# COMPUTER EXPERIMENTS FOR QUALITY CONTROL BY PARAMETER DESIGN

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# COMPUTER EXPERIMENTS FOR QUALITY CONTROL BY PARAMETER DESIGN

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*ABSTRACT*

Taguchi's off-line quality control methods for product and process improvement emphasize experiments to design quality "into" products and processes. These experiments search for values of engineering parameters that define products insensitive to sources of variability (noise). Typically, separate experimental designs are used for the engineering parameters and the noise parameters. Data from the noise design are collapsed to loss statistics, or signal-to-noise ratios, and these statistics are modeled and used to predict good levels of the engineering parameters. In addition to physical experiments, these ideas have been applied to computer experiments where the observations are generated by a computer model or simulation. In Very Large Scale Integrated (VLSI) circuit design, the application of interest here, computer modeling is invariably quicker and cheaper than physical experimentation. Nevertheless, the cost simulation can still prohibit the large number of experimental runs often required for a Taguchi experiment.

Our approach models quality characteristics generated by the computer simulation as functions of both the engineering and noise parameters. The *single* experimental design for both types of parameters typically requires far fewer runs. The model is used to predict the quality characteristics, from which loss statistics can also be predicted and optimized. In the VLSI applications described, we obtain effective prediction of product performance with comparatively few observations.

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# Introduction

Taguchi's off-line quality control methods for product and process improvement (Taguchi and Wu 1980, Taguchi 1986) have generated considerable industrial and academic interest. They emphasise designing quality "into" products and processes, so that they are insensitive to sources of variability or noise, rather than achieving quality after the fact by on-line inspection (inspecting in quality). Parameter design, an important step in off-line quality control, is the search for levels of engineering parameters that lead to a product or process robust to the noise factors. These engineering parameters, such as nominal dimensions, are often called control factors, because they can easily be changed or controlled (unlike noise factors, which are typically expensive to control). Kackar (1985) gave a very readable account of the main ideas; Kackar and Shoemaker (1986) and Phadke (1986) provided several examples.

A key feature of these ideas is the separation of the control factors $x_{\text{con}}$ and the noise factors $x_{\text{noise}}$. (Sometimes a control factor and a noise factor may relate to the same variable: for instance, a nominal value and a deviation from nominal due to manufacturing variablility.) For example, in the main application of this article, a Very Large Scale Integrated (VLSI) circuit, the control factors are transistor dimensions to be chosen. The noise factors correspond to manufacturing-process variability, including variablity about the selected nominal dimensions. The objective of the experiment is to choose the transistor dimensions so that the circuit performance is robust to the noise variation.

Approaches by Taguchi employ two experimental designs (or arrays): one for the control factors and another for the noise factors. Observations on a quality characteristic are taken for every combination of $x_{\text{con}}$ in the control array and $x_{\text{noise}}$ in the noise array. For given $x_{\text{con}}$ in the control array, the "replicate" observations generated by the noise array are reduced to a loss statistic (or a related signal-to-noise ratio), measuring the performance of the engineering design defined by $x_{\text{con}}$. The loss statistic averages a measure of loss over the distribution of the noise factors. In the main example below the target for the simulation output $Y(x_{\text{con}}, x_{\text{noise}})$ is zero, and we use an expected squared-error loss, following Taguchi,

$$L(x_{\text{con}}) = \int Y^2(x_{\text{con}}, x_{\text{noise}}) g(x_{\text{noise}}) d\, x_{\text{noise}}, \tag{1}$$

where $g(x_{\text{noise}})$ is the noise probability density function. The objective is to find an engineering design $x_{\text{con}}$ that minimizes this loss. Usually this is done by modeling the observed losses, often including only the main effects of $x_{\text{con}}$, and optimizing over $x_{\text{con}}$. Even if the engineer has carefully parameterized the problem to reduce interaction effects on the underlying quality characteristic $Y$, it is not clear why complicated loss statistics or signal-to-noise ratios should admit such simple, main-effects models.

VLSI circuits may be simulated by computer models or codes, in which case the simulation output is a deterministic function of all the factors, both control and noise, if noise factors are included in the inputs. Many parameter-design experiments in electrical engineering are of this type: for example, Phadke's (1986) differential operational amplifier experiment and even Taguchi's Wheatstone bridge example (Taguchi and Wu 1980, Chapter 5.2). In VLSI-circuit design, computer modeling is invariably quicker and cheaper than physical experimentation. Nonetheless, the cost of the simulation can still prohibit the large number of experimental runs often required by Taguchi's "crossed-array" plans. Similarly, direct optimization of $L(x_{\text{con}})$ can require large numbers of evaluations of $Y(x_{\text{con}}, x_{\text{noise}})$ in order to evaluate $L(x_{\text{con}})$ at every $x_{\text{con}}$ tried.

In contrast, our approach models a quality characteristic $Y(x_{\text{con}}, x_{\text{noise}})$ generated by the computer simulation as a function, typically polynomial, of both $x_{\text{con}}$ and $x_{\text{noise}}$. The *single* experimental design for both types of parameters typically requires far fewer runs than crossed arrays. The model provides a computationally-cheap surrogate $\hat{Y}(x_{\text{con}}, x_{\text{noise}})$ for the simulator. The loss statistic, for example (1), can in turn be predicted by replacing $Y(x_{\text{con}}, x_{\text{noise}})$ by the approximating $\hat{Y}(x_{\text{con}}, x_{\text{noise}})$. We then search for $x_{\text{con}}$ minimizing the predicted loss statistic. In the VLSI applications described, as well as requiring comparatively few observations, we obtain more-effective prediction of product performance than crossed-array experiments, probably because the underlying quality characteristic $Y$ admits a simpler model than the loss statistic, and is therefore easier to predict accurately.

The next section defines our method in greater detail. Much of the remainder of the article is concerned with application to the main VLSI example.

## Optimizing Loss Statistics Via Modeling the Underlying Response

The proposed method involves six steps.

1. Design an experiment to predict the response of interest as a function of the control factors $x_{\text{con}}$ and the noise factors $x_{\text{noise}}$:

$$\text{Response} = \text{linear model} + \text{error},$$

or

$$Y = \sum \beta_j f_j(x_{\text{con}}, x_{\text{noise}}) + \text{error}.$$

The $f_j$'s are assumed-known functions of $x_{\text{con}}$ and $x_{\text{noise}}$ (for example, a polynomial model), and the $\beta_j$'s are unknown constants to be estimated from the data. The error term represents systematic departure from the assumed model, because the computer response $Y$ is deterministic. As noted in the introduction, we do not use separate control and noise arrays; considerable economy in the number of observations can result from designing a single experiment for both types of factors.

2. Predict $Y(x_{\text{con}}, x_{\text{noise}})$ by the least squares estimate $\hat{Y}(x_{\text{con}}, x_{\text{noise}}) = \sum \hat{\beta}_j f_j(x_{\text{con}}, x_{\text{noise}})$.

3. For given $x_{\text{con}}$, predict a loss statistic $L(x_{\text{con}})$ from the estimated response function. For example, the prediction of the loss statistic (1) is

$$\hat{L}(x_{\text{con}}) = \int \hat{Y}^2(x_{\text{con}}, x_{\text{noise}})g(x_{\text{noise}})dx_{\text{noise}}.$$

In the application below the density $g(x_{\text{noise}})$ is taken as uniform on a finite set of noise levels representing typical and extreme processing conditions. This is mainly for convenience.

We prefer to minimize expected loss, because it underlies the Taguchi philosophy, rather than maximize a signal-to-noise ratio. Leon, Shoemaker, and Kacker (1987) discussed the connections between these two optimization problems.

4. Minimize $\hat{L}(x_{\text{con}})$ as a function of $x_{\text{con}}$. In practice, the mathematical optimization will be tempered by engineering and cost considerations. For instance, VLSI designs with small values of $x_{\text{con}}$ and hence chip area are often of interest.

5. Conduct a confirmatory experiment to evaluate $L(x_{\text{con}})$ by fixing $x_{\text{con}}$ at the value(s) found in step 4 and varying $x_{\text{noise}}$ according to $g(x_{\text{noise}})$.

6. Iterate if necessary. For instance, if the optimization step suggests values of $x_{\text{con}}$ outside the design region which are technically and economically feasible then the design region might be shifted.

## Example: VLSI Clock Driver Design

### Background

Clock drivers play a critical role in digital integrated circuits and systems. Figure 1 shows such a clock driver circuit. From the master clock $\mathrm{CLK}_{\mathrm{M}}$, the circuit generates CLK and $\overline{\mathrm{CLK}}$, clocks with pulses of opposite polarity, to control data flow. Usually it is important that CLK and $\overline{\mathrm{CLK}}$ have the same transition times (zero clock skew). However, due to process variability and other factors this is often difficult to attain. Because each clock signal switches twice per machine cycle, two clock skews $S_1$ and $S_2$ can be measured (in units of nanoseconds), as illustrated in Figure 2.

The main goal is to determine widths $w_1, \ldots, w_6$ for the transistors $M_1, \ldots,$ $M_6$ in Figure 1 that give the smallest clock skews in the presence of process variability. To allow for quadratic effects, the experiment is carried out with each width at three levels, denoted by -1, 0, and 1.

For this type of circuit, process variability is often characterized by extracting three sets of process parameters associated with high (H), medium (M), and low (L) current-driving capabilities in the transistors. The transistors are of two types, P ($M_1$, $M_3$, and $M_5$ in Figure 1) and N ($M_2$, $M_4$, and $M_6$). A P-type transistor is always paired with an N type (*e.g.*, $M_1$ and $M_2$). Because the three P-type transistors are manufactured simultaneously the process affects them in the same way, and one noise factor is sufficient for the P-type transistors. Let PH, PM, and PL denote high, medium, and low current-driving capabilities for the P-type transistors. Similarly, a second noise factor with levels NH, NM, and NL is sufficient for the three N-type transistors. Following Shoji (1986) we experiment with the five combinations PH-NH, PH-NL, PM-NM, PL-NH, and PL-NL to represent both typical and extreme processing conditions. These noise combinations are coded $1, \ldots, 5$ below.

We will consider loss statistics that combine the two skews. For fixed $w = (w_1, \ldots, w_6)$, there are five pairs of skews corresponding to the five noise levels. Denote these skews by $Y_1(w), \ldots, Y_{10}(w)$. The target skew is zero, and the logarithm of the average squared-error loss is

$$L_{\text{sq}}(w) = \log[\frac{1}{10} \sum_{i=1}^{10} Y_i^2(w)]. \tag{2}$$

The logarithmic transformation, being monotonic, does not affect the optimal $w$ but is relevant below when $L_{\text{sq}}$ is modeled directly. A more-conservative performance measure is the worst-case skew

$$L_{\text{wc}}(w) = \max[|Y_1(w)|, \ldots, |Y_{10}(w)|]. \tag{3}$$

## Minimizing loss via modeling the clock skews

Several considerations determined the choice of model for the clock skews as functions of the control and noise factors.

- Because curvature and interactions cannot be ruled out we adopt a second-order polynomial model.

- There are only five combinations of the two noise factors, whereas a full second-order model in two factors has six unknown constants. For simplicity, therefore, we treat the noise combinations as a single qualitative factor at the five levels $1, \ldots, 5$. (As a referee pointed out, an alternative parameterization is to have $N$ and $P$ effects, the $N \times P$ interaction, and a contrast between the center point and the corner points. This would facilitate interpretation, though predictions would be identical.)

- There are two chains of transistors in Figure 1: $(M_1, \ldots, M_4)$ and $(M_5, M_6)$. This suggests that only the seven interactions $w_1w_2$, $w_1w_3$, $w_1w_4$, $w_2w_3$, $w_2w_4$, $w_3w_4$, and $w_5w_6$ involving transistors in the same chain need be considered.

- No similar reasoning leads to a reduction in the number of interactions between the widths and the noise factor, however. Although one might suspect, for example, that there would be no interaction between the widths of the P-type transistors and the component of the noise factor representing variability in the current-driving capabilities of the N-type transistors, this is not the case. The two types of transistors are interconnected. These interactions between control and noise factors allow the noise effects (and thus sensitivity to noise) to differ at the various control-factor configurations. Easterling (1985) pointed out this connection between interactions and robustness to noise.

Thus, we model each of the two skews as a function of the widths $w$ and noise level $j$ by

$$\begin{aligned} Y(w,j) &= \beta_0 + \beta_1 w_1 + ... + \beta_6 w_6 + \beta_{11} w_1^2 + \cdots + \beta_{66} w_6^2 \\ &+ \beta_{12} w_1 w_2 + \beta_{13} w_1 w_3 + \beta_{14} w_1 w_4 + \beta_{23} w_2 w_3 + \beta_{24} w_2 w_4 + \beta_{34} w_3 w_4 \\ &+ \beta_{56} w_5 w_6 + \gamma_j + \delta_{1j} w_1 + \cdots + \delta_{6j} w_6 + Z. \end{aligned} \tag{4}$$

The unknown constants $\gamma_1, \ldots, \gamma_5$ and $\delta_{11}, \ldots, \delta_{65}$ are the main effects for the qualitative noise factor and the interaction effects between the control and noise factors. Because observations derive from a deterministic circuit simulator there is no random error, and $Z$ represents systematic departure from the assumed linear model. Not all of the unknown constants are identifiable, so we arbitrarily set $\gamma_5 = \delta_{15} = \cdots = \delta_{65} = 0$. This leaves 48 unknown constants to be estimated.

We designed a 60-observation experiment to estimate the 48 unknown constants in model (4). The number of runs is fairly arbitrary; clearly at least 48 are needed, and we wanted a modest number of degrees of freedom to assess potential lack of fit. The ACED package (Welch 1985) was used to obtain the design. This package can construct experiments according to various optimality criteria. Here, the observations are computer simulations not subject to random error. Thus, the mean-squared-error criterion (Welch 1983) in ACED, which addresses bias in prediction arising from model inadequacy, is appropriate. (This criterion also includes the variance arising from random error; we weighted the bias component to be dominant.) The use of a computer package like ACED also circumvents some difficulties in designing this experiment: only some of the interactions need to be estimated and the five-level noise space is not a regular factorial arrangement. The experimental design and the resulting data are given in Table 1.

The two skews are separately modeled via (4). Least squares estimation of the unknown constants allows us to predict the two skews at untried levels of the control and noise factors. In the presence of systematic error rather than random error, statistical testing is inappropriate. Nonetheless, the root mean squared errors of the least squares analyses for the two skews are .03 and .08 (relative to data ranges of about $-3.9$ to 0.2 and $-2.2$ to 3.8), suggesting that the models fit well. For brevity the least squares estimates of the unknown constants are not given. However, we note that

- The first-order effects for both the control and noise factors are all large.
- Many of the second-order (quadratic and interaction) effects are moderately large.
- The contrast between high and low levels of the N-channel noise factor is larger than that for P. It is often found that the N-channel variability is

more critical.

For fixed $w$, the two models are used to predict five pairs of skews corresponding to the five noise levels. Denote these 10 predictions by $\hat{Y}_1(w), \ldots, \hat{Y}_{10}(w)$. The loss statistics (2) and (3) can then be estimated by

$$\hat{L}_{\text{sq}}(w) = \log[\frac{1}{10}\sum_{i=1}^{10}\hat{Y}_i^2(w)]$$

and

$$\hat{L}_{\text{wc}}(w) = \max[|\hat{Y}_1(w)|, \ldots, |\hat{Y}_{10}(w)|].$$

The next step is to minimize either of these loss statistics with respect to $w$. Discussion of this and the validation from confirmatory experiments is deferred to a comparison with alternative experimental design and modeling strategies.

## Modeling loss statistics directly

For comparison, we also conducted an experiment with separate control and noise arrays, as has been advocated by Taguchi for optimizing through direct modeling of a performance measure.

The choice of a model for a loss statistic $L$ is problematic. Whereas the engineer may have substantial background knowledge concerning the underlying response, approximate models for complex loss functions are typically not so intuitive. In this example, when modeling the skews there is an engineering basis for omitting some control-factor interactions. However, this need not imply that the same interactions are negligible when modeling the loss statistics, which are nonlinear functions of the skews. Indeed, these interactions turn out to have fairly large effects. As a simple illustration, $Y = x_1 + x_2$ has no interaction between two factors $x_1$ and $x_2$, but the loss $Y^2$, for example, clearly does. A log transformation is often suggested to reduce interaction effects in signal-to-noise ratios similar to the squared-error loss (2) (see, for example, Kackar 1985). In the absence of engineering intuition, however, we adopt a full second-order model in $w$ for both loss statistics (though hesitantly for the non-smooth $L_{WC}$):

$$L(w) = \beta_0 + \sum_{i=1}^{6} \beta_i w_i + \sum_{i=1}^{6} \sum_{j=i}^{6} \beta_{ij} w_i w_j + Z. \tag{5}$$

There are 28 unknown constants, and we designed a 40-run experiment for the control factors, again using ACED. As when modeling the clock skews, the size of the control array is somewhat arbitrary but allows lack of fit to be measured. Crossing with the noise array of size five leads to a total of 200 runs. Part of the experimental design and data are given in Table 2. For given $w$ in the control array the data generated across the noise array are collapsed to the loss statistics $L_{SQ}(w)$ and $L_{WC}(w)$ in (2) and (3). Fitting model (5) to these observed statistics provides direct predictions of $L_{SQ}(w)$ and $L_{WC}(w)$ at untried $w$'s, which can be optimized with respect to $w$. The root mean squared errors for the $L_{SQ}$ and $L_{WC}$ fits are 0.1 and 0.25 relative to data ranges of $-.5$ to $.8$ and 1.0 to 3.9. Qualitatively, then, the models do not appear to fit quite as well as those for the skews.

This experiment does not strictly follow the pattern of analysis in the examples given by Taguchi (1986). That paradigm fits additive models to the loss statistics (ignoring interactions) and would optimize the level of each transistor width separately. In this example, using the data from the $40 \times 5$-run experiment, such an approach leads to extremely poor predictions (given below). Also, Taguchi's $L_{18}$ experimental design might be used for the control factors. The comparison we do make is more consistent with our methods.

## Results and comparisons

We now give results for:

(I) the 60-run experiment for the skew models (4), from which loss statistics are predicted indirectly, and

(II) the $(40 \times 5)$-run, crossed-array experiment for modeling the loss statistics directly via (5).

We also consider a hybrid strategy:

(III) the $(40 \times 5)$ experiment from strategy II but modeling the skews to predict the loss statistics as in strategy I.

Table 3 gives results for the loss statistic $L_{\rm SQ}$. Listed there are the best-three sets of transistor dimensions $w$ over the $3^6$ grid $\{-1,0,1\}^6$ as predicted by each of the three strategies. The last column gives the true $L_{\rm SQ}$'s from confirmatory experiments. Similarly, Table 4 presents the results for the loss statistic $L_{\rm WC}$. Key features of these results are:

- The confirmatory experiments indicate that strategy I predicts the skews accurately enough to give predictions very close to the actual losses.

- Strategy I gives more reliable predictions and superior circuit designs (smaller actual losses) than strategy II. These differences are of importance; for example, a reduction of the worst-case skew from 1.07 *ns* to 0.53 *ns* is of practical significance.

- Strategies I and III give virtually identical results. This shows that a well-chosen, small experiment can be adequate.

- Comparing the results for all three strategies indicates the superiority of modeling the skews rather than modeling the loss statistics directly.

- Fitting additive models (ignoring interactions) to the loss statistics from the $(40 \times 5)$-run, crossed-array experiment and optimizing the level of each transistor width separately produces even worse predictions. For example, the predicted best squared-error loss is $-2.39$, but the actual loss computed from a confirmation experiment is $-0.58$.

- Strategy I gives the same best-three circuit designs for the two loss statistics. There is some indication that even better performance might be obtained by another experiment changing the ranges of the last four transistors widths. This is borne out by the results of minimizing the predicted loss statistics over the continuous region $[-1, 1]^6$ rather than the discrete region

$\{-1, 0, 1\}^6$. Such optimization of the predictor of $L_{\text{sq}}$ from strategy I leads to $w = \{-.07, .10, -1, 1, 1, 1\}$, so that the last four optimal widths are on the boundary. The implication is that going beyond the boundary could lead to further improvement.

## Discussion

One reason why our proposed method gives reliable predictions with few observations here is that the skews admit simple models. Moreover, engineering understanding of the underlying response facilitates model identification. Another, more general, advantage of modeling the underlying response rather than a loss statistic is that collapsing the data to a loss statistic could hide important relationships in the data.

Clearly, more empirical experience is required to determine the general usefulness of the proposed strategy. In another application, a sense-amplifier circuit, the proposed method with 48 observations gives more accurate predictions of the losses than modeling the loss statistics with a crossed-array experiment of 140 runs. Again, about two thirds of the observations are saved. In this example there is little difference in the actual performances of the optimized circuit designs.

We used a computer-aided statistical design package to automatically generate the experimental plans. This kind of tool avoids many of the complications often experienced when using catalogued experimental designs. For example, the user can concentrate on the model without worrying about matching the desired interactions to the aliasing structure. We believe that the widespread adoption of these tools and increased attention to modeling rather than combinatorics would encourage the experimentation needed to improve quality. Similarly, model fitting and the minimization of predictions over a grid are straightforward using, for example, SAS.

These methods can be extended to physical experiments with random error. In such experiments noise variability is due to unmodeled sources (measurement error, omitted variables, etc.) as well as the noise variables being manipulated. If the unmodeled sources are unimportant or lead to a noise component with constant variance there is little technical difficulty in extending our methods. Nonconstant variance requires further study, however.

Returning to computer experiments with no random error, Sacks, Schiller and Welch (1989) discussed several examples and treated the systematic departure from a linear model as a realization of a stochastic process. This approach replaces least squares prediction by flexible functions that interpolate the data points. Applying these techniques to parameter-design problems is the subject of ongoing research and will be reported elsewhere.

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Key Words: *Computer Experiment, Off-Line Quality Control, Parameter Design, Response Surface, Taguchi Method.*

TABLE 1. Experimental Design and Data for Modeling Clock Skews (Two Skews Are Observed for Each Run)

| Run | Transistor Widths $w$ | | | | | | Noise Level | Skews | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | −1 | −1 | −1 | −1 | −1 | −1 | 3 | −1.289 | −0.307 |
| 2 | −1 | −1 | −1 | −1 | −1 | 0 | 4 | −0.636 | −1.199 |
| 3 | −1 | −1 | −1 | −1 | 1 | −1 | 1 | −1.219 | 0.907 |
| 4 | −1 | −1 | −1 | 1 | 1 | −1 | 2 | −1.151 | 1.678 |
| 5 | −1 | −1 | −1 | 1 | 1 | 1 | 3 | −0.449 | −0.422 |
| 6 | −1 | −1 | −1 | 1 | 1 | 1 | 5 | −0.510 | −0.343 |
| 7 | −1 | −1 | 0 | −1 | −1 | −1 | 5 | −2.758 | 0.157 |
| 8 | −1 | −1 | 1 | −1 | −1 | 1 | 2 | −2.414 | −1.309 |
| 9 | −1 | −1 | 1 | 0 | −1 | 1 | 5 | −1.920 | −1.633 |
| 10 | −1 | −1 | 1 | 1 | −1 | 1 | 4 | −0.809 | −1.546 |
| 11 | −1 | −1 | 1 | 1 | 0 | 0 | 1 | −1.227 | −0.496 |
| 12 | −1 | 0 | −1 | −1 | 0 | 1 | 2 | −1.412 | 0.041 |
| 13 | −1 | 0 | −1 | 1 | 0 | 1 | 4 | −0.452 | −0.628 |
| 14 | −1 | 0 | 0 | 0 | −1 | −1 | 1 | −1.127 | 0.062 |
| 15 | −1 | 0 | 1 | −1 | 1 | 0 | 5 | −3.860 | 2.011 |
| 16 | −1 | 0 | 1 | 0 | 0 | −1 | 4 | −2.107 | 0.863 |
| 17 | −1 | 1 | −1 | −1 | −1 | −1 | 2 | −2.300 | 1.350 |
| 18 | −1 | 1 | −1 | −1 | −1 | 1 | 3 | −1.118 | −0.466 |
| 19 | −1 | 1 | −1 | 0 | −1 | 0 | 5 | −1.495 | 0.070 |
| 20 | −1 | 1 | −1 | 1 | 0 | 1 | 1 | −0.512 | −0.236 |
| 21 | −1 | 1 | −1 | 1 | 1 | −1 | 4 | −1.184 | 1.592 |
| 22 | −1 | 1 | 1 | −1 | −1 | −1 | 1 | −2.126 | 0.479 |
| 23 | −1 | 1 | 1 | −1 | 1 | 1 | 4 | −2.504 | 0.931 |
| 24 | −1 | 1 | 1 | 0 | 1 | −1 | 3 | −2.769 | 2.567 |
| 25 | −1 | 1 | 1 | 1 | 1 | −1 | 5 | −3.315 | 3.759 |
| 26 | −1 | 1 | 1 | 1 | 1 | 1 | 2 | −1.982 | 1.149 |
| 27 | 0 | −1 | −1 | −1 | 0 | −1 | 5 | −1.927 | 0.365 |
| 28 | 0 | −1 | −1 | 0 | 1 | 1 | 4 | −0.452 | −0.922 |
| 29 | 0 | −1 | 0 | 1 | −1 | 0 | 5 | −0.855 | −2.175 |
| 30 | 0 | −1 | 1 | −1 | 0 | 0 | 4 | −1.768 | −0.748 |

TABLE 1. Continued

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 31 | 0 | −1 | 1 | 1 | −1 | 1 | 3 | −0.715 | −2.177 |
| 32 | 0 | 0 | −1 | 0 | 1 | 1 | 1 | −0.510 | −0.283 |
| 33 | 0 | 0 | 0 | 0 | −1 | 0 | 2 | −1.401 | −0.715 |
| 34 | 0 | 0 | 1 | 1 | 0 | 1 | 5 | −1.576 | −0.639 |
| 35 | 0 | 1 | −1 | −1 | 1 | −1 | 3 | −1.841 | 2.004 |
| 36 | 0 | 1 | 0 | 0 | 0 | 1 | 5 | −1.704 | 0.004 |
| 37 | 0 | 1 | 1 | 1 | 1 | 0 | 4 | −1.491 | 0.724 |
| 38 | 1 | −1 | −1 | −1 | 1 | 1 | 2 | −0.852 | −1.439 |
| 39 | 1 | −1 | −1 | 0 | 0 | 1 | 3 | −0.266 | −2.196 |
| 40 | 1 | −1 | −1 | 1 | −1 | −1 | 1 | −0.067 | −1.691 |
| 41 | 1 | −1 | 0 | −1 | 1 | 0 | 1 | −1.156 | −0.583 |
| 42 | 1 | −1 | 0 | 1 | 1 | −1 | 3 | −1.096 | 0.053 |
| 43 | 1 | −1 | 0 | 1 | 1 | 0 | 4 | −0.649 | −1.010 |
| 44 | 1 | −1 | 1 | −1 | −1 | 1 | 1 | −0.958 | −1.978 |
| 45 | 1 | −1 | 1 | 0 | 1 | 0 | 5 | −2.049 | −0.687 |
| 46 | 1 | −1 | 1 | 1 | −1 | −1 | 2 | −1.545 | −1.504 |
| 47 | 1 | 0 | −1 | −1 | −1 | 1 | 5 | −0.928 | −2.081 |
| 48 | 1 | 0 | 0 | −1 | 1 | 1 | 4 | −1.197 | −0.423 |
| 49 | 1 | 0 | 1 | −1 | 1 | 1 | 3 | −2.007 | −0.161 |
| 50 | 1 | 1 | −1 | −1 | 0 | 0 | 1 | −0.889 | −0.391 |
| 51 | 1 | 1 | −1 | 1 | −1 | −1 | 3 | −0.659 | −0.482 |
| 52 | 1 | 1 | −1 | 1 | −1 | −1 | 5 | −1.166 | −0.121 |
| 53 | 1 | 1 | −1 | 1 | −1 | 1 | 2 | −0.285 | −1.440 |
| 54 | 1 | 1 | −1 | 1 | −1 | 1 | 4 | 0.229 | −1.841 |
| 55 | 1 | 1 | 0 | −1 | 1 | 1 | 5 | −2.269 | 0.539 |
| 56 | 1 | 1 | 0 | 1 | −1 | 1 | 1 | −0.236 | −1.312 |
| 57 | 1 | 1 | 1 | −1 | −1 | −1 | 4 | −1.642 | −0.343 |
| 58 | 1 | 1 | 1 | −1 | −1 | 0 | 5 | −3.055 | −0.533 |
| 59 | 1 | 1 | 1 | −1 | 1 | −1 | 2 | −3.522 | 2.585 |
| 60 | 1 | 1 | 1 | 1 | 0 | 0 | 3 | −1.440 | −0.175 |

TABLE 2. Part of the Crossed-Array Experimental Design and Data for Modeling Loss Statistics Directly (Two Skews Are Observed for Each Run)

| Runs | Transistor Widths $w$ | | | | | | Skew at Noise Level 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1–5 | −1 | −1 | −1 | −1 | 0 | 1 | −0.73 | −1.12 | −1.01 | −0.83 | −1.30 |
| | | | | | | | −0.76 | −0.78 | −0.76 | −0.86 | −0.78 |
| 6–10 | −1 | −1 | −1 | 1 | 1 | −1 | −0.74 | −1.15 | −1.01 | −0.88 | −1.43 |
| | | | | | | | 0.60 | 1.68 | 1.16 | 0.72 | 1.97 |
| 11–15 | −1 | −1 | 0 | 0 | −1 | −1 | −0.98 | −1.85 | −1.44 | −1.03 | −2.06 |
| | | | | | | | −0.46 | 0.01 | −0.34 | −0.57 | −0.13 |
| 16–20 | −1 | −1 | 1 | −1 | 0 | −1 | −2.11 | −3.45 | −2.85 | −2.39 | −3.91 |
| | | | | | | | 0.39 | 1.22 | 0.79 | 0.47 | 1.43 |
| 21–25 | −1 | −1 | 1 | 1 | 1 | 1 | −1.15 | −1.44 | −1.46 | −1.39 | −1.79 |
| | | | | | | | −0.29 | −0.29 | −0.19 | −0.22 | −0.09 |
| ⋮ | | | | | | | | | | | |

TABLE 3. Predicted Best Three Circuit Designs for the Squared-Error Loss Statistic

| | | | | | | | | $L_{sq}$ | |
|---|---|---|---|---|---|---|---|---|---|
| Experiment | Modeling | Best $w$ on $3^6$ Cube | | | | | | Predicted | Actual |
| 60 runs | skews via (4) | 0 | 0 | −1 | 1 | 1 | 1 | −.90 | −.90 |
| | | 1 | 1 | −1 | 1 | 1 | 1 | −.76 | −.77 |
| | | −1 | −1 | −1 | 1 | 1 | 1 | −.68 | −.72 |
| 40 × 5 runs | $L_{sq}$ via (5) | −1 | 1 | −1 | 1 | 0 | 1 | −.38 | −.50 |
| | | −1 | 0 | −1 | 1 | 0 | 1 | −.31 | −.61 |
| | | −1 | 1 | −1 | 1 | −1 | 1 | −.30 | −.41 |
| 40 × 5 runs | skews via (4) | 0 | 0 | −1 | 1 | 1 | 1 | −.89 | −.90 |
| | | 1 | 1 | −1 | 1 | 1 | 1 | −.76 | −.77 |
| | | −1 | −1 | −1 | 1 | 1 | 1 | −.69 | −.72 |

TABLE 4. Predicted Best Three Circuit Designs for the Worst-Case Loss Statistic

| | | | | | | | | $L_{wc}$ | |
|---|---|---|---|---|---|---|---|---|---|
| Experiment | Modeling | Best $w$ on $3^6$ Cube | | | | | | Predicted | Actual |
| 60 runs | skews via (4) | 0 | 0 | −1 | 1 | 1 | 1 | .50 | .53 |
| | | −1 | −1 | −1 | 1 | 1 | 1 | .56 | .51 |
| | | 1 | 1 | −1 | 1 | 1 | 1 | .63 | .66 |
| $40 \times 5$ runs | $L_{wc}$ via (5) | −1 | 0 | −1 | 0 | 0 | 1 | .93 | 1.07 |
| | | −1 | 0 | −1 | 0 | 1 | 1 | 1.05 | 1.17 |
| | | −1 | 1 | −1 | 0 | 0 | 1 | 1.06 | 1.36 |
| $40 \times 5$ runs | skews via (4) | 0 | 0 | −1 | 1 | 1 | 1 | .50 | .53 |
| | | −1 | −1 | −1 | 1 | 1 | 1 | .52 | .51 |
| | | 1 | 1 | −1 | 1 | 1 | 1 | .63 | .66 |

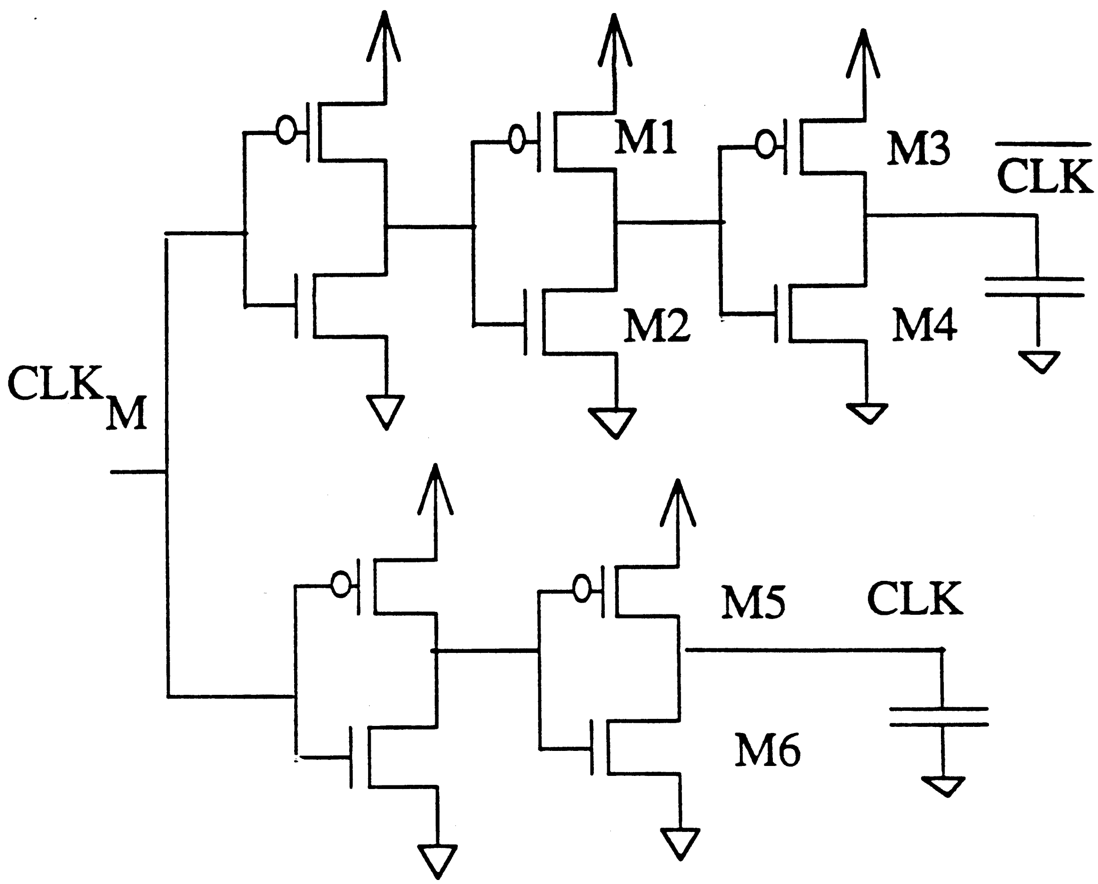


**Figure 1. CMOS Clock-Driver Circuit**

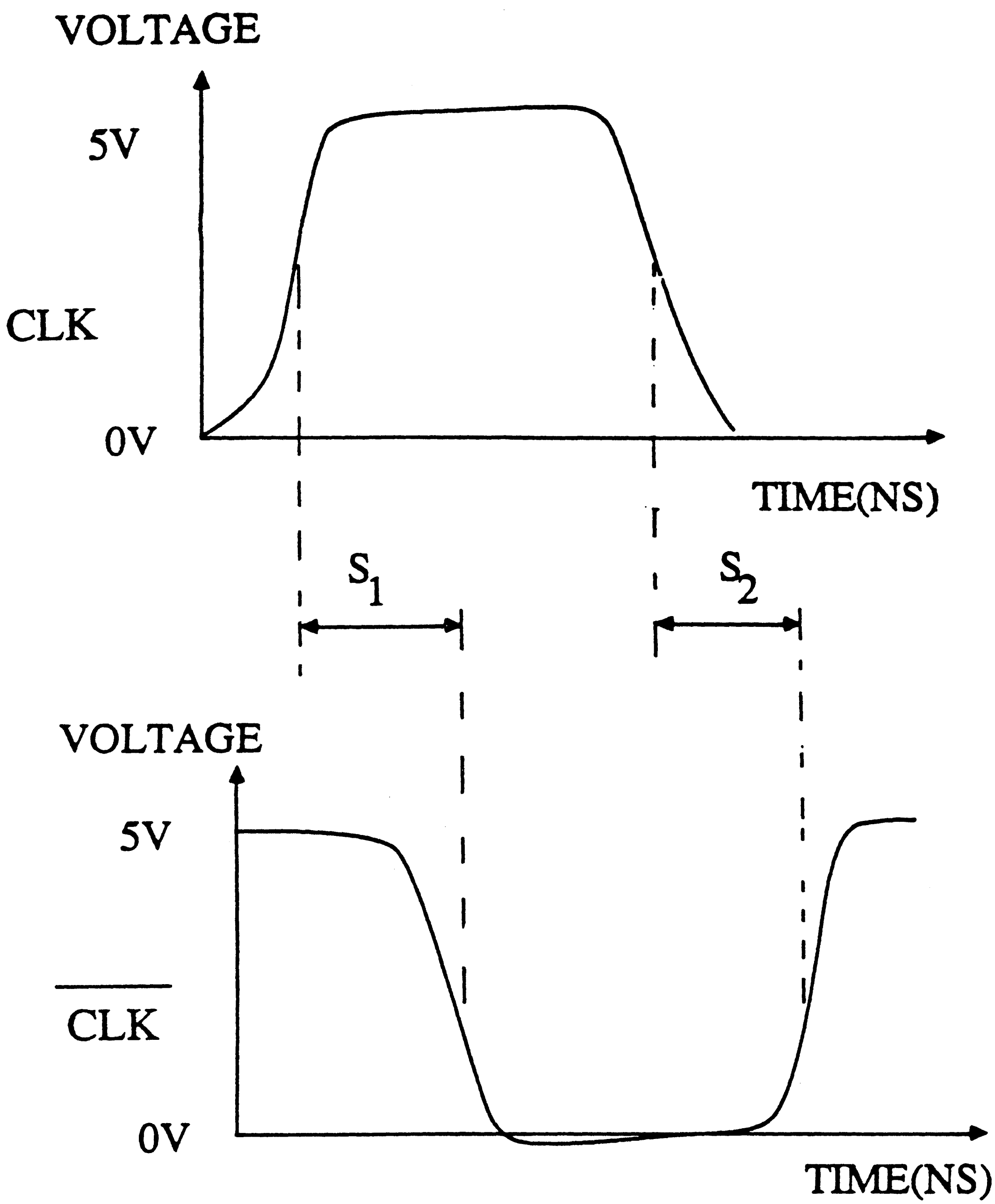


**Figure 2. Clock Signals and Skews**