RR-89-04
Revised

Economical Experimentation Methods for Robust Parameter Design

A. Shoemaker, K. Tsui, C.F. Wu
University of Waterloo

RR-89-04
March 1989
Revised October 1989

Economical Experimentation Methods for Robust Parameter Design

Anne C. Shoemaker
Kwok-Leung Tsui

AT&T Bell Laboratories

C. F. Jeff Wu

University of Waterloo

Abstract

Taguchi's parameter design technique has proven effective for improving the robustness of product and process designs, but in some applications his method for introducing noise leads to unnecessarily expensive experiments. In fact, the expense comes from estimation of a large number of interactions.

Taguchi's "product array" experimental setup consists of a control array (i.e., inner array) completely crossed with a noise array (i.e., outer array). This dictates estimation of many control-by-noise factor interactions, and often higher order interactions as well.

In this paper, we abandon the "product array" formulation and instead use a single array for both control factors and noise factors, an idea also proposed by Welch et al.(1989). This "combined array" approach includes Taguchi's product arrays as a special case, but does not dictate that all control factor-by-noise factor interactions be estimated. Instead, some of these degrees of freedom can be re-directed, allowing far greater flexibility for estimating effects which may be important for physical reasons. At the same time, the experiment size is often dramatically reduced.

Using a combined array instead of a product array requires a different data analysis approach for robust design problems, however. This new approach is based on modeling the response, *Y*, then using this model to infer the control factor levels that minimize the objective function. An informal procedure for doing this is illustrated here. A more formal procedure is given by Welch et al. (1989) in the context of computer experiments.

1. *INTRODUCTION*

1.1 *Robust Parameter Design*

Robust parameter design is an approach to reducing performance variation in products and processes. Products and their manufacturing processes are influenced both by factors that are controlled by designers and by difficult-to-control factors such as environmental conditions, raw material properties, and aging. The idea of robust parameter design is to select the levels of the easy-to-control factors (called "control factors" or "design parameters") to minimize the effects of the hard-to-control factors (called "noise factors").

Robust parameter design ideas were brought to the attention of statisticians by Japanese quality expert, Dr. Genichi Taguchi (see Taguchi,1986). Taguchi formulates the robust parameter design problem as follows:

Choose the levels of the control factors, θ to minimize the expected loss caused by the noise factors, ε. That is,

$$\min_{\theta} \; R(\theta) = E_{\varepsilon} \, L(Y,\tau) \quad ,$$

where Y is the product or process response and τ is the target for that response. $L(Y,\tau)$ is a measure of the loss caused when Y deviates from τ, and is usually approximated by $(Y-\tau)^2$.

In practice expected loss might be replaced as an objective function by the variance of Y or coefficient of variation of Y (Taguchi's "signal-to-noise ratio"), or by some other appropriate measure. Taguchi and Phadke (1984) and Leon, Shoemaker, and Kacker (1987) discuss reasons why performance measures other than expected loss might be used. For example, Leon et al. show that if an adjustment control factor exists, either the variance or coefficient of variation may be a more useful optimization criterion than the average squared error loss. In this paper we will not address the problem of choosing an appropriate performance measure. R will refer to the performance measure chosen for the given problem.

1.2 Taguchi's Experimental Set-up: The Product Array

Experiments are often needed to solve the robust parameter design problem because the functional relationship between R and the control factors is not known. Taguchi recommends a two-part experimentation strategy, as illustrated in Figure 1.1. The control factors are varied according to a "control array", which is an orthogonal array. For each row in the control array, the noise factors are varied according to a "noise array", also an orthogonal array. Each noise array provides an estimate of the optimization criterion, R. The control factor levels that minimize R are chosen by analyzing the control factor main effects and, sometimes, two-factor interactions.

Since the noise array is run for every row in the control array, we call this set-up a product array and write it as CAxNA, where "CA" refers to the control array and "NA" refers to the noise array.

Taguchi's product array approach has three major disadvantages:

1. It can require a very large number of runs because the noise array is repeated for every row in the control array.

   In some situations the noise array is really just a plan for taking systematic multiple measurements of the response to approximate the effects of a number of noise factors, and is relatively inexpensive. In other situations, however, the noise array may actually represent separate process runs. For example, in a study to improve a wave soldering process, the noise factors might concern the type of circuit board being soldered: whether the board was single or double sided, whether it was multiwire or not, and whether it was multilayer or not. The objective might be to find a soldering process that worked well for all types of boards, and if that were not possible, to identify optimum process settings for each type of board. In this case, each type of board would have to be run separately through the process, and the 3-factor, 4-run noise array would quadruple the number of runs in the experiment.

2. The product array approach focuses on modeling R, which is often a nonlinear, many-to-one transformation of the response, Y. Even when Y follows a linear model in the control and noise factors, it is unlikely that R can be modeled well by a low order linear model, even if data

transformation is employed.

3. The product array experiment uses a large number of its degrees of freedom to estimate interactions between control factors and noise factors (see Section 2). Because of the structure of the product array, there is no flexibility to use some of these degrees of freedom to estimate other effects, such as control factor-by-control factor interactions.

1.3 Overview

In this paper we explore an alternative experimental setup for robust parameter design. This setup, which combines the control factors and noise factors in a single array, does not share the three disadvantages of the product array approach. The "combined array" approach focuses on modeling the response *Y* rather than *R*, so it is easier to postulate and fit a model. This approach includes the product array approach as a special case since the CAxNA setup can be written as one very large array including both control and noise factors. But the combined array approach does not force the estimation of all control factor-by-noise factor interactions, so the experiment size is usually greatly reduced and there is more flexibility for estimating other interactions that might be important.

In Section 2 we characterize the model implied by the product array approach, and see that it includes (i) the control factor main effects and those control factor-by-control factor interactions estimable in the control array, (ii) the noise factor main effects and those noise factor-by-noise factor interactions estimable in the noise array, and (iii) all the generalized interactions between the effects in (i) and the effects in (ii).

In Section 3 we introduce the combined array approach and illustrate through several examples the flexibility for effects estimation and economy of runs that this approach brings.

In Section 4 we show how combined arrays can be used to solve the robust design problem. This will involve modeling the response, *Y*, rather than the objective function, *R*, and using the response model to identify control factor levels that should improve *R*. We will also discuss related work by Hamami, Hooper, and Nazaret (1987) and Welch, Yu, Kang, and Sacks (1989).

In Section 5 we use the silicon wafer epitaxial growth process example of Kackar and Shoemaker (1986) to compare the performance of three alternative strategies:

- modeling the objective function, R, directly from the data using a product array design (i.e., similar to Taguchi's approach),
- modeling the response using a product array design (to study the benefits of modeling the response and not the objective function),
- and modeling the response using a smaller combined array design (to see if a smaller model and experiment can work as well).

2. *The Model Implied by the Product Array Approach*

Product array designs used by Taguchi for Robust Parameter Design have a special structure that dictates estimation of certain effects, regardless of whether these effects are likely to be present for physical reasons. In this section, we give a general characterization of the capacity of product array designs for estimating main effects and interactions in a model for the response Y. The general characterization is first motivated by an example.

(In Sections 2 and 3 we use capital letters $A, B, C, \cdots$ to denote control factors and small letters $a, b, c, \cdots$ to denote noise factors. To abuse the notation we also use them to denote their main effects.)

Example 1. Suppose the control array (CA) is a 4-run fractional factorial design with three 2-level factors given by $A, B, C(=AB)$, and the noise array (NA) is a similar design given by $a, b, c(=ab)$. Then the product array corresponding to CA and NA, denoted by CAxNA, has 16 runs; 6 factors, A, B, C, a, b, c; and the following defining relations:

$$I=ABC=abc=ABCabc.$$

It is a 1/4 -fraction of the 2^6 full factorial design.

From the defining relations above, we can see that if all the 2-factor and higher-order interactions between the control factors and between the noise factors (which we call CxC interactions and NxN interactions respectively) are negligible, the six main effects are estimable. In addition, all nine 2-factor interactions between control factors and noise factors (which we call CxN interactions) are estimable if the 3rd- and higher-order interactions are negligible. So nine degrees of freedom in the product array CA x NA are used to estimate the nine CxN interactions.

The capacity in CA x NA for estimating the CxN interactions, as demonstrated in Example 1, is a special case of the following general result. Let $\mathbf{d_1}$ and $\mathbf{d_2}$ be two orthogonal arrays with n and m runs performed at $\underset{\sim}{x}_1, \ldots, \underset{\sim}{x}_n$ and $\underset{\sim}{z}_1, \ldots, \underset{\sim}{z}_m$, respectively. The product array of $\mathbf{d_1}$ and $\mathbf{d_2}$, denoted by $\mathbf{d_1} \times \mathbf{d_2}$ has nm runs performed at $(\underset{\sim}{x}_p, \underset{\sim}{z}_q)$, $p=1, \ldots, n$, $q=1, \ldots, m$.

Assume that the expected response E(y) at $\underset{\sim}{x}$ can be described by the linear model

$$E(y)=\alpha_o+\sum_{i=1}^{k} f_i(\underset{\sim}{x})\alpha_i, \tag{2.1}$$

where $\alpha_i, i=0, \ldots, k$, depend on the level of $\underset{\sim}{z}$ at which y is observed. Similarly, we assume that E(y) at $\underset{\sim}{z}$ can be described by

$$E(y)=\beta_o+\sum_{j=1}^{l} g_j(\underset{\sim}{z})\beta_j, \tag{2.2}$$

where $\beta_j, j=0, \ldots, l$, depend on the level of $\underset{\sim}{x}$ at which y is observed.

Given (2.1) and (2.2), it is reasonable to assume that the expected response E(y) at $(\underset{\sim}{x}, \underset{\sim}{z})$ can be adequately described by the linear model

$$E(y)=\gamma_o+\sum_{i=1}^{k} f_i(\underset{\sim}{x})\gamma_{io}+\sum_{j=1}^{l} g_j(\underset{\sim}{z})\gamma_{oj}+\sum_{i=1}^{k}\sum_{j=1}^{l} f_i(\underset{\sim}{x})g_j(\underset{\sim}{z})\gamma_{ij}, \tag{2.3}$$

where $\sum_{p=1}^{n} f_i(\underset{\sim}{x}_p) = \sum_{q=1}^{m} g_j(\underset{\sim}{z}_q) = 0$, for all i, j, γ_{io} are the factorial effects (i.e., main effects, two-factor interactions, etc.) of $\underset{\sim}{x}$, γ_{oj} are the factorial effects of $\underset{\sim}{z}$, and γ_{ij} are their generalized interactions (John, 1971). Note that, if γ_{io} and γ_{oj} are both main effects, γ_{ij} is a 2-factor interaction, and if γ_{io} is a main effect and γ_{oj} a 2-factor interaction, γ_{ij} is a 3-factor interaction, etc. In general, $\gamma_{io} \neq \alpha_i$ and $\gamma_{oj} \neq \beta_j$. However, if y in (2.1) is interpreted as an average over $\underline{z}_1, \ldots, \underline{z}_m$, then $\gamma_{io} = \alpha_i$, and likewise $\gamma_{oj} = \beta_j$ if y in (2.2) is interpreted as an average over $\underline{x}_1, \ldots, \underline{x}_n$.

If $\mathbf{d_1}$ and $\mathbf{d_2}$ are chosen so that the factorial effects α_i and β_j in (2.1) and (2.2), respectively, are estimable, then the factorial effects γ_{io}, γ_{oj}, and γ_{ij} in (2.3) are estimable for $\mathbf{d_1} \times \mathbf{d_2}$ since the matrix of coefficients for (2.3) is the Kronecker product of the corresponding matrices for (2.1) and (2.2). Hence the estimation capacity of $\mathbf{d_1} \times \mathbf{d_2}$ can be summarized as follows.

Lemma. Assume (2.1), (2.2) and (2.3) hold. Suppose orthogonal arrays are used for $\mathbf{d_1}$ and $\mathbf{d_2}$ with $\alpha_1, \ldots, \alpha_k$ being the estimable factorial effects for $\mathbf{d_1}$ and $\beta_1, \ldots, \beta_l$ being the estimable factorial effects for $\mathbf{d_2}$. Then in the product array $\mathbf{d_1} \times \mathbf{d_2}$, γ_{io}, γ_{oj} and their generalized interactions γ_{ij}, $i=1, \ldots, k$, $j=1, \ldots, l$ are estimable.

In the applications of this Lemma, we do not distinguish between the effects γ_{io} for the product array $\mathbf{d_1} \times \mathbf{d_2}$ and the effects α_i for the control array, $\mathbf{d_1}$, and so forth. For simplicity, we use the same notation for both.

Example 1 is a special case of the Lemma. In the CA, the main effects A, B, C are estimable. In the NA, the main effects a, b, c are estimable. In CAxNA, A, B, C, a, b, c and their interactions Aa, Ab, Ac, Ba, Bb, Bc, Ca, Cb and Cc are estimable.

One can see clearly from the Lemma the deficiencies of Taguchi's product array approach to robust parameter design. The additional (n-1)(m-1) degrees of freedom in CA x NA are used for estimating CxN and higher order interactions. There is no flexibility for using them to estimate CxC interactions which may be important, or for estimating quadratic effects of control or noise factors. Furthermore, in most practical situations it might be sensible to assume apriori that higher order interactions and certain CxN interactions are negligible. It seems wasteful in these situations to have a large run size just to ensure that all the CxN interactions are estimable.

To rectify these deficiencies of the product array approach, an alternative approach will be proposed in the next section.

3. A More Economical and Flexible Approach: The Combined Array

We can avoid the inflexibility and often reduce the expense inherent in the product array approach by combining control factors and noise factors in a single array. The idea of using a single matrix for both types of factors is also considered by Welch et al.(1989). In the combined array approach, the experimenter constructs a single array to estimate those effects considered most likely to be important apriori. In particular, the experimenter now has the flexibility to rank-order all possible interaction effects according to their likely importance, and construct a plan that can estimate those at the top of the list. As we will see, the combined array approach often also leads to a smaller experiment.

Note that the product array approach is a special case of the combined array approach since the product array can be viewed as a single, albeit large, array for the combined set of control and noise factors.

The combined array approach does not share the two deficiencies of the product array approach pointed out at the end of Section 2. It allows great *flexibility* in the estimation of important effects, and the run size can be much smaller than that of the product array. This *economy* in run size results from avoiding estimation of unimportant effects.

Flexibility will be illustrated in Examples 2 and 3 below. Both economy and flexibility will be illustrated in Examples 4 and 5. In all the examples interactions among three and more factors are assumed negligible.

The factors and run size in Examples 2 and 3 are the same as in Example 1.

Example 2. Use as the combined array a 16-run fractional factorial design with the defining relations, $I = ABCa = abc = ABCbc$. From the relation ABCa=I, we have

$$AB = Ca,\ AC = Ba,\ BC = Aa,$$

that is, the three CxC interactions are estimable if the three CxN interactions Aa, Ba and Ca are negligible, and vice versa. In comparison with Example 1, this combined array experiment provides more flexibility for estimating the CxC interactions since some of the CxN interactions are likely to be unimportant. On the other hand the three CxN interactions, *Aa*, *Ba* and *Ca*, are estimable if the CxC interactions are negligible. As in Example 1 the other six CxN interactions are estimable.

Although this combined array has the same size as the product array in Example 1, the main effects of the control factors *A*, *B*, *C* are estimable here without the stringent assumption needed in Example 1 that 2-factor interactions are negligible.

Example 3. The combined array is a 16-run fractional factorial design with the defining relations, $I = ABCa = BCbc = Aabc$. The 2-factor interactions are aliased with each other as shown below:

$$AB = Ca,\ AC = Ba,\ BC = Aa = bc,$$
$$Bb = Cc,\ Bc = Cb,\ Ab = ac,\ Ac = ab. \tag{3.1}$$

The main effects *A*, *B*, *C* are estimable. At most seven out of nine CxN interactions are estimable if their aliases in (3.1) are negligible. Two advantages are that the noise effects a, b, c are estimable and that the NxN interactions are estimable if their aliases in (3.1) are negligible. The last property is not particularly attractive since NxN are usually less important than CxN or CxC for robust design purposes.

The estimation capacities for the product array of Example 1 and the two combined arrays of Examples 2 and 3 are summarized and compared in Table 3.1. If we want to estimate some CxC and CxN interactions, plans 2 and 3 are preferred.

TABLE 3.1 ABOUT HERE

In the product array approach, some degrees of freedom may be used for estimating high order interactions induced by multiplication of CxC and NxN interactions. Examples 4 and 5 illustrate how the combined array approach reduces the number of runs by avoiding estimation of high order interactions that are unlikely to be important.

Example 4. Suppose there are four 2-level control factors, *A*, *B*, *C*, and *D*, and two 2-level noise factors, *a* and *b*. Assume the CxC interactions *AB*, *AC* and *AD* are potentially important and we wish to estimate them. If we use the product array approach, we first construct a control array that estimates all main effects, *A*, *B*, *C*, and *D* and the three important interactions, *AB*, *AC* and *AD*. We then construct a noise array that estimates two main effects, *a* and *b* and the interaction *ab*. Figure 1.1 shows the product array of the corresponding 8-run control array and 4-run noise array. The defining relation of this plan is $I = ABCD$. According to the Lemma in Section 2, the resulting 32-run product array allows us to estimate six main effects: *A*, *B*, *C*, *D*, *a* and *b*, twelve 2-factor interactions: *AB*, *AC*, *AD*, *ab*, *Aa*, *Ba*, *Ca*, *Db*, *Ab*, *Bb*, *Cb*, and *Db*, ten 3-factor interactions and three 4-factor interactions.

Since the 3-factor and 4-factor interactions are not likely to be important, it is more efficient to plan a single experiment that estimates only the six main effects and the twelve 2-factor interactions. Since a 16-run fractional factorial design is too small for the number of effects to be estimated, we can use a D-optimal design algorithm to find the combined array. Three such designs of size 20, 22 and 24 have been generated from an optimal design algorithm, DETMAX (Mitchell, 1977), used in the software system RS/DISCOVER. Only the 22-run design is shown in Table 3.2. The determinants of the X^TX matrices for these three designs are respectively 9.4×10^{22}, 44.7×10^{22}, and 177.8×10^{22}, which increase roughly by a constant multiple. These figures suggest that the gain in D-efficiency from each additional run is roughly constant, so choice among these designs is a compromise between efficiency and run size. All three designs are approximately two-thirds the size of the product array, but allow estimation of all those potentially important effects estimable from the product array experiment.

TABLES 3.2 ABOUT HERE

In practice, engineering knowledge may allow us to go further in eliminating the need to estimate superfluous effects. Even quite limited knowledge of underlying physical mechanisms may imply that certain interactions can be ruled out. For example, if we believe that the noise factor *a* may interact only with the control factors *A* and *B*, and the noise factor *b* may interact only with the control factors *C* and *D*, then in addition to the three control-by-control interactions, *AB*, *AC* and *AD*, only four control-by-noise interactions, *Aa*, *Ba*, *Cb*, and *Db*, need to be estimated. Using the combined array approach, we can construct a 16-run fractional factorial plan to estimate the six main effects and the seven 2-factor interactions. One such plan is given by the defining relations, $I = ABCD = ABab = CDab$.

In an experiment plan that involves 3-level factors, the first order effects are the linear terms of the main effects. The second order effects include both the quadratic terms of the main effects and the linear-by-linear terms of the 2-factor interaction effects. In some situations we are willing to assume that effects higher than second order are unlikely to be important. The product array approach, however, always results in a plan that estimates many third or higher order effects. The following example illustrates the savings in run size that is possible by using a combined array.

Example 5. Suppose there are two 3-level control factors, A and B and two 3-level noise factors, a and b. Assume we wish to estimate the linear-by-linear term of the 2-factor interaction, $A_L B_L$, but do not believe that higher order interactions are likely to be large. Using the product array approach, we first construct a 3^2 full factorial plan for the control array that can estimate the main effects, A_L, B_L, A_Q, and B_Q, and the interaction effect, $A_L B_L$. We again choose a 3^2 full factorial plan for the noise array that estimates the two main effects a and b and their interaction. According to the Lemma, the resulting 81-run product array allows us to estimate a total of eighty effects, including four first order effects: A_L, B_L, a_L and b_L, ten second order effects: A_Q, B_Q, a_Q, b_Q, $A_L B_L$, $A_L a_L$, $A_L b_L$, $B_L a_L$, $B_L b_L$, and $a_L b_L$, and sixty-six third or higher order effects.

Since only the fourteen first and second order terms are likely to be important, it is more efficient to use the combined array approach to plan a smaller experiment that can estimate these effects. A standard central composite design of 25 runs, approximately one third of the size of the product array, will allow us to estimate these fourteen effects, in addition to some other second order effects (see Box and Draper, 1987). Another alternative is to use an optimal design algorithm to generate even smaller plans which estimate only the fourteen desired effects.

4. A Response Model Procedure for Analyzing Combined Array Experiments

The examples in Section 3 illustrate the greater flexibility for effects estimation and greater economy in run size that is possible using the combined array approach. But how does this approach help solve the robust parameter design problem defined in Section 1?

The combined array approach implies a totally different procedure for robust parameter design than the one used with product array experiments. This procedure, called the "Response Model Procedure" has five steps:

1. Specify a preliminary model relating the response Y to the control factors θ and noise factors ε.
2. Find an experiment plan that allows this model to be estimated. This plan will be a combined array, containing both control factors and noise factors.
3. Conduct the experiment. From the observed data y, select an appropriate model relating y to control and noise factors.
4. Use the model from Step 3 to identify improved control factor settings.
5. Conduct a confirmation experiment to compare the performances of the settings from Step 4 and the original setting. If necessary, iterate Steps 1 through 4.

In specifying the model in Step 1, use any available knowledge of the problem under investigation. This knowledge might allow some control-by-noise interactions and curvature effects to be estimated.

Model selection in Step 3 can be facilitated by the use of such techniques as half-normal plots, ANOVA, stepwise regression, Cp and PRESS statistics, data transformation, or regression diagnostics. These techniques should be augmented by examination of interaction plots, especially CxN interactions, since these interactions offer the potential for reducing variation. However, the size of a CxN interaction alone is not necessarily a measure of the potential for improving robustness. The two scenarios illustrated in Figure 4.1 demonstrate that a modest interaction may be more useful in robust design than a large, significant interaction. This point is illustrated in the real example in Section 5.

In Step 4 the improved settings can be identified by informal interpretation of the model parameters using main effects and interaction plots. This method, illustrated in the example of Section 5, is particularly suitable if R is a measure of variability. It is simple, graphical and often provides insight into physical mechanisms which allow the response variation to be reduced.

A more formal version of this procedure proposed by Welch et al.(1989) may be required if different plots point to conflicting control factor settings or if the fitted response model is too complex. The formal procedure can be generically described as follows:

Approximate the expected loss $R(\theta) = E(L(Y, \tau) \mid \theta)$ by using the estimated model for Y obtained in Step 3 and taking the expectation of $L(Y, \tau)$ with respect to the distribution of the noise factors ε. Denote the resulting function $R(\theta)$. Then take θ^* that minimizes $R(\theta)$ to be the improved control factor setting. The expectation of L can be approximated either by simulations from the joint distribution of the noise factors or by using a noise array to obtain a rough estimate.

The model building and optimization processes in Steps 1 through 4 may be iterated to attain a better value of θ^*. This may mean using techniques such as fold-over designs and central composite designs to refine the model of Step 3, perhaps adding higher order terms. Or it may mean moving in the direction of largest improvement of $R(\theta)$ and conducting another experiment in a new region of the control factor space.

In the context of computer experimentation, the ideas of modeling the response and estimating the objective function by simulating from the response model were proposed by Hamami, Hooper and Nazaret(1987) and formally laid out in Welch et al.(1989).

Taguchi's approach, on the other hand, differs dramatically from the "Response Model Procedure" outlined above. Rather than first modeling Y, Taguchi models loss directly, using a noise array to obtain an estimate of average loss for each combination of control factor values. The resulting experiment plan is a product array. The corresponding "Loss Model Procedure" is:

1. Specify a preliminary model relating R to the control factors θ.

2. Find a control array that allows the model in Step 1 to be estimated. Find a noise array for the approximation of R. The resulting plan is a product of the two arrays.

3. Conduct the experiment, compute the approximated values, $\hat{R}$, and based on them select an

appropriate model relating $\hat{R}$ to the control factors θ.

4. Find θ^* to minimize $\hat{R}$, where $\hat{R}$ is the estimate of R obtained from the model in Step 3.

5. Conduct a confirmation experiment to compare the performances of the settings from Step 4 and the original setting.

Each approach has its advantages and disadvantages:

I. Ease of modeling:

Postulating a model for Y in Step 1 of the Response Model Procedure should be easier than postulating a model for R in Step 1 of the Loss Model Procedure. The experimenter is more likely to have intuition or knowledge about the relationship between the factors and Y than about the relationship between the factors and R. In addition, even if Y can be described by a fairly simple regression model, the same is not true for R since R is obtained from Y through a nonlinear and many-to-one transformation. Therefore it would be quite unusual to find a transformation h (for example, from the power family) for which h(R) can be described by a simple regression model in the control factors. A similar point has been made in Phadke and Taguchi(1988) and Welch et al.(1989).

II. Additional information revealed by control-by-noise interactions:

In the Response Model Procedure, CxN interactions plots can suggest which control factors can be used to dampen the effects of noise factors, and may also provide insight into physical mechanisms which allow the response variation to be reduced. On the other hand, the Loss Model Approach masks these relationships by aggregating over all the noise factors. This will be illustrated in Section 5.

III. Consequences of model misspecification:

In general, the Response Model Procedure depends more critically than the Loss Model Approach on how well the model fits. Since control factor levels are determined from the fitted model, misspecification of the model could lead to control factor levels that are far from optimal. The confirmation experiment provides some insurance against this scenario, but model diagnosis methods clearly should play a big role in the Response Model Procedure.

Regardless of the modeling procedure, a product array provides some built-in insurance against modeling difficulties. If either the response model or loss model is inadequate and does not lead to improved control factor levels, estimates of R obtained directly from the data are available if a product array was used. If all else fails, the control factor levels which gave the best value of R in the control array can be adopted, if they are better than the initial control factor levels.

5. *Example: Improvement of Epitaxial Growth Process*

This example illustrates the potential advantages of the response model approach, both in terms of information obtained from modeling and in terms of the economics of the experiment. The example is based on a real experiment reported by Kacker and Shoemaker (1986). Because the raw data is no longer available, however, pseudo-data has been imputed from the actual marginal averages, as explained below.

The example concerns growth of uniform layers of silicon on top of silicon wafers. It is one of the earliest steps in integrated circuit fabrication. Of primary importance is that the layers be of uniform thickness. Kacker and Shoemaker describe the process in more detail.

There are eight control factors of interest as shown in Table 5.1. Deposition time is customarily used to adjust the average thickness to the target thickness, which may be different for different types of integrated circuits. Because deposition time was used in this way, variance was used as the objective function rather than mean squared error (see Leon, Shoemaker, and Kackar (1987) for an explanation of why variance might be used as an objective function when an adjustment parameter exists).

TABLE 5.1 ABOUT HERE

During the growth process, the wafers are mounted on the sides of a fixture called a susceptor. This susceptor has seven sides, each with a top and bottom position, so that a total of 14 wafers are processed at one time. We will consider two noise factors: location of the wafer on the susceptor (top or bottom), and facet (1-6) (there were actually 7 facets, but data from one of them is missing). To simplify the analysis, we will consider only four of the facets, numbers 1, 2, 4, and 6. Omission of the remaining facets doesn't qualitatively change the results of the analysis.

The imputed data were obtained from

$$y_{ijk} = \bar{y}_{ij.} + \bar{y}_{i.k} - \bar{y}_{i..} + \delta_{ijk} \quad , \tag{5.1}$$

where y is the thickness of the epitaxial layer, i corresponds to a particular combination of control factor levels, j is the index for location, k is the index for facet, δ_{ijk} is normally distributed with mean 0 and standard deviation .1, and

$$\bar{y}_{ij.} = \frac{1}{6}\sum_{k=1}^{6} y_{ijk} \quad , \quad \bar{y}_{i.k} = \frac{1}{2}\sum_{j=1}^{2} y_{ijk} \quad , \quad \bar{y}_{i..} = \frac{1}{12}\sum_{j=1}^{2}\sum_{k=1}^{6} y_{ijk} \quad .$$

This amounts to assuming that location and facet are independent in their effects on the epitaxial thickness response. Tables 5.2 and 5.3 shows marginal means corresponding to the 16 control factor combinations in the experiment.

TABLES 5.2 and 5.3 ABOUT HERE

We will compare the results obtained using

1. Loss Model Procedure with Product Array
2. Response Model Procedure with Product Array
3. Response Model Procedure with a Combined Array that is a fraction of the Product Array.

5.1 Loss Model Procedure with Product Array

Following Taguchi's product array formulation and the original experiment as reported by Kacker and Shoemaker, we use as the control array a 16-run fractional factorial design with the following generators: $D=ABC$, $F=ABE$, $G=ACE$, $H=BCE$. We use an 8-run full factorial noise array, crossing the 4-level facet noise factor with the 2-level location noise factor.

Figure 5.1 shows the half-normal plot of the estimable effects of the control factors on *log Var*(y). The only obviously significant variance effect is the control factor H, nozzle position. Although the effect of the control factor A (rotation method) is not clearly significant, the level of this factor might still be changed in an effort to reduce *Var*(Y), since the change doesn't involve any additional manufacturing cost.

Based on this analysis the following control factors would be changed from their initial settings:

	Factor	Initial Setting	New Setting
A	Rotation Method	Oscillating	Continuous
H	Nozzle Position	4	6

Since these are the same conclusions as in the original study, we know that these changes gave a 37% reduction in the epitaxial thickness standard deviation.

5.2 Response Model Procedure with Product Array

The response model implied by the product array used above allow estimation of all control and noise main effects, and all control-by-noise interactions.

We represent the facet noise factor, which has 4 levels, by three orthogonal contrasts,

$$
\begin{aligned}
M_1 &= F_1+F_2-(F_4+F_6) \\
M_2 &= F_1+F_4-(F_2+F_6) \\
M_3 &= F_1+F_6-(F_2+F_4)
\end{aligned}
$$

A half-normal plot of the estimable response effects is given in Figure 5.2.

As expected, deposition time has a large effect on thickness, but so does location. Of special interest are the control-by-noise interactions because they can indicate potential for achieving a robust control factor combination. The large HL (location-by-nozzle position) interaction may indicate nozzle position's potential for reducing sensitivity to location. The interaction plot in Figure 5.3 bears this out. Clearly, nozzle position 6 gives far less thickness variability between top and bottom locations than does nozzle position 2.

Despite the fact that none of the facet-by-control interaction effects seem significant, the interaction plots in Figures 5.4(i) through 5.4(v) show some opportunities for improving cross-facet uniformity. Judicious selection of levels for factors A and F could reduce $\log Var(Y)$ since in each case one of the control factor levels gives a much flatter line than the other. On the other hand, the facet-by-C and facet-by-H interactions have large mean squares, but do not have obvious potential for improving robustness.

Based on this analysis, the settings of control factors H, A, and F might be changed to increase uniformity in epitaxial thickness:

	Factor	Initial Setting	New Setting
A	Rotation Method	Oscillating	Continuous
F	HCl Etch Temp	1200	1215
H	Nozzle Position	4	6

As noted earlier, factor D is used to adjust the average thickness to target, and indeed it has a very large main effect for thickness. Using D to adjust the mean in this way will, however, have some effect on the thickness variability.

The response model approach has given us several important pieces of information that we could not obtain from the loss model approach. First, it was revealed that nozzle position (H) affects variability because it reduces cross-location variation; it does not seem to have any potential for reducing cross-facet variation. Second, the large (but not significant) rotation method effect observed in the Loss Model Approach is apparently present because continuous rotation of the susceptor gives better uniformity across facets than oscillation of the susceptor. Finally, the interaction plots also showed that a longer deposition time and the higher HCl etch temperature give greater thickness uniformity. All of these observations give the experimenter greater understanding of the physical mechanisms behind the epitaxial growth process than would have been realized with the loss model approach.

The results are qualitatively the same if the standard deviation used in equation (5.1) is increased to 0.2. However, the significant effects change substantially if it is increased to 0.5.

5.3 Response Model Procedure with a Fractionated Combined Array

If the response model approach is being used, there is no reason to run a product array. Instead, the design matrix containing both noise and control factors can be fractionated.

Table 5.4 shows several ways in which the product array for the epitaxial growth experiment could have been fractionated. In comparison with the 128-run product array (Design #1), Design #2 requires only 64 runs and still allows estimation of most control-by-noise interactions. However, it confounds E with LM_1, L with EM_1, and M_1 with EL. Design #3, on the other hand, allows estimation of all control and noise factor main effects, plus 12 of the control-by-control interactions, 19 control-by-noise interactions, and 2 out of the 3 possible noise-by-noise interactions, all in 64 runs.

TABLE 5.4 ABOUT HERE

Noting that Design #2 is a subset of the product array design, we have analyzed the data that would correspond to this 64-wafer experiment. This analysis leads to qualitatively the same results as presented in Section 5.2 for the product array design. Because of the confounding mentioned above, however, we cannot tell that the large $L=EM_1$ effect is actually due to a large location effect, and not to an EM_1 interaction. If engineering knowledge could not clear up this ambiguity, additional runs would be needed to de-alias these effects (see Box, Hunter, and Hunter (1978), page 413 for a way to do this economically).

The fractionated combined array designs suggested for this example in Table 5.4 do not substantially reduce the cost of the experiment. This is because they all still require that 16 different processing conditions be run, just as in the product array design. For each processing condition, the noise factors location and facet are fairly inexpensive to study by simply measuring epitaxial thickness of wafers processed at different locations and facets. On the other hand, fractionating the combined array in this example does reduce material and measurement costs, if dummy (scrap) wafers can be placed in the unneeded positions on the susceptor.

However, the options for fractionating the combined array do illustrate that significant run and cost savings could be possible in applications in which noise is truly expensive to study.

6. Discussion

The most obvious benefit of the combined array approach to robust parameter design is a dramatic reduction in the number of runs over the product array approach.

In addition, the combined array approach gives the experimenter flexibility to use degrees of freedom to estimate the effects most likely to be important. These might include control factor-by-control factor interactions and curvature effects in some control factors, in addition to certain control factor-by-noise factor interactions. In contrast, the product array approach requires that a large number of degrees of freedom be used to estimate all possible control factor-by-noise factor interactions, and often control factor-by-control factor-by-noise factor interactions, even though many of these effects could be ruled out apriori using even limited knowledge of underlying physical mechanisms.

The third major advantage of the combined array approach is its focus on modeling Y, as described in Section 4. This is usually a simpler job than modeling R. To make the modeling job as easy as possible, we should try to choose a response Y that has an additive relationship with the control and noise factors. Phadke and Taguchi (1987) encourage use of knowledge about underlying physical mechanisms to choose an appropriate response, but data transformations such as those used by Box and Cox (1964) could also be used to good effect.

Another advantage of modeling Y is additional information about specific CxN interactions that may allow reduction of response variability induced by noise factors. This was illustrated in Section 5, where different control factors were revealed to have a dampening effect on each of the two noise factors.

As described in Section 4, a potential drawback of the combined array approach is its dependence on the fitted model for Y. For this reason, the confirmation experiment and model diagnostics play an especially important role in combined array experiments. Model selection and checking techniques such as stepwise regression and normal probability plots of residuals can be used. (see Daniel (1976) and Box, Hunter and Hunter (1978))

The combined array/response model approach is perhaps not as simple conceptually as the product array/loss model approach. As demonstrated in Section 5, analysis of response models requires not only identification of large factor effects, but also detailed examination of interaction plots.

Sometimes use of the combined array does not lead to much financial savings because noise can be relatively easily and inexpensively introduced through replication. For example, noise was actually relatively cheap to study in the epitaxial growth example of Section 5. However, in many other situations (the wave soldering process mentioned in Section 1, for example) introducing certain noise factors does multiply the cost of the experiment and the combined array is very valuable.

The situations considered in this paper involve control and noise factors whose values can be fixed at various levels in an experiment. If some control or noise factors are random (for example, if control factors drift from their nominal levels and are thus sources of noise themselves), the response model may be a mixed- or random-effects model. These cases would require different estimation and analysis techniques than those presented here.

Acknowledgement

The authors wish to acknowledge the referees, William Q. Meeker, and V. N. Nair for helpful comments.

The work of A. C. Shoemaker and K. L. Tsui was supported by the Manufacturing and Development Council of AT&T Bell Laboratories. The work of C. F. J. Wu was supported by the National Science Foundation grant DMS-8420968, AT&T Bell Laboratories, and the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

Box, G.E.P. and Cox, D.R. (1964), "An Analysis of Transformations", *Journal of Royal Statistical Society, Ser. B*, 26, 211-252.

Box, G.E.P. and Draper, N.R. (1987), *Empirical Model-Building and Response Surfaces*, John Wiley & Sons, New York.

Box, G.E.P. and Hunter, W.G., and Hunter, J.S. (1978), *Statistics for Experimenters*, John Wiley & Sons, New York.

Daniel, C. (1976), *Applications of Statistics to Industrial Experimentation*, John Wiley & Sons, New York.

Hamami, M., Hooper, J.H., and Nazaret, W.A. (1987), *Computed Aided Design for Quality*, presented at the Northern New Jersey Chaper of the American Statistical Association's Spring Symposium.

John, P.W.M. (1971), *Statistical Design and Analysis of Experiments*, The Macmillan Company, New York.

Kackar, R.N. and Shoemaker, A.S. (1986), "Robust Design: A Cost-Effective Method for Improving Manufacturing Processes", *AT&T Technical Journal*, Vol. 65, Issue 2, 39-50.

Leon, R.V., Shoemaker, A.C., and Kackar, R.N. (1987), "Performance Measure Independent of Adjustment: An Explanation and Extension of Taguchi's Signal to Noise Ratio", *Technometrics*, Vol. 29, No. 3, 253-85.

Mitchell, T.J. (1974), "An Algorithm for the Construction of 'D-Optimal' Experimental Designs", *Technometrics*, Vol. 16, No. 2, 203-10.

Phadke, M.S. and Taguchi, G. (1987), "Selection of Quality Characteristics and S/N Rations for Robust Design", presented at IEEE GLOBECOM-87 Meetings, Tokyo, Japan.

Taguchi, G. (1986), *Introduction to Quality Engineering: Designing Quality into Products and Processes*, Asian Productivity Organization, Tokyo, Japan.

Taguchi, G. and Phadke, M.S. (1984), "Quality Engineering Through Design Optimization", IEEE Global Communications Conference, GLOBECOM-84 Meetings, Atlanta, GA, 1106-13.

Welch, W.J., Yu, T.K., Kang, S.M. and Sacks, J. (1989), "Computer Experiments for Quality Control by Parameter Design", to appear in the *Journal of Quality Technology*.

Examples	C	CxC	CxN	N	NxN
1	+	-	9/9 ++	+	-
2	++	+	6/9 ++ & 3/9 +	+	-
3	++	+	7/9 +	++	+

(i) C and N denote respectively the main effects of control factors and of noise factors.
(ii) ++ : estimable if 3- and higher-order interactions are negligible.
+ : estimable if other 2-factor interactions are negligible.
- : not estimable.
(iii) 6/9 ++ : 6 out of 9 effects are ++.

Table 3.1. Comparison of estimation capacities for the plans in Examples 1-3

Run	A	B	C	D	a	b
1	1	1	1	1	1	-1
2	1	-1	-1	1	1	-1
3	1	-1	-1	-1	-1	-1
4	1	1	-1	1	-1	-1
5	1	1	1	1	-1	1
6	1	-1	1	1	1	1
7	-1	1	1	-1	-1	-1
8	1	-1	1	1	-1	-1
9	1	-1	1	-1	1	-1
10	-1	-1	1	-1	-1	1
11	1	1	-1	-1	1	1
12	-1	-1	-1	-1	1	1
13	-1	1	-1	-1	1	-1
14	-1	1	1	1	-1	-1
15	1	-1	-1	1	-1	1
16	-1	1	-1	1	1	1
17	-1	-1	-1	1	-1	-1
18	1	1	1	-1	-1	1
19	-1	-1	1	1	1	-1
20	-1	1	1	-1	1	1
21	-1	1	-1	-1	-1	1
22	-1	-1	1	1	-1	1

Table 3.2: 22-run D-optimal design

	Factor	Initial Setting	Experimental Settings			
	CONTROL FACTORS					
A	Rotation Method	Oscillating	Continuous	Oscillating		
B	Wafer Code	-	668G4	678D4		
C	Deposition Temperature	1215	1210	1220		
D	Deposition Time	Low	High	Low		
E	Arsenic Flow Rate	57%	55%	59%		
F	HCl Etch Temperature	1200	1180	1215		
G	HCl Flow Rate	12%	10%	14%		
H	Nozzle Position	4	2	6		
	NOISE FACTORS					
L	Location		Top	Bottom		
M	Facet		1	2	4	6

Table 5.1: Control and Noise Factors and Their Settings

Run	Facet 1	Facet 2	Facet 4	Facet 5	Facet 6	Facet 7
1	14.812	14.774	14.772	14.794	14.860	14.914
2	14.886	14.810	14.868	14.876	14.958	14.932
3	13.996	13.988	14.044	14.028	14.108	14.060
4	13.860	13.876	13.932	13.846	13.896	13.870
5	14.182	14.172	14.126	14.274	14.154	14.082
6	13.768	13.778	13.870	13.896	13.932	13.914
7	14.722	14.736	14.774	14.778	14.682	14.850
8	14.758	14.784	15.054	15.058	14.938	14.936
9	14.324	14.092	13.536	13.588	13.964	14.328
10	13.970	14.448	14.326	13.970	13.738	13.738
11	14.184	14.402	15.544	15.424	15.036	14.470
12	13.918	14.044	14.926	14.962	14.504	14.136
13	14.648	14.350	14.682	15.034	15.384	15.170
14	15.272	14.656	14.258	14.718	15.198	15.490
15	13.614	13.202	13.704	14.264	14.432	14.228
16	13.866	14.130	14.256	14.000	13.640	13.592

Table 5.2: Marginal means for Facet.

Run	Top Location	Bottom Location
1	15.352	14.290
2	14.993	14.783
3	14.059	14.016
4	14.307	13.453
5	14.191	14.139
6	14.341	13.379
7	15.363	14.151
8	15.300	14.543
9	14.652	13.291
10	14.013	14.050
11	14.914	14.773
12	14.885	13.945
13	15.000	14.756
14	15.511	14.353
15	14.529	13.286
16	13.987	13.841

Table 5.3: Marginal means for Location.

Design	Generators	Number of Estimable Effects (Assuming no 3-factor interactions)				
		C	N	CxC	CxN	NxN
#1 128 Wafers Product Array	D=ABC F=ABE G=ACE H=BCE	All 8	All 4	0	All 32	All 3
#2 64 Wafers	Above and M_1=EL	7	2	0	28	0
#3 64 Wafers	F=ABE G=ACEL H=ACDE M_1=BCE M_2=ABCDL	All 8	All 4	12	19	2

Table 5.4: Three alternative combined array designs for the epitaxial growth example

CONTROL ARRAY

ROW	A	B	C	D
1	-1	-1	-1	-1
2	-1	-1	1	1
3	-1	1	-1	1
4	-1	1	1	-1
5	1	-1	-1	1
6	1	-1	1	-1
7	1	1	-1	-1
8	1	1	1	1

NOISE ARRAY

ROW	a	b
1	-1	-1
2	-1	1
3	1	-1
4	1	1

ROW	a	b
1	-1	-1
2	-1	1
3	1	-1
4	1	1

Figure 1.1: Two-part experimentation strategy recommended by Taguchi

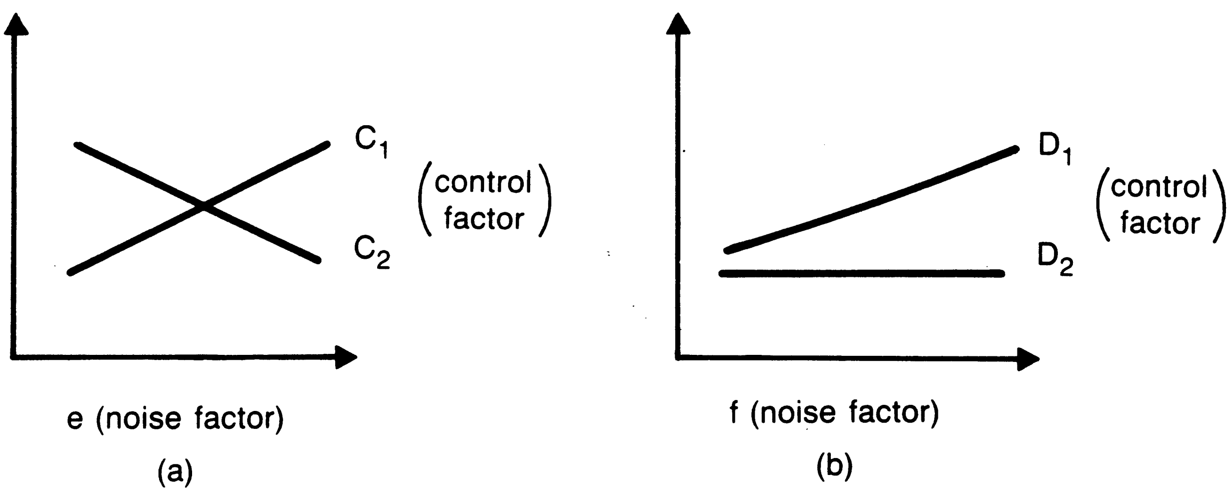

Figure 4.1: Two hypothetical $C \times N$ interaction plots. In (a), $C \times e$ is large but does not give an opportunity to improve robustness. In (b) $D = D_2$ gives better robustness to noise factor f than $D = D_1$.

Figure 5.1: Half Normal Plot of Log Variance Effects

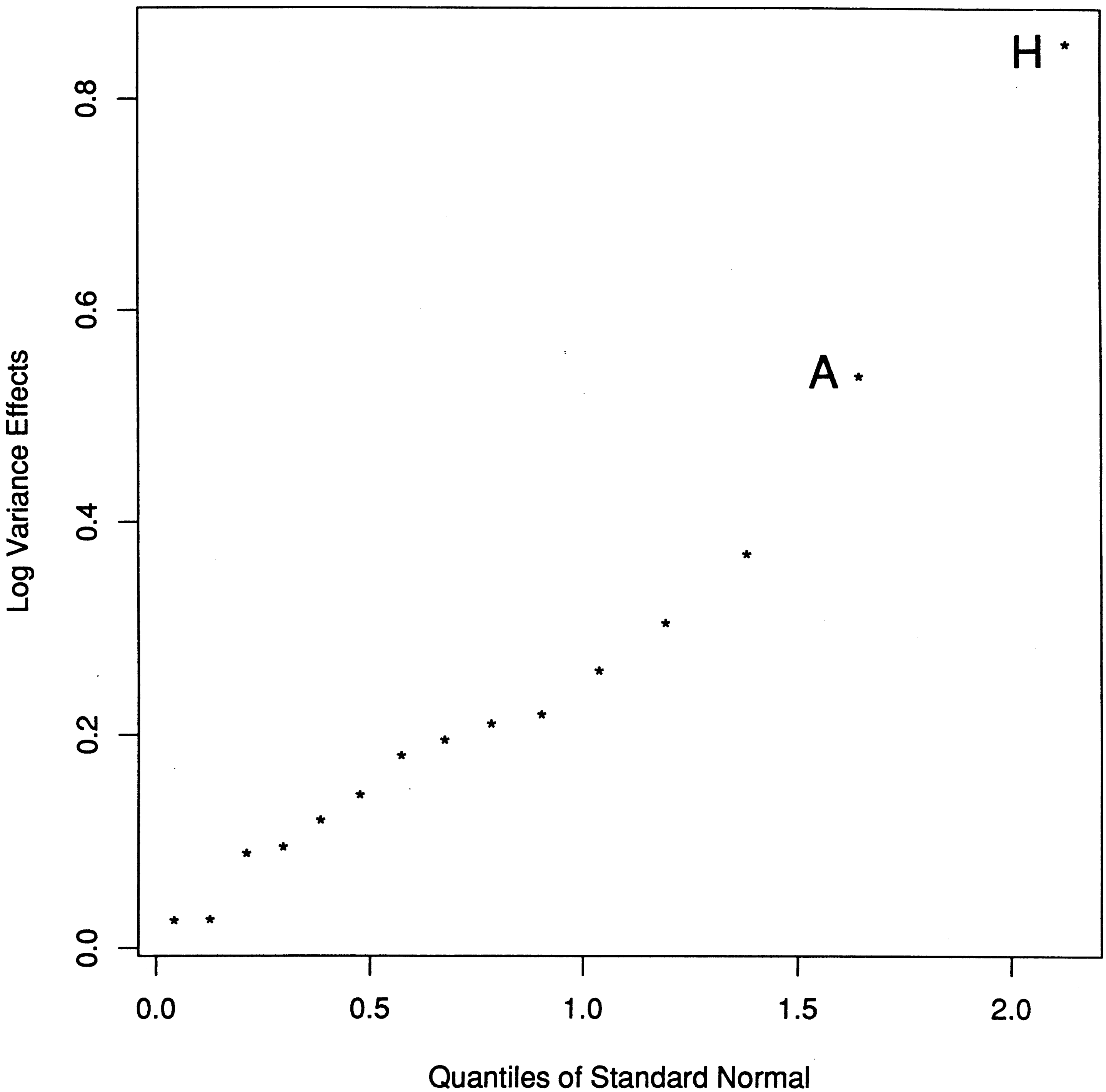

Figure 5.2: Half Normal Plot of Response Model Effects

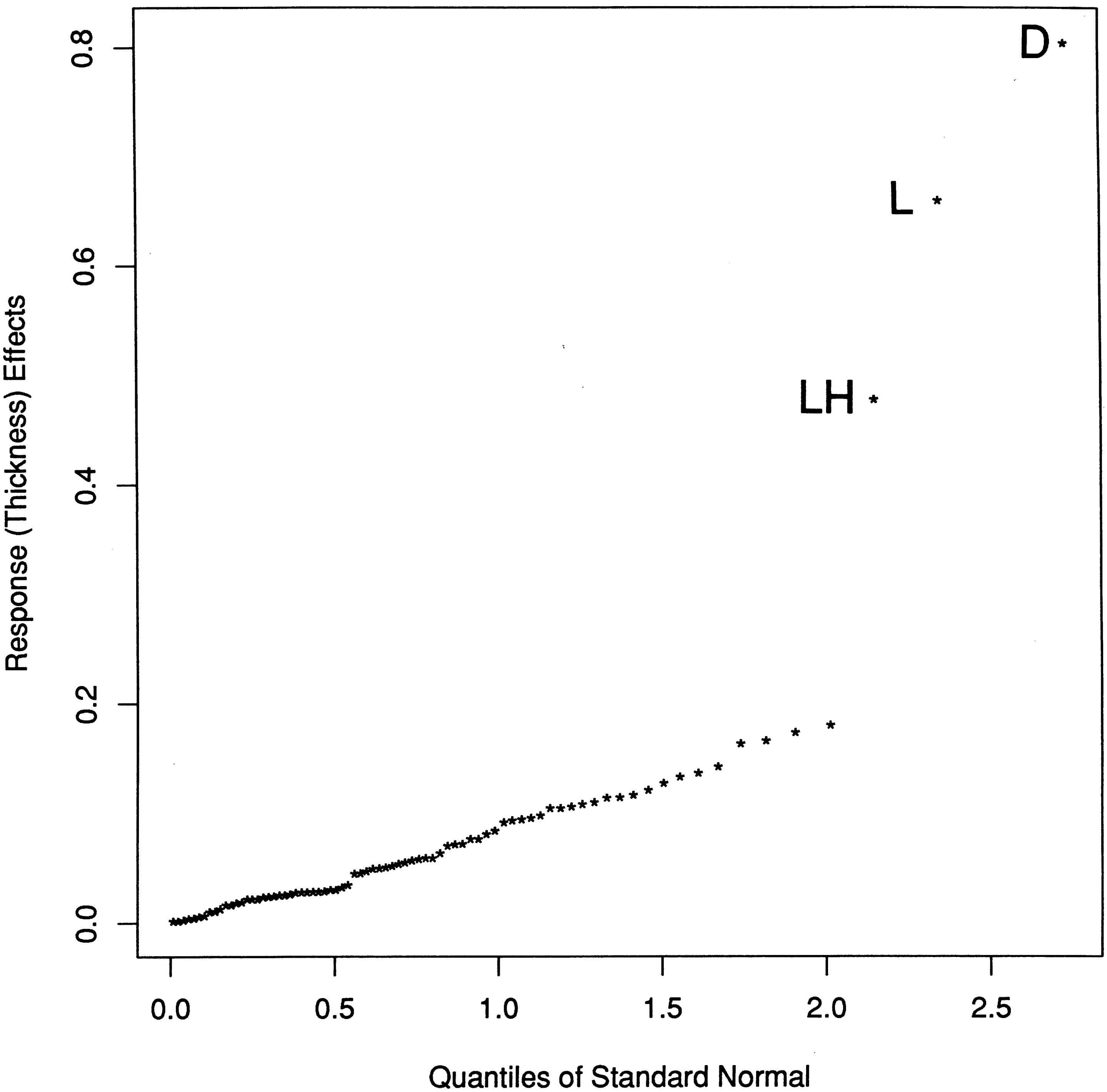

Figure 5.3 : Interaction Plot for Nozzle Position (H)

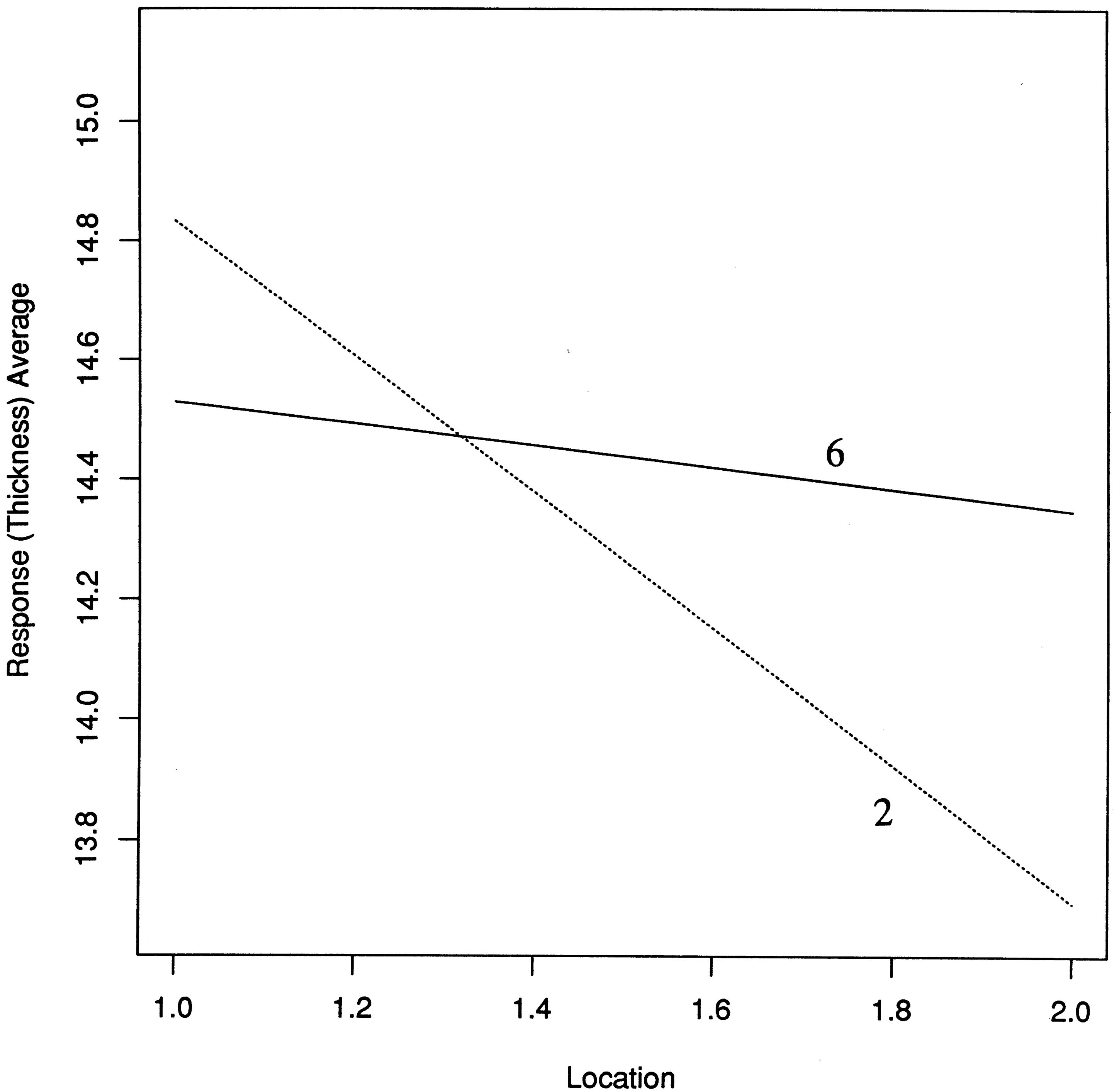

Figure 5.4 (i) :Interaction Plot for Rotation Method (A)
(Mean Square = .132)

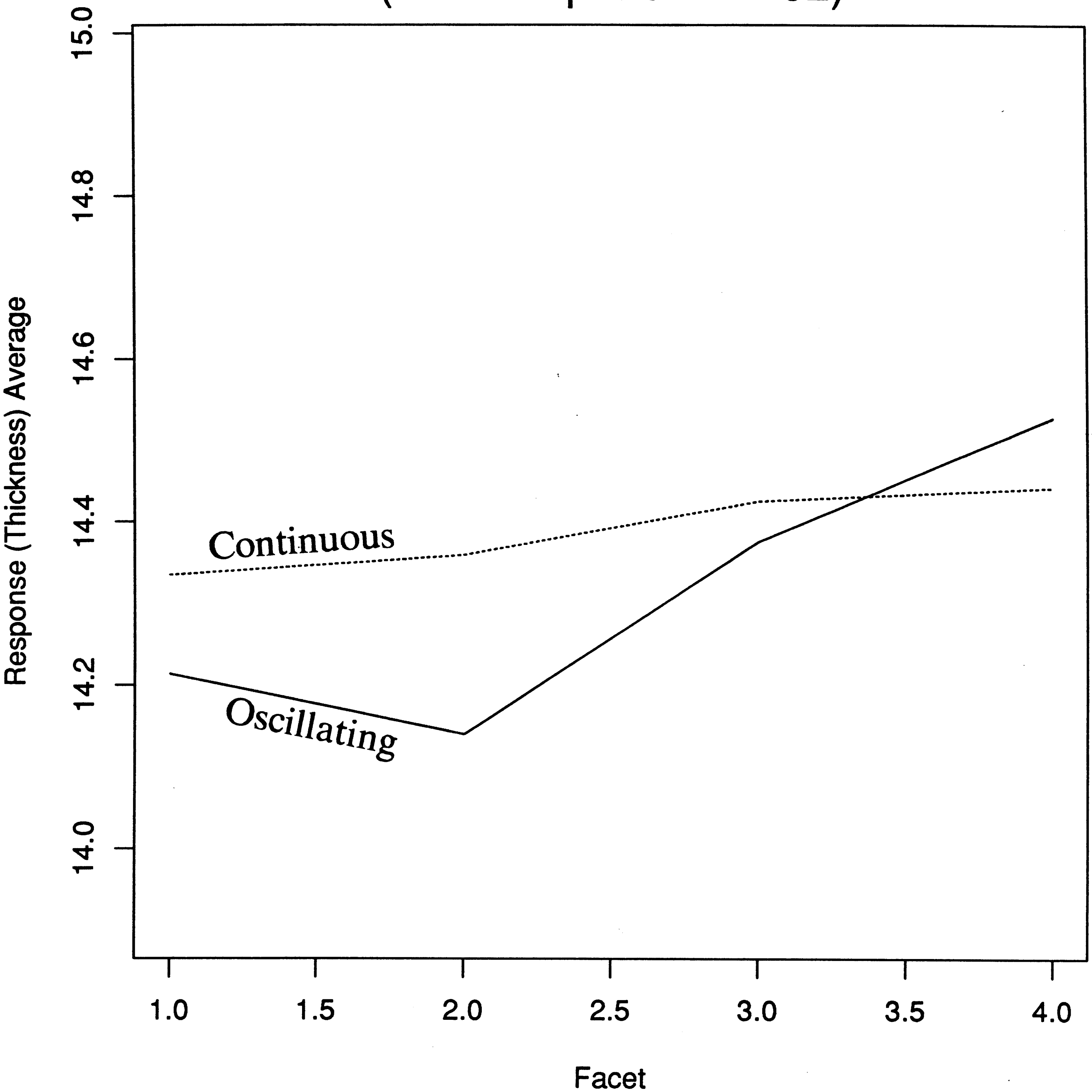

Figure 5.4(ii) :Interaction Plot for Deposition Temp (C)
(Mean Square = .446)

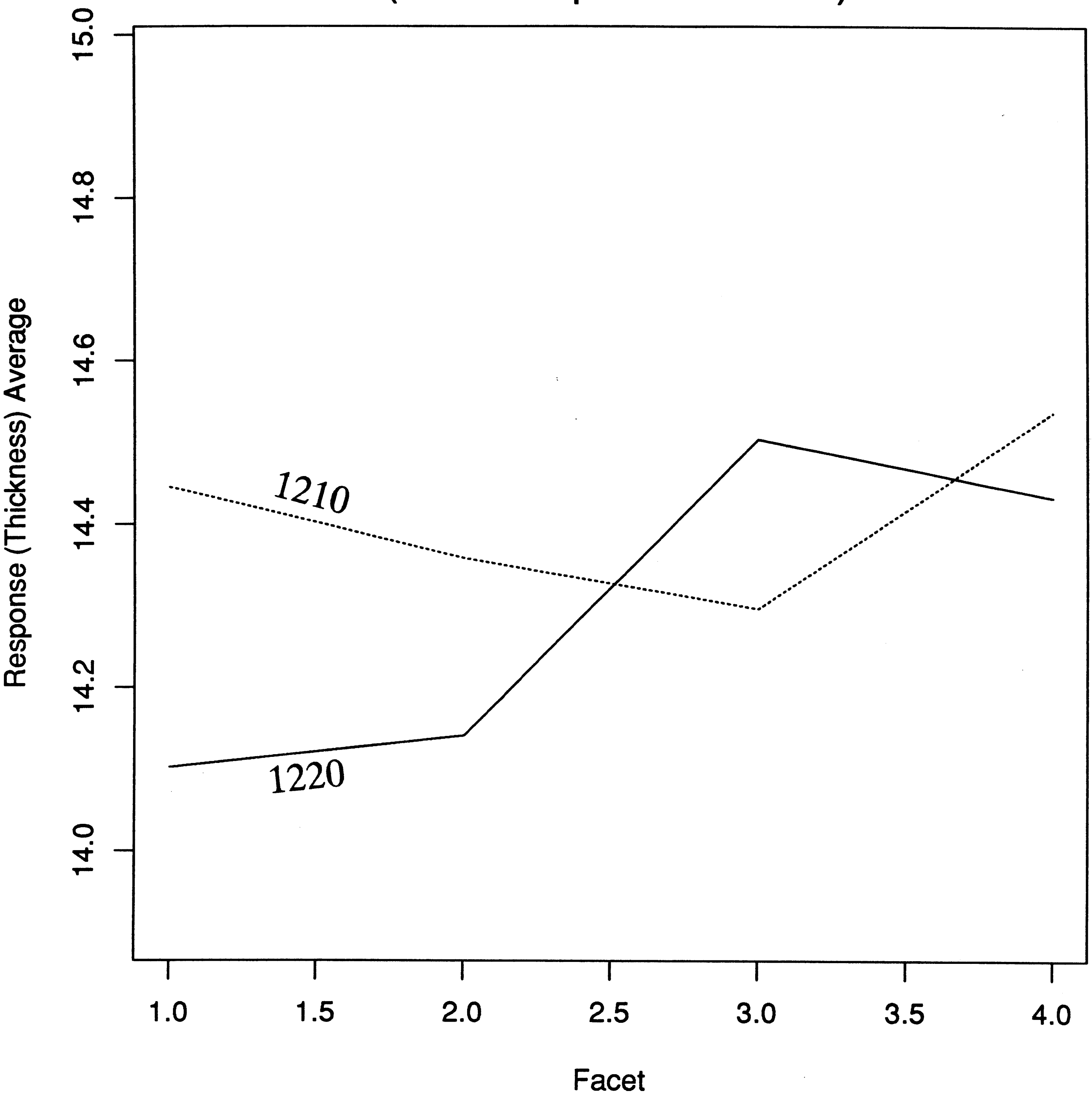

Figure 5.4(iii):Interaction Plot for Deposition Time (D)
(Mean Square = .213)

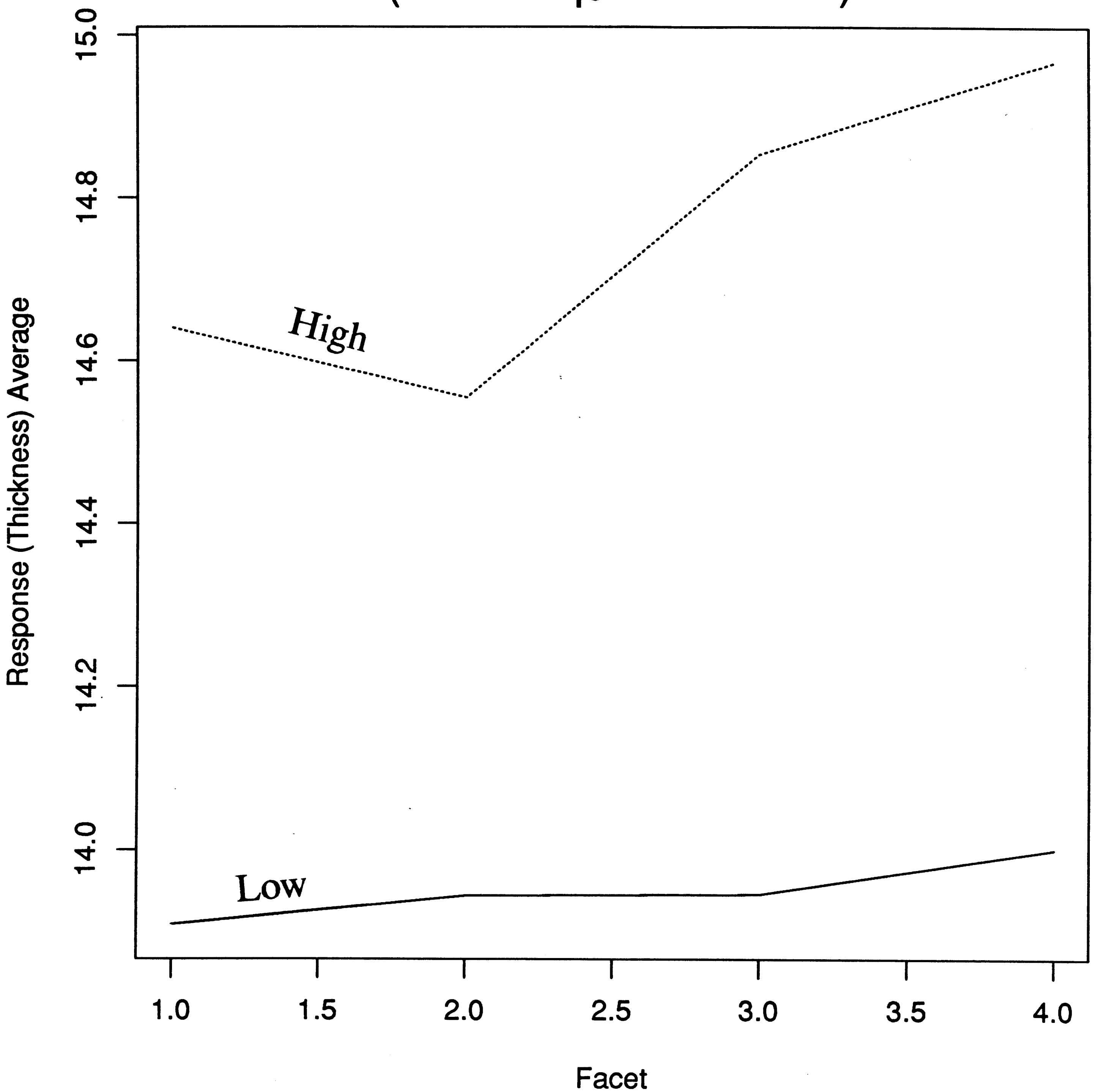

Figure 5.4(iv):Interaction Plot for HCL Etch Temp (F)
(Mean Square = .150)

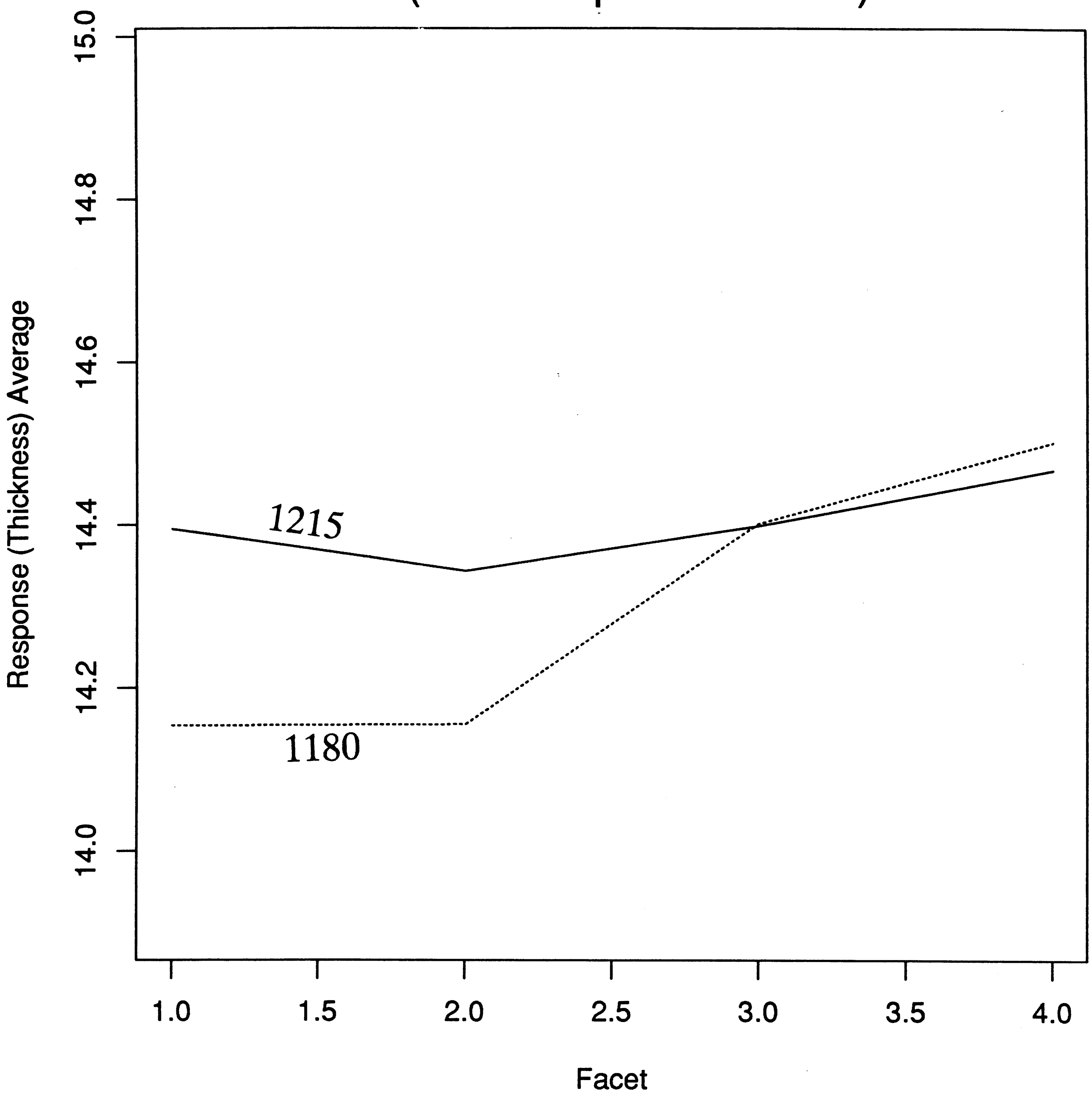

Figure 5.4 (v):Interaction Plot for Nozzle Position (H)
(Mean Square = .295)