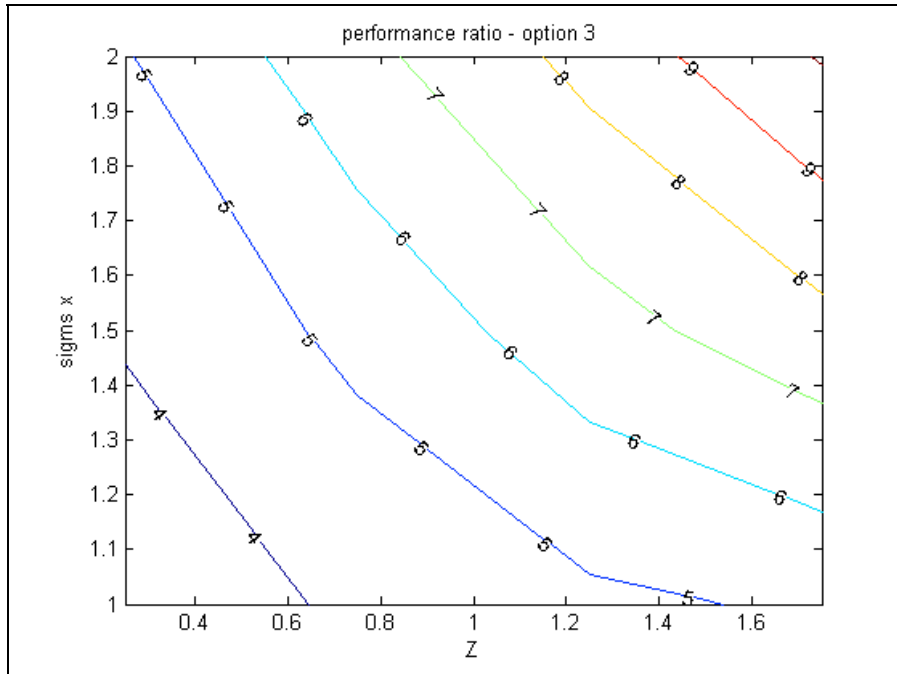


Figure 3.5: Performance ratio in situation 3

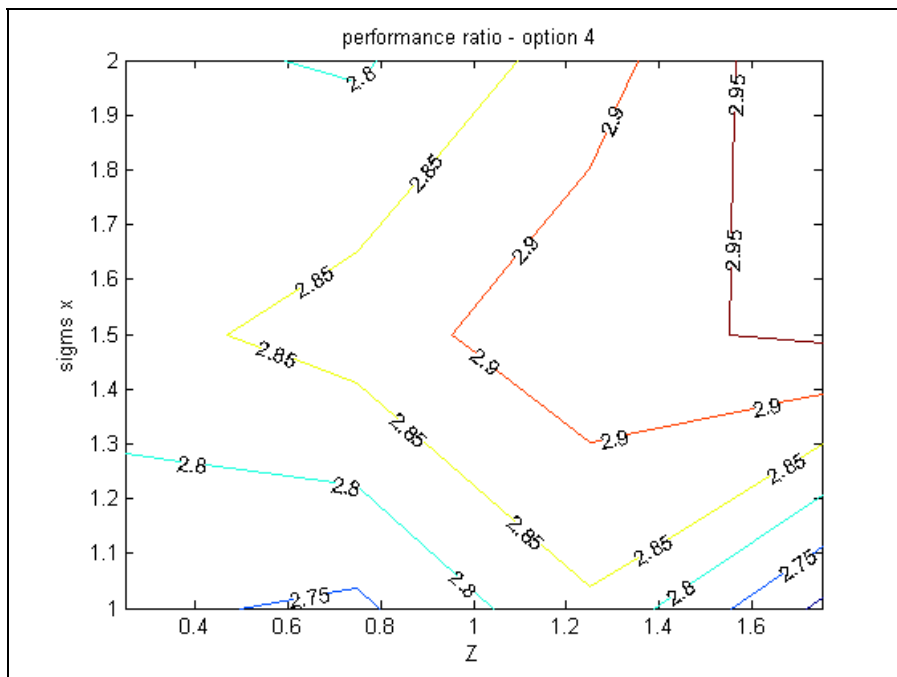


Figure 3.6: Performance ratio in situation 4

Figures 3.3 to 3.6 demonstrate that the performance ratio is bigger than 1 in all situations which validates and generalize, on some aspects, the theoretical results given earlier in Section 3.2.3.

The Figures also indicate that when the values of z and σ_x increase the performance ratio increases as well. The reason is that the standard deviation due to dominant cause (the value of $(\beta_1 + \beta_3 z)^2 \sigma_x^2$ in Equation (3.3)) grows when the value of z and/or σ_x increase. In other words, the effect of dominant cause in the output variation increases and we have a dominant cause that has higher importance. Thus, the desensitization experiment is more effective when the dominant cause has a greater effect.

3.3 Case Study: Geometric Distortion of Drive Gears

In this section, all three approaches are applied to the experiment introduced earlier in Section 2.2. We use a simulation study to compare three different experiments (i.e. robustness, Taguchi style, and desensitization experiments) for solving the Gear example problem. As described in the Gear example, there are five control factors and three noise variables. The main effect plot for dishing of the gear (Figure 3.7) suggests factor “H” as a dominant cause and scatter plots of the response versus noise factors (Figure 3.8) confirm this suggestion.

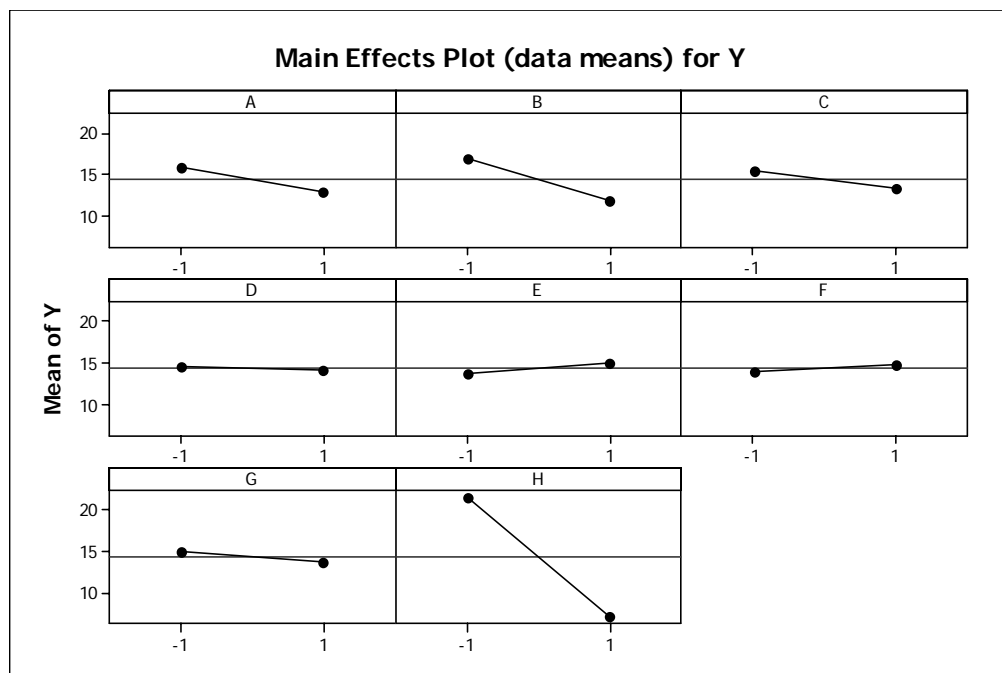


Figure 3.7: Main effects plot for the Gear experiment

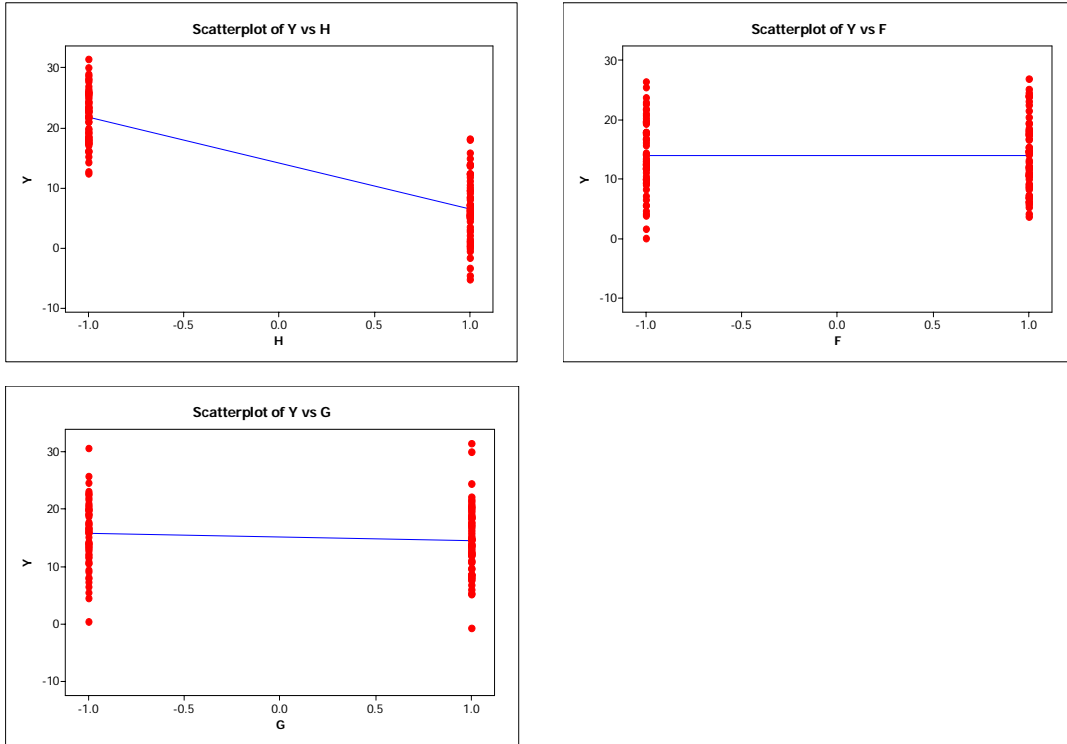


Figure 3.8: Scatter plot of the response versus factor H, F, and G in the Gear experiment

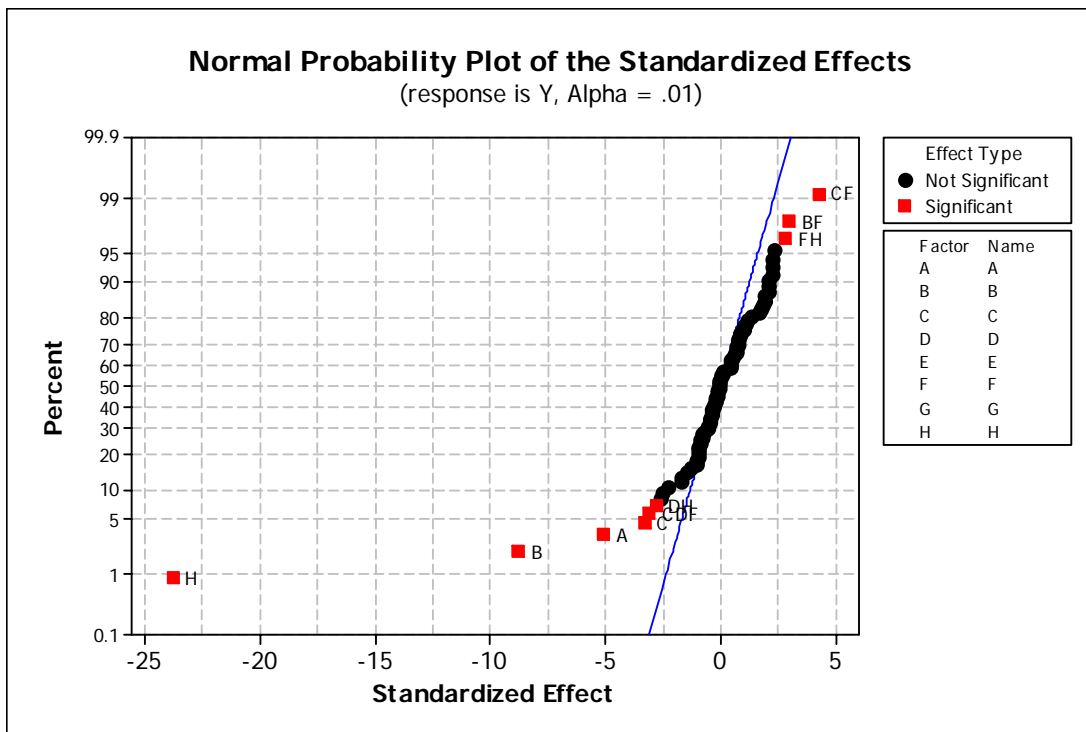


Figure 3.9: Normal probability plot in the Gear experiment (when response is Y)

Considering the normal probability plot (Figure 3.9), a reduced model is constructed as:

$$Y = 14.336 - 1.523x_A - 2.648x_B - 0.992x_C - 0.312x_D + 0.625x_E + 0.422x_F - 0.695x_G - 7.195x_H + 1.297x_Cx_F + 0.922x_Bx_F + 0.859x_Fx_H - 0.844x_Dx_H - 0.93x_Cx_Dx_F + R \quad (3.12)$$

The model in Equation (3.12) is assumed the true model of the process and is used in simulation program to generate the data. Given the model we can generate a response surface model for the process variance:

$$\sigma_y^2 = (-7.195 - 0.844x_D)^2 \sigma_H^2 + (0.422 + 1.297x_C + 0.922x_B - 0.93x_Cx_D)^2 \sigma_F^2 + (-0.695)^2 \sigma_G^2 + (0.859)^2 \sigma_F^2 \sigma_H^2 + \sigma_r^2 \quad (3.13)$$

Here, it is assumed that F, G, H are uncorrelated random variables and $\mu_F = \mu_G = \mu_H = 0$. (Note that $\text{Var}(XY) = \text{Var}(X) \times \text{Var}(Y)$ where X and Y are independent and $E(X) = E(Y) = 0$). The standard deviations of noise factors are also assumed to be all equal to 0.5 ($\sigma_F = \sigma_G = \sigma_H = 0.5$) and the value of $\sigma_r^2 = 11.72$ is taken from the ANOVA table (see MS of Residual Error in Table 3.2).

Table 3.2: ANOVA table in the Gear example

Analysis of Variance for Y						
Source	DF	Seq SS	Adj SS	MS	F	P
Main Effects	8	8094.9	8094.9	1011.87	86.34	0.000
2-Way Interactions	28	877.7	877.7	31.35	2.67	0.002
3-Way Interactions	46	793.0	793.0	17.24	1.47	0.099
Residual Error	45	527.4	527.4	11.72		
Total	127	10293.1				

Considering Equation (3.12) as the model that describes the real process, the simulator runs three different experiments (robustness, desensitization, and Taguchi style) and then analyzes the resulting data to determine the optimum treatment combination recommended by each experiment.

Following the desensitization approach an experiment is designed to include only the dominant cause (H) and five control factors (A, B, C, D, and E). The desensitization experiment includes a 2^{5-1} fractional factorial design for the control array and for each treatment combination of the candidates there are runs for both levels of the dominant cause. The dominant cause, factor H, is fixed at extreme levels (± 1 i.e. $\pm 2\sigma_H$) and the total number of runs is determined based on the number of replicates. For example for just one replicate there will be 32 runs ($1 \times 2^{5-1} \times 2$) and for two replicates we will have 64 runs. According to the Section 2.3, in the analysis a regression model is constructed based on the experiment results. The regression function models the response (output) in terms of the control factors and the interactions between the control factors and the dominant cause. This regression model is used to generate a response surface model for the process variance. For each simulation run, the solution is the setting that minimizes the process variance as predicted by the fitted response model.

The robustness experiment is a 2^{5-1} fractional factorial with only the five control factors. Each control factor is fixed at its low and high levels (± 1) and the total numbers of runs are determined based on the number of replicates. To fairly compare the desensitization and the robustness experiment the same

number of runs is considered for the two experiments. So, the number of replicates in the robustness experiment is two times of the number of replicates in the desensitization case. For two replicates in the desensitization case ($2 \times 2^{5-1} \times 2 = 64$ runs), for instance, there would be four replicates in the robustness experiment ($4 \times 2^{5-1} = 64$ runs). Noise factors are varied during the experiment as three random variables. The plan of the Taguchi experiment is the same as described for the Gear example in the Section 2.2. A 2^{5-1} fractional factorial design is used for the control array and a 2^3 full factorial design is used for the noise array. Using this plan the number of runs is 128 ($2^{5-1} \times 2^3 = 128$). So, given the described experimental plans, the number of runs in the simulated Taguchi experiment can not be less than 128, but for robustness experiment the number of runs can be the same as in the desensitization experiment. Table 3.3 shows σ_y (i.e. square root of Equation (3.13)) for all 16 combinations of factors A to E in a 2^{5-1} fractional factorial design.

Table 3.3: Standard deviation of output for all candidates' combinations

Treatment	A	B	C	D	E	σ_y
1	-1	-1	-1	-1	1	4.8816
2	-1	-1	-1	1	-1	5.3133
3	-1	-1	1	-1	-1	4.7662
4	-1	-1	1	1	1	5.2960
5	-1	1	-1	-1	-1	4.7080
6	-1	1	-1	1	1	5.3181
7	-1	1	1	-1	1	5.0158
8	-1	1	1	1	-1	5.3642
9	1	-1	-1	-1	-1	4.8816
10	1	-1	-1	1	1	5.3133
11	1	-1	1	-1	1	4.7662
12	1	-1	1	1	-1	5.2960
13	1	1	-1	-1	1	4.7080
14	1	1	-1	1	-1	5.3181
15	1	1	1	-1	-1	5.0158
16	1	1	1	1	1	5.3642

As you can see in Table 3.3 and from Equation, the smallest output variation (4.7080) is obtained when we have either treatment 5 or 13 as the setting of fixed inputs. So optimum setting can be determined as:

A: high or low B: high C: low D: low E: high or low

Note that the most important control factor is D. We will use this optimum setting later to compare the suggested settings from the desensitization, robustness and Taguchi experiments.

The simulation program runs each test experiment 1000 times. For each simulation run, the proposed new process settings suggested by each experiment are evaluated using Equation (3.13) (i.e. using the true model). Then, the mean and standard deviation of all 1000 σ_y s for each type of experiments are recorded. Suggested settings are summarized in Tables 3.4 and 3.5. Looking at these tables we can say that the robustness experiment, for example, suggests factor A at its high level for 510 out of 1000 runs and at its low level for 490 times of simulation runs and suggests treatment #1 for 74 times of simulation runs. Table 3.4 also compares these recommendations with the optimum setting given by Table 3.3.

Table 3.4: Recommended settings by each method per 1000 runs of simulation

Method	Levels	A	B	C	D	E
Robustness	H	0.5100	0.4680	0.4820	0.3260	0.5020
	L	0.4900	0.5320	0.5180	0.6740	0.4980
	interpretation	high or low	high or low	high or low	high or low	high or low
Desensitization	H	0.4920	0.4820	0.5010	0.0090	0.5280
	L	0.5080	0.5180	0.4990	0.9910	0.4720
	interpretation	high or low	high or low	high or low	low	high or low
Taguchi	H	0.2880	0.1930	0.3960	0.4450	0.5360
	L	0.7120	0.8070	0.6040	0.5550	0.4640
	interpretation	low	low	high or low	high or low	high or low
Optimum setting		high or low	high	low	low	high or low

Table 3.5: Number of each treatment combination recommended by each experiment for 1000 runs of simulation

Treatment	A	B	C	D	E	σ_y	Desensitization	Robustness	Taguchi
1	-1	-1	-1	-1	1	4.8816	134	74	152
2	-1	-1	-1	1	-1	5.3133	0	39	149
3	-1	-1	1	-1	-1	4.7662	131	74	138
4	-1	-1	1	1	1	5.2960	2	47	112
5	-1	1	-1	-1	-1	4.7080	120	77	67
6	-1	1	-1	1	1	5.3181	1	54	50
7	-1	1	1	-1	1	5.0158	141	82	15
8	-1	1	1	1	-1	5.3642	0	49	11
9	1	-1	-1	-1	-1	4.8816	132	94	0
10	1	-1	-1	1	1	5.3133	0	50	85
11	1	-1	1	-1	1	4.7662	117	89	87
12	1	-1	1	1	-1	5.2960	0	48	31
13	1	1	-1	-1	1	4.7080	110	72	29
14	1	1	-1	1	-1	5.3181	2	36	7
15	1	1	1	-1	-1	5.0158	110	79	2
16	1	1	1	1	1	5.3642	0	36	1
Possible values for σ_y							4.7080, 4.7662, 4.8816, 5.0158	4.7080, 4.7662, 4.8816, 5.0158, 5.2960, 5.3133, 5.3181, 5.3642	4.7080, 4.7662, 5.0158, 5.2960, 5.3133, 5.3181

The mean and standard deviation of calculated σ_y s for each experiment are shown in Table 3.6.

Table 3.6: Calculated performance measures in each method

Number of Runs	Robustness		Desensitization		Taguchi method	
	Mean of σ_y s	Standard deviation of σ_y s	Mean of σ_y s	Standard deviation of σ_y s	Mean of σ_y s	Standard deviation of σ_y s
32	5.0590	0.2523	4.9140	0.1957		
64	4.9930	0.2456	4.8745	0.1627		
128	4.9603	0.2356	4.8458	0.1229	4.9047	0.1717

Table 3.4 indicates that the desensitization experiment suggests the level of the most important factor (i.e. factor D) correctly in 99 percent of simulation runs. The reason that D is the most important factor (for making the process insensitive to the variation in the dominant cause) is that factor D is the only fixed input that has interaction with the dominant cause (see Equation(3.12)). From Table 3.5, it we see that the desensitization experiment more likely leads to small values of σ_y compared with the robustness and Taguchi experiments. The largest possible value of σ_y using the desensitization experiment is 5.0158 while it is 5.3642 and 5.3181 in the robustness and Taguchi experiments respectively.

Table 3.6 summarizes the simulation results. Note that the best method will yield the lowest average and the least variation in σ_y s. The results in Table 3.6 shoe that the desensitization experiment has the lowest average of the σ_y s and thus the highest efficiency comparing with the robustness and Taguchi experiments regardless of the number of runs. Moreover, if we compare the desensitization experiment in the case that has only 32 runs with the Taguchi experiment (with 128 runs); it is revealed that the desensitization experiment with 4 times fewer runs has almost the same efficiency of the Taguchi experiment. In other words, using the knowledge of dominant cause, a desensitization experiment which is smaller, easier and consequently cheaper (in desensitization experiment you need to fix fewer noise factors than in a Taguchi experiment) can be conducted and the same efficiency and results of a much larger Taguchi experiment can be expected. Equally important, the choice of noise factors in a Taguchi experiment is a critical issue. As mentioned in Section 2, Taguchi recommends using engineering judgment to select the noise factors and assumes that the choice includes all important noise factors. However, if the dominant cause is not known there is a risk of excluding the dominant cause from the outer array. This risk is one of the Taguchi method’s main drawbacks. To assess the consequences of risk we decided to exclude the dominant cause (e.g. H) from Taguchi experiment plan and then rerun the simulation and analyze the obtained data. Note that as we now have a 22 full factorial design for the outer array, we can also use a 64-run Taguchi experiment. Comparing the results in Table 3.7 with those in Table 3.6 shows that without the dominant cause in the noise array the Taguchi approach is the weakest approach.

Table 3.7: Performance measures of Taguchi method (the dominant cause H is excluded)

Number of Runs	Taguchi method	
	Mean of σ_y s	Standard deviation of σ_y s
64	5.0443	0.2647
128	5.0356	0.2556

So, a potential drawback of the Taguchi method experiments is that it depends critically on how well the noise factors are chosen. If the dominant cause is absent from the experimenter choice of noise factors, the experiment will likely fail. In the desensitization case, however, the dominant cause is known and we do not need to worry about the selection of noise factors.

4 CONCLUSIONS

A qualitative and quantitative comparison of the desensitization approach versus robustness and Taguchi approaches was presented and both kinds of comparisons suggested the desensitization method is the cheapest, the most convenient, and the most effective approach to the RPD at the manufacturing stage of a product development life cycle. This result was reconfirmed by considering a real world problem and comparing the three different approaches in the context of that problem. To run a desensitization experiment we need knowledge of dominant cause(s) of output variation. As a result, searching for the dominant cause of variation is highly recommended before proceeding to any experimental investigation to look for a robust solution. After finding the dominant cause, if an obvious solution is not evident and the dominant cause can be controlled temporarily, we suggest conducting a desensitization experiment to find a robust solution. The robustness approach can be selected as a last hope when it is hard to fix the levels of the dominant cause during a desensitization experiment or when we can't find the dominant cause.

Appendix

Performance Indexes (“m” noise factors and “n” control factors)

- Performance index in desensitization case ($\text{Std}(P_{\text{des}})$)

k : # of replicates

$i = 1, 2, \dots, m$: # of noise factors

$j = 1, 2, \dots, n$: # of control factors

$$Y = \beta_0 + \sum_{i=1}^m \beta_i x_i + \sum_{j=1}^n \beta_{m+1} z_j + \sum_{i=1}^m \sum_{j=1}^n \beta_{ij} x_i z_j + R$$

$$x_i \sim N(0, \sigma_{x_i}^2) \quad R \sim N(0, \sigma_r^2)$$

$$x_i = \begin{cases} +2\sigma_{x_i} \\ -2\sigma_{x_i} \end{cases} \quad z_j = \begin{cases} +a_j \\ -a_j \end{cases}$$

$$P_{\text{des}} = \sum_{i=1}^m (\beta_i + \underbrace{\sum_{j=1}^n \hat{\beta}_{ij} z_j}_{A_i})^2 \sigma_{x_i}^2 + \sigma_r^2$$

$$\text{VAR}(\hat{\beta}_{ij}) = \frac{\sigma_r^2}{(2\sigma_{x_i} a_j)^2 (2^n \times 2^m) k}$$

$$\text{VAR}(P_{\text{des}}) = \sum_{i=1}^m \sigma_{x_i}^4 \text{VAR}(A_i^2) \quad , A_i \sim N(\mu_{A_i}, \sigma_{A_i}^2)$$

$$A_i = \beta_i + \sum_{j=1}^n \hat{\beta}_{ij} z_j$$

$$\mu_{A_i} = \beta_i + \sum_{j=1}^n \beta_{ij} z_j$$

$$\sigma_{A_i}^2 = \sum_{j=1}^n \text{VAR}(\hat{\beta}_{ij}) z_j^2 = \frac{n\sigma_r^2}{(2\sigma_{x_i})^2 (2^n \times 2^m)k}$$

A_i s are independent, normally distributed random variables with means μ_{A_i} and variances $\sigma_{A_i}^2$. Thus,

$$\sum_{i=1}^m \left(\frac{A_i}{\sigma_{A_i}} \right)^2 \sim \chi_m^2(\lambda) \text{ and } \lambda = \sum_{i=1}^m \left(\frac{\mu_{A_i}}{\sigma_{A_i}} \right)^2, \text{VAR}(\chi_m^2(\lambda)) = 2(m + 2\lambda)$$

$$\text{VAR}(P_{des}) = \sum_{i=1}^m \sigma_{x_i}^4 \text{VAR}(A_i^2)$$

$$\sigma_{A_i}^2 = \frac{n\sigma_r^2}{(2\sigma_{x_i})^2 (2^n \times 2^m)k}$$

Considering above equations and following similar procedure that we had in the case of one noise and one control factors, we can formulate the performance index in the desensitization case as

$$\text{VAR}(P_{des}) = \frac{n^2 \sigma_r^4 2(m + 2\lambda)}{16 (2^n \times 2^m k)^2}$$

$$\therefore \text{Std}(P_{des}) = \sqrt{\frac{n^2 \sigma_r^4 2(m + 2\lambda)}{16 (2^n \times 2^m k)^2}}$$

- Performance index in robustness case($\text{Std}(P_{rob})$)

$$P_{rob} = \text{VAR}(Y) = s^2$$

$$s^2 \sim \frac{\sigma_y^2}{n-1} \chi_{n-1}^2 \Rightarrow \text{VAR}(s^2) = \text{VAR}(P_{rob}) = \left(\frac{\sigma_y^2}{n-1} \right)^2 2(n-1) = \frac{2\sigma_y^4}{n-1}$$

where n (# of observations used in the calculation of s^2) for this case is equal to $2^m \times k$. So :

$$\text{VAR}(P_{rob}) = \frac{2\sigma_y^4}{2^m k - 1} \text{ where } \sigma_y^2 = \left(\sum_{i=1}^m \mu_{A_i}^2 \sigma_{x_i}^2 \right) + \sigma_r^2$$

$$\therefore \text{Std}(P_{rob}) = \sqrt{\frac{2 \left[\left(\sum_{i=1}^m \mu_{A_i}^2 \sigma_{x_i}^2 \right) + \sigma_r^2 \right]^2}{2^m k - 1}}$$

To prove that performance index in the case of desensitization is less than performance index in the case of robustness (i.e. desensitization is more efficient than robustness), we need to show:

$$\frac{2\sigma_y^4}{2^m k - 1} > \frac{n^2 \sigma_r^4 2(m + 2\lambda)}{16 (2^n \times 2^m k)^2} \quad (\text{A.1})$$

$$\text{or } \frac{2 \left[\left(\sum_{i=1}^m \mu_{A_i}^2 \sigma_{x_i}^2 \right) + \sigma_r^2 \right]^2}{2^m k - 1} \rangle \quad \frac{n^2 \sigma_r^4 2(m + 2\lambda)}{16(2^n \times 2^m k)^2}$$

and denoting $\sum_{i=1}^m \mu_{A_i}^2 (2\sigma_{x_i})^2$ by d we need to prove that

$$2 \left[(d + 4\sigma_r^2)^2 \right] (2^n \times 2^m k)^2 \rangle \quad 2n\sigma_r^2 (mn\sigma_r^2 + 2(2^n \times 2^m)kd)(2^m k - 1)$$

or

$$2(2^n \times 2^m k) \left[\frac{1}{2} (2^n \times 2^m k) d^2 + 8(2^n \times 2^m k) \sigma_r^4 + 4(2^n \times 2^m) kd \sigma_r^2 \right] \rangle$$

$$n(2^m k - 1)(mn\sigma_r^4 + 4(2^n \times 2^m)kd\sigma_r^2)$$

As $2(2^n \times 2^m k)$ is greater than $n(2^m k - 1)$ for all natural numbers (it can be proved using Mathematical Induction), we still have a true expression if we do not have $4(2^n \times 2^m)kd\sigma_r^2$ in the both sides brackets of above inequality:

$$2(2^n \times 2^m k) \left[\frac{1}{2} (2^n \times 2^m k) d^2 + 8(2^n \times 2^m k) \sigma_r^4 \right] \rangle \quad \text{(A.2)}$$

$$n(2^m k - 1)(mn\sigma_r^4)$$

In this expression as it is mentioned before $2(2^n \times 2^m k)$ is greater than $n(2^m k - 1)$ and by Mathematical Induction we can also prove that $8(2^n \times 2^m k) \sigma_r^4 \rangle mn^2 \sigma_r^4$ (or in other words, $8(2^n \times 2^m) \rangle mn^2$) is true of all natural numbers (see next subsection). So, expression (A.2) and consequently expression (A.1) is true.

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