DETECTING SPATIAL EFFECTS FROM FACTORIAL EXPERIMENTS: AN APPLICATION IN INTEGRATED-CIRCUIT MANUFACTURING

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DETECTING SPATIAL EFFECTS FROM FACTORIAL EXPERIMENTS: AN APPLICATION IN INTEGRATED-CIRCUIT MANUFACTURING

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ABSTRACT

We propose a method to detect spatial effects in integrated-circuit fabrication processes through factorial experimentation. The method assumes that a large number of chips is available on each silicon wafer and that at least one wafer is produced for each experimental setting. In this article, we consider the binary (functional/non-functional chip) response case and identify spatially influential factors that affect the clustering of functional chips on a wafer by using a suitable measure of clustering.

1 Introduction

The manufacturing process of integrated circuits (IC) or very large scale integrated (VLSI) circuits consists of multiple stages. At each stage there can be a number of factors (parameters) that affect the IC or VLSI chip quality. Among various measures of quality, the IC manufacturing industry has concentrated on process yield. For simultaneously and efficiently studying the effect of various factors on quality and yield, design of experiments is an indispensable tool. Phadke, Kackar, Speeney and Grieco (1983) performed designed experiments to study factors that influence specific chip characteristics, but did not consider the spatial relationships of the chips on a wafer. On the other hand, Mallory, Perloff, Hasan and Stanley (1983) and other similar papers studied spatial patterns on wafers with graphical tools but paid little attention to the combined effect that several factors might have on such patterns. In this article, we examine the use of factorial experiments to identify both spatially and yield influential factors for the binary response case, i.e., a chip is functional or non-functional, good or bad.

Although modeling yield from existing IC manufacturing processes is a passive application of statistics, its role in monitoring such processes is important. A historical review of yield modeling can be found in Stapper (1989b) and over 120 cited references there. Most of these models do not address the dependence between any two chips or any two neighboring chips, however. Actively taking steps to identify spatially influential factors and process variables leads to better understanding of the fabrication process and to potential quality improvement.

While studying yield alone can lead to improved production conditions, considering spatial clustering can uncover process settings which produce more clustering of functional or nonfunctional chips. Although various communications with IC manufacturing engineers indicate that yield remains the primary concern, they all agreed that spatial clustering is important in distinguishing between randomly and nonrandomly scattered defects. Electrostatic interference, dust particle accumulation during transportation, dirt build up from tool wear and nonuniform crystalline properties of substrates are some examples of causes of nonrandom clustering of nonfunctional chips. Through well planned factorial experiments, we seek to identify and avoid some of these potential causes of defective clustering.

In order to identify spatial effects, a suitable measure of clustering is required. See Filliben, Kafadar and Shier (1983) for a discussion of various definitions of spatial nonrandomness or dependence. In Section 2, we examine one such measure of spatial clustering and demonstrate its usefulness.

In Section 3, we propose a method to detect spatially influential factors by designed experiments. Throughout, it is assumed that each silicon wafer contains a large number of chips. Numerous examples of wafers with many chips appear in the IC manufacturing literature. Mallory et al. (1983) conducted a study involving a complete audit of 402 chips on each of 35 wafers. Perloff, Hasan and Blome (1980) examined all 118 chips on each wafer in their study. Also, Ferris-Prabhu, Smith, Bonges and Paulsen (1987) collected data from more than 1000 wafers; each wafer contained 137 chips. The large number of chips on a wafer allow large sample properties of spatial statistics to be used so that standard analyses of factorial experiments can be applied. A simulation study is presented which provides empirical validation of the asymptotic normality assumption.

In Section 4, we analyze data from a real experiment which has been generously provided by the Intel Corporation. A detailed description of the experiment and our analysis using the proposed method is given. Our analysis reveals a spatially influential effect for this particular process.

Section 5 concludes with a number of remarks. Many studies on spatial pattern using graphical tools can be found in the engineering literature, e.g., Markert and Current (1983), Mallory, Perloff, Hasan and Stanley (1983) and Perloff, Wahl and Reimer (1977). A study of radial variation of yield is reported by Ferris-Prabhu, Smith, Bonges and Paulsen (1987). We view the proposed method not as a replacement for these graphical tools but rather as a supplement for enhancing the understanding of the IC manufacturing process.

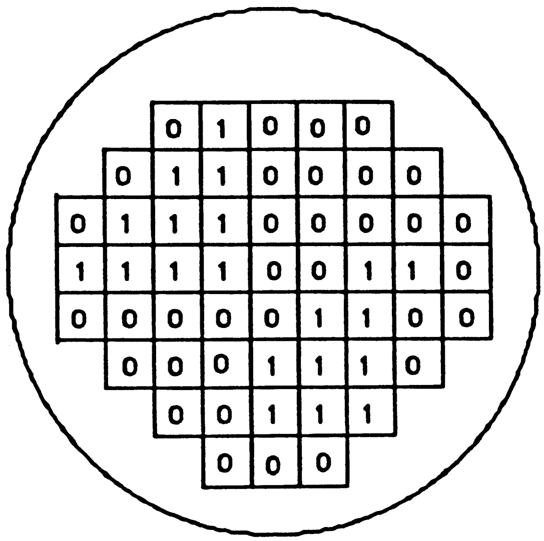

Figure 1: A Wafer Configuration with Functional Chips

2 A Measure of Spatial Clustering

We introduce a spatial measure of clustering by considering a simplified example. Suppose that a complete audit of each chip on a wafer is obtained, where each chip is recorded as being functional or nonfunctional according to a specific characteristic. For example, the characteristic may be a properly formed logical gate or resistance below a certain range. One configuration of such a wafer is shown in Figure 1. Each "1" represents a functional or good chip. Although the wafer in this figure has only 54 chips, the number of chips on a wafer can be in the hundreds as indicated by the examples given in the Introduction.

Since the response variable is binary, the dependence among chips can be measured by a join-count statistic. A join is formed when two chips lie in a neighborhood of one another. When functional chips cluster together on a wafer, the number of joins connecting functional chips is larger than when the functional chips are scattered all over.

In this article, we define the neighborhood structure to be the king move neighborhood

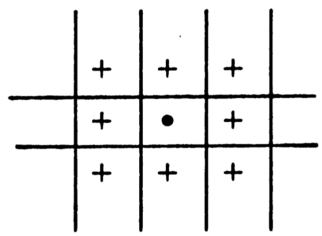

Figure 2: King Move Neighborhood

from chess, where the neighbors of "•" are denoted by "+" in Figure 2. We chose this neighborhood structure because it treats the diagonally adjacent chips as neighbors, which is essential in capturing the circular boundary of a wafer. There are other choices of neighborhood structure but this one is adequate for illustrating our methodology.

Assume that each wafer under study has the same number of chips, say n. Then, three measures of spatial correlation (clustering) for binary responses are given by:

J(GG) = number of joins for king move neighbors that connect two functional (good) chips

J(GB) = number of joins for king move neighbors that connect a functional (good) and a non-functional (bad) chip

J(BB) = number of joins for king move neighbors that connect two non-functional (bad) chips

Letting $X_i = 1$ for a "good" chip at site i ($X_i = 0$ otherwise), and $\delta_{ij} = 1$ if X_i and X_j

Table 1: Join-Count Statistics for Figure 3 Patterns

Pattern	1	2	3	4	5	6	7	8	9	10	11	12
J(GG)	6	4	0	3	28	11	1	22	53	20	15	42
J(GB)	26	15	42	22	40	33	74	42	41	52	95	63
J(BB)	145	158	135	152	109	133	102	113	83	105	67	72

are king move neighbors ($\delta_{ij} = 0$ otherwise), then

$$J(GG) = \sum_i \sum_j \delta_{ij} X_i X_j$$

$$J(GB) = \sum_i \sum_j \delta_{ij} (1 - X_i) X_j$$

$$J(BB) = \sum_i \sum_j \delta_{ij} (1 - X_i)(1 - X_j).$$

Also, let $y = \sum_i X_i$ denote the number of functional chips on a wafer and define raw yield to be the proportion of functional chips on a wafer, i.e., y/n. Note that this definition of yield is different from that given in the engineering literature (Stapper (1989a)) which models the number of defects on each chip by a compound Poisson distribution.

These join-count statistics (Cliff and Ord (1981)) were introduced to test for the existence of spatial correlation among binary responses in a region. Their moments under two sampling schemes and their large sample properties were given in Moran (1948) and Krishna-Iyer (1949, 1950). Furthermore, this statistic is a weighted U-statistic whose large sample results can be found in Shapiro and Hubert (1979).

To illustrate some possible configurations of functional and non-functional chips, consider several simple examples. Suppose that there are $n = 54$ chips on a wafer and for some experimental runs there were wafers with 5, 12, and 20 "good" chips, labeled by "1" in Figure 3. Patterns 1 to 4 have 5 "good" chips, patterns 5 to 8 have 12 and patterns 9 to 12 have 20.

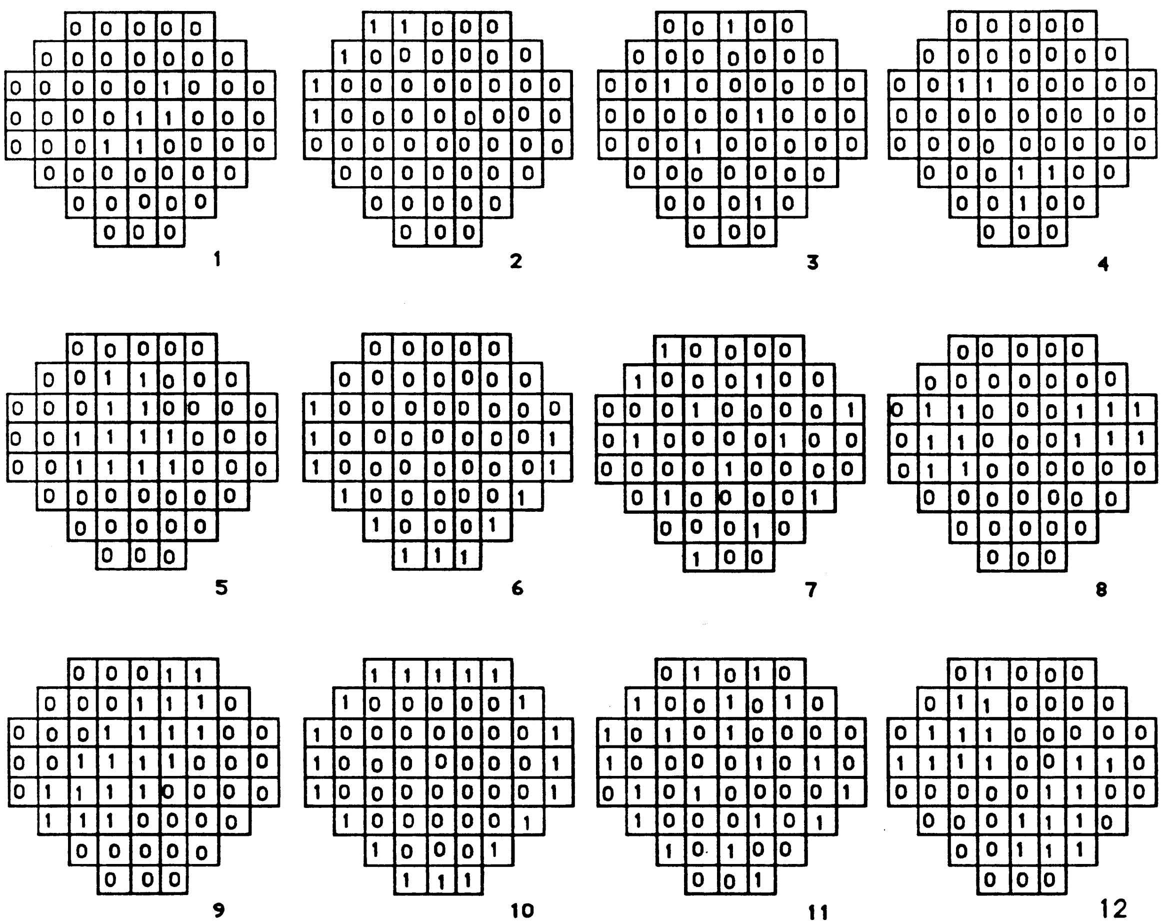

Figure 3: 54 Chip Wafers Configurations with 5, 12, 20 Good Chips

Table 1 summarizes the join-count statistics for these patterns and shows how the clustering measures reflect the spatial patterns seen in these wafers. Note that join-count statistics for patterns 1 to 4, 5 to 8 and 9 to 12 vary according to how much the "good" chips are clustered. For example, given the same number of "good" chips, J(GG) for pattern 1 is larger than that for pattern 2 which in turn is larger than that for pattern 3. A similar ordering is also seen for patterns 5, 6, 7 and patterns 9, 10, 11. Patterns 4, 8 and 12 contain two separate clusters, but have J(GG) counts close to patterns 1, 5 and 9, respectively. Finally, the large values of J(GB) for patterns 3, 7 and 11 indicate that J(GB) measures how much the "good" and "bad" chips are mixed. For these patterns the "good" and "bad" chips are mixed too much, the spatial analog of negative autocorrelation in a time series.

Two other measures of spatial clustering can be found in Appendix II of Bartlett (1967) and in Stapper (1986), respectively. The first uses a conditional autoregressive model and requires a good moment estimator for a clustering parameter α. The second measure requires successive partitionings of a wafer by grids of various sizes and uses nonlinear regression to estimate the clustering parameter α. The first measure relies on the accuracy of the moment estimator and the assumption of the conditional autoregressive model. The second is computationally intensive and requires a scheme to partition the wafer. On the other hand, the join-count statistic is relatively easy to compute. Moreover, its large sample properties allow a simple method of analysis.

3 The Proposed Method

Suppose an engineer wants to investigate the spatial effects of three factors (A, B, C) for an IC fabrication process. For example, these factors might be the viscosity of a special chemical, spindle speed and bake temperature, respectively. After choosing two levels for each factor, he could study these factors by performing a full factorial experiment in eight runs. The experimental plan is given in Table 2, where + represents the high level setting of the factor and − represents the low level. At least one silicon wafer per run is assumed

Table 2: Full Factorial Design for Three Factors

Experimental Run (Wafer)	Factors A	B	C
1	-	-	-
2	+	-	-
3	-	+	-
4	+	+	-
5	-	-	+
6	+	-	+
7	-	+	+
8	+	+	+

to be made under the conditions specified by the run. For simplicity, we consider the case when only one wafer per run is made. Note that if the engineer wants to study more than three factors, then a fractional factorial experiment can be performed.

The join-count statistics are then calculated for each wafer. For wafers that contain clusters of functional chips, the J(GG) statistic will be larger than that expected if the functional chips had been randomly scattered. Similarly, if the functional and nonfunctional chips are mixed too much, then the J(GG) statistic will be smaller than expected and the J(GB) statistic will be larger than expected.

Note that clustering of functional chips is more likely to occur when there is a large number of functional chips on a fixed size wafer. Consequently, the yield's influence on the clustering measures needs to be accounted for so that real spatial effects can be detected. The approach that we take is to standardized the join-count statistics by their conditional means and variances (i.e., given y functional chips) under spatial independence. The means

Table 3: Conditional Means and Variances of Join-Count Statistics Under Spatial Independence

Join-Count	Mean	Variance
J(GG)	$\mu(GG) = T\frac{(y)(y-1)}{(n)(n-1)}$	$T_1\frac{(y)(y-1)}{4(n)(n-1)} + (T_2 - 2T_1)\frac{y(y-1)(y-2)}{4n(n-1)(n-2)}$ $+(4T^2 + T_1 - T_2)\frac{y(y-1)(y-2)(y-3)}{4n(n-1)(n-2)(n-3)} - \mu(GG)^2$
J(GB)	$\mu(GB) = 2T\frac{y(n-y)}{n(n-1)}$	$T_1\frac{y(n-y)}{2n(n-1)} + (T_2 - 2T_1)\frac{y(n-y)(n-2)}{4n(n-1)(n-2)}$ $+(4T^2 + T_1 - T_2)\frac{y(y-1)(n-y)(n-y-1)}{n(n-1)(n-2)(n-3)} - \mu(GB)^2$

and variances under spatial independence are given in Moran (1948) and Krishna-Iyer (1949, 1950) and presented in Table 3. The mean and variance of J(BB) are obtained by replacing y by $n - y$ in the mean and variance of J(GG).

Note that T = total number of joins = $(1/2)\sum_i \sum_j \delta_{ij}$, where δ_{ij} was defined in the preceding section. Also, $\delta_{ij} = \delta_{ji}$ and $T_1 = (1/2)\sum_i \sum_j(\delta_{ij}+\delta_{ji})^2 = 4T$ and $T_2 = \sum_i(\delta_{i.}+\delta_{.i})^2$ $= 4\sum_i \delta_{i.}^2$, where $\delta_{i.} = \sum_j \delta_{ij}$ = number of neighbors for the ith element. Because boundary elements have a different number of neighbors than the interior elements, $\delta_{i.}$ is not constant for all elements and consequently T_2 is influenced by the configuration of the wafer. For example, in a n_1 by n_2 rectangular lattice, $\delta_{i.} = 3, 5, 8$ for corner, edge and interior elements of the lattice, respectively and $T_2 = 4[3^2 * 4 + 5^2 * (2 * (n_1 - 2 + n_2 - 2)) + 8^2 * (n_1 - 2) * (n_2 - 2)]$.

The exact joint distributions of these join-count statistics and yield are generally quite complicated and shape dependent. Evaluation of the conditional probabilities associated with all possible $J(\ )$ values for a given y requires computing all possible arrangements of "1" on a wafer whose shape could be rectangular or more unevenly shaped. Krishna-Iyer (1950) gave some exact distributions for several small rectangular lattices. Nevertheless, the moments of these random variables are available and large sample distributions have been developed.

When y/n is not too close to zero or one, the asymptotic distribution of each join-count statistic is normal. If y/n converges to a constant far from zero or one, then the fact that

$Var(J(GG)) = O(n)$ and a version of the sufficient condition given in Noether (1970) can be used to show that $J(GG)$ approaches normality. Alternatively, the results on join-count statistics in Shapiro and Hubert (1979) can be used. Therefore, the conditional standardized join-count statistics behave like standard normal variables and are free of the influence of y. These conditional standardized join-count statistics then become *approximate* measures of spatial clustering. That is, $J(GG)|y \approx N(\mu(y), \sigma^2(y))$ and given y, $[J(GG)-\mu(y)]/\sigma(y) \approx N(0,1)$.

For y/n close to zero, the asymptotic distribution for J(GG) is Poisson. Barton and David (1966) proved this result in the context of random intersections of graphs where they were concerned with the adjacency of time rather than that of physical location. Because a Poisson distribution's mean is equal to its variance, one can conditionally standardize the join-count statistics by $[J(GG) - \mu(y)]/\sqrt{\mu(y)}$. Similarly, for y/n near one, the J(BB) statistic behaves like a Poisson random variable since $(n-y)/n$ is near zero. J(BB) can then be standardized as above.

Next we address the validity of the normal approximation for the small sample case. The exact conditional distribution of the join-count statistics given $y = 0, 1, 2, \ldots n$ requires the enumeration of all n choose y patterns for each y which can be computationally prohibitive even for moderate n. Approximate conditional distributions can be obtained by taking a sample of the n choose y patterns, however. By this method, we obtained approximate conditional distributions of J(GG) for a 10 by 10 rectangular grid of spatially independent binary random variables for each $y = 0, 1, 2, \ldots 100$. An examination of the conditional distributions for normality showed that normality appears to hold for $10 \leq y \leq 90$. Moreover, the conditional means and standard deviations were comparable to those computed from the equations in Table 3. This suggests the following empirical rule of thumb for using the standardized join-count statistics based on the conditional moments from Table 3; use them for $0.1 \leq y/n \leq 0.9$.

Recall the twelve patterns given in Figure 3. Tables 4 and 5 present the conditional

Table 4: Conditional Means and Variances of Join-Count Statistics for Various Yields Under Spatial Independence

Yield y/n	37%		22%		9%	
Join-Count	Mean	Variance	Mean	Variance	Mean	Variance
J(BB)	29.3899	21.3274	106.4969	19.1652	145.4591	10.6709
J(GB)	84.1090	36.5719	62.3396	26.0884	30.3040	11.9537
J(GG)	23.5010	12.7983	8.1635	5.6190	1.2369	1.0129

means and variances of the join-count statistics for these yields under spatial independence and summarize the conditional standardized join-count statistics for these patterns. The conditional means and variances were computed from the expressions given in Table 3.

Next we compare the standardized spatial measures for these patterns. Patterns 1, 4, 5, 8, 9 and 12, which have all functional chips clustered either in the middle or in two group, are those with the largest standardized J(GG) for each given yield. Thus, the standardized J(GG), which quantifies the amount of clustering, is useful for distinguishing these patterns from the others. Note that while pattern 12 has more *GG* joins, the standardized J(GG) for pattern 8 is slightly larger than that for pattern 12; although these two patterns appear indistinguishable, the standardized $J(GG)$ reveal that the "good" chips are slightly more clustered in pattern 8.

The standardized J(BB) identify those patterns with the most clustering of "bad" chips. For example, patterns 2, 6 and 10 have the largest standardized J(BB) and are those with "good" chips on the boundary and "bad" ones in the center.

The standardized J(GB) identify those patterns with the most mixing of "good" and "bad" chips. Patterns 3, 7 and 11 with negative standardized J(GG) and J(BB) all have positive standardized J(GB). That is, functional chips are likely to have nonfunctional chips as their neighbors, the extreme opposite of clustering.

Since J(GG) is a useful measure for clustering and is approximately normal, we propose

Table 5: Conditional Standardized Join-Count Statistics for Figure 3 Patterns

Pattern	J(BB)	Std. J(BB)	J(GB)	Std. J(GB)	J(GG)	Std. J(GG)
1	145	-0.1405	26	-1.2449	6	4.7326
2	158	3.8391	15	-4.4264	4	2.7454
3	135	-3.2018	42	3.3829	0	-1.2290
4	152	2.0023	22	-2.4018	3	1.7518
5	109	0.5718	40	-4.3737	28	8.3683
6	133	6.0540	33	-5.7442	11	1.1966
7	102	-1.0272	74	2.2829	1	-3.0220
8	113	1.4855	42	-3.9822	22	5.8371
9	83	2.9471	41	-7.1284	53	8.2458
10	105	7.7109	52	-5.3095	20	-0.9786
11	67	-0.5175	95	1.8009	15	-2.3763
12	72	0.5652	63	-3.4906	42	5.1710

using the conditional standardized J(GG) as the response in the factorial experiment described at the beginning of this section. These responses can then be analyzed by standard methods for factorial designs. Since the yields are usually different between experimental runs (wafers), the conditional standardized measures account for yield differences and allow significant spatial factors to be detected which have no effect on yield. Thus, knowing such factors allows engineers to find conditions which improve the clustering of functional chips on a wafer without sacrificing yield.

There are several scenarios that may be encountered in analyzing cluster measures in these experiments.

- When the yields are similar across all runs and none is near 0% or 100%, then spatially influential factors can be identified by analyzing the conditional standardized join-count statistics as the response variable. The effects can first be calculated using Yates' algorithm and then plotted on normal probability paper to identify the spatially significant effects. A positive significant effect implies that functional chips cluster more at the high level of the factor; a negative significant effect means that the good chips cluster more at the low level of the factor.

- When the yields vary substantially across the experimental runs and none is near 0% or 100%, both the yield and standardized join-count statistics should be analyzed to determine significant yield and spatial clustering factors. If some factors affect yield but not clustering, while other factors affect clustering but not yield, then settings can be recommended that achieve a high yield and a large degree of clustering. Situations where factors affect yield and clustering in opposite directions may very well occur. In this case, process engineering knowledge and process costs play important roles in recommending levels for these factors.

- If some of the experimental runs (wafers) have yields near 0%, the standard analysis should be avoided since it would be combining Poisson and normal responses. In

Table 6: Number of Wafers Per Lot for the Real Experiment

Lot	1	2	3	4	5	6	7	8	9	10
# Wafers	21	19	23	24	21	22	21	22	21	24

this case, those runs with larger yields could still be analyzed for spatial effects using regression. Also, an investigation of the poor yield runs is warranted to determine the causes for such low yields.

A confirmatory experiment to verify the spatial effects of these factors is also recommended. Since J(BB) can be found by subtracting J(GB) and J(GG) from the total possible joins T, the standardized J(BB) statistics can also be analyzed. Note that once significant spatial effects are identified, graphical techniques are needed to reveal the nature of the clustering patterns.

4 A Real Experiment

Recently, an experiment designed to study the effect of three manufacturing process parameters on wafer yield was made available to us by Walt Flom of the Intel Corporation. Each parameter (factor) had two desired settings, the standard setting (low level) and a new setting (high level). Neither the description of the factors nor the levels used were released because they are proprietary information. Ten lots of 24 wafers were produced for this experiment. For each lot, the three factor full factorial design, the set of eight factorial combinations previously given in Table 2, was replicated three times. Because of various unforeseen reasons, all 24 wafers were not available in every lot, although there was at least one wafer for each of the eight combinations. See Table 6 for the actual number of wafers in each lot. Thus, the experimental plan was a factorial design with blocking (lot) and unbalanced replication.

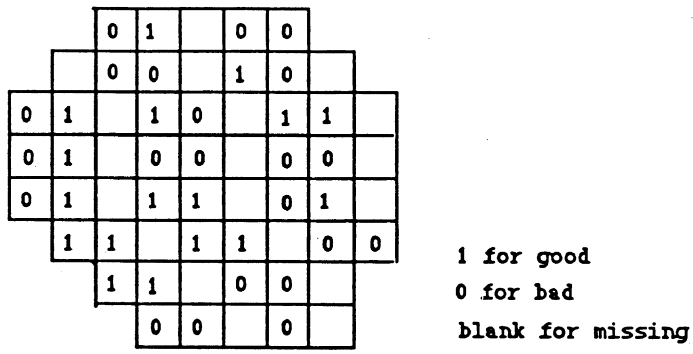

Figure 4: One Wafer from Real Experiment Showing Locations of Unreleased Data

The wafers produced in the experiment were test vehicles, i.e., they were not production wafers. In such situations, it is not uncommon to have several types of devices (circuits) on each wafer. For reasons of confidentiality, we were given the results for only one device. Figure 4 displays the results for one wafer, where the blank cells correspond to the unreleased information. Note that each wafer had the same pattern of unavailable data.

Because the device locations are not contiguous, the method proposed in the previous section cannot be used directly. However, the idea of analyzing conditional standardized spatial statistics applies. While this pattern of missing chips does not affect the calculation of the spatial statistics, it does affect their distribution under spatial independence. Nevertheless, the approximate conditional distribution of these join-count statistics can easily be simulated for each $y = 0, 1, \ldots, 39$. These distributions appear to be normal for $5 \leq y \leq 34$. Since 216 out of the 218 wafers had $5 \leq y \leq 34$, we standardized the join-count statistics by the empirical conditional means and standard deviations obtained from the simulation. Somewhat surprisingly, the conditional means and standard deviations are comparable to

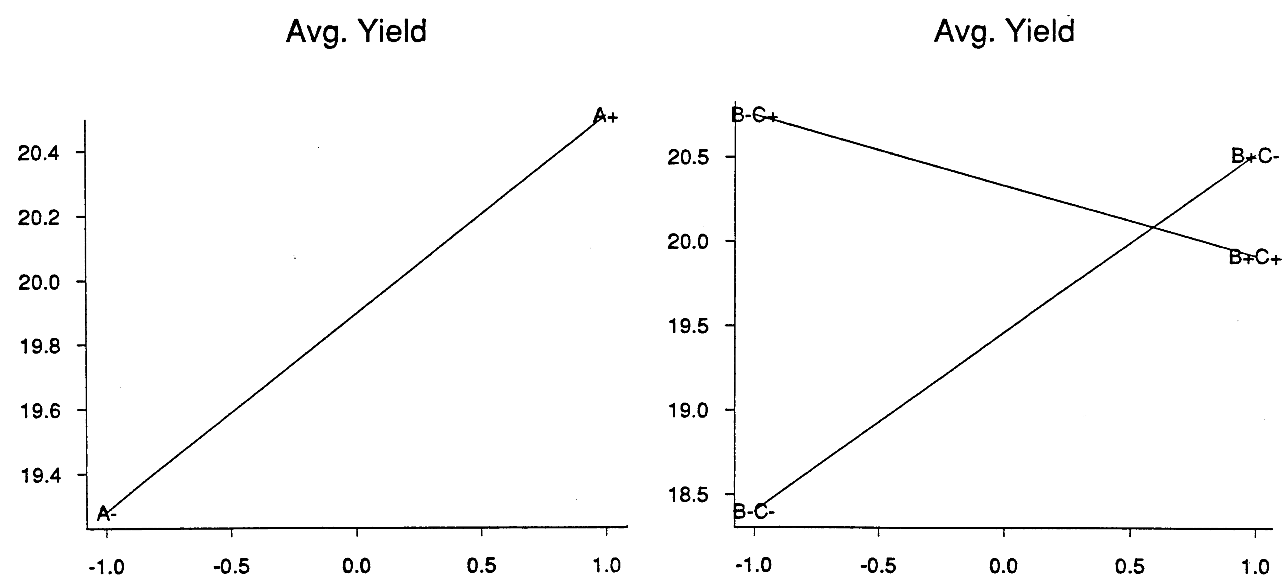

Figure 5: Mean Yield for Factor A and Mean Yield for Factor B and C Combinations

those obtained by the equations from Table 3. Important yield and spatial effects were then investigated by fitting linear models to the yields and standardized spatial statistics. The GLM procedures in MINITAB and SAS were used to handle the unbalanced nature of the experimental design. A check of the fitted model residuals revealed no anomalies.

The main effect A and BC interaction were significant for yield. Figure 5 displays the average yields corresponding to the levels of A and BC. The yield is higher for new A setting and is higher for the combination of standard B setting with new C setting.

Moreover, the AB interaction was significant for clustering. Figure 6 presents this spatial interaction effect by plotting the average values of the standardized join-count statistics for the four A and B combinations. The figure indicates that clustering can be increased at the new setting of A and standard setting of B.

Thus, the overall recommendation is new A setting, standard B setting and new C setting. A confirmatory experiment is needed to verify that both yield and clustering increase at these recommended settings. Although the manufacturing process engineer could not provide an

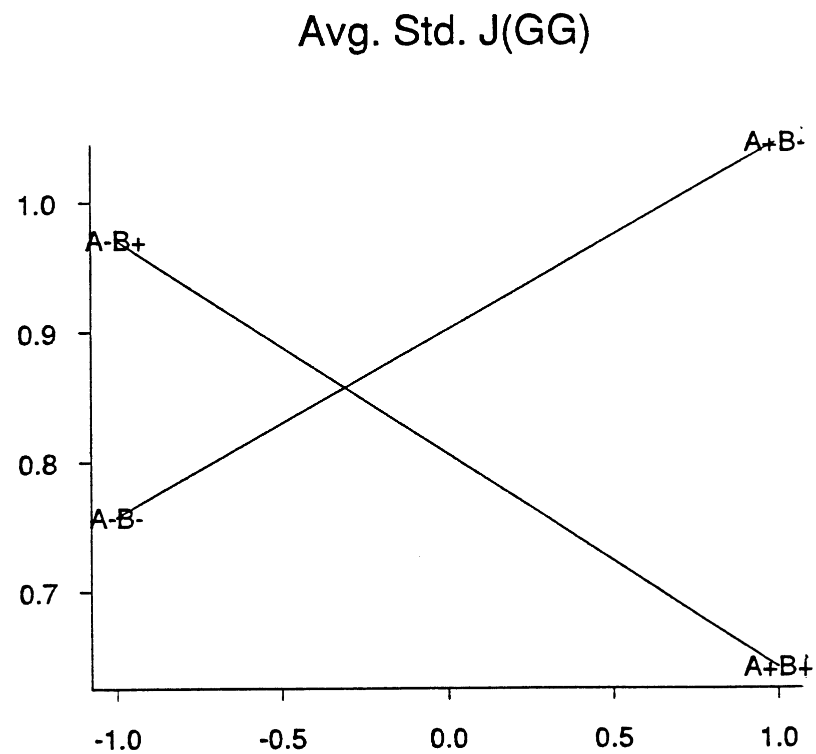

Figure 6: Mean Standardized J(GG) for Factor A and B Combinations

explanation for the clustering effect (AB interaction), identifying the existence of such an effect acts as a catalyst for learning more about the process and for improving the process.

5 Discussion

We conclude this article by making some remarks regarding the application of the proposed method, its connection with other methods, and areas of ongoing and future research.

The proposed method supplements other available methods. For example exploratory studies like those given in Mallory et al. (1983) and Perloff et al. (1977) used plotted contour maps and histograms. In contrast, our method uses quantitative measures of clustering so that several factors can be studied simultaneously through designed experiments.

The method is readily available to the IC manufacturing industry since it is easy to implement on lap-top or personal computers. That is, standard techniques for analyzing factorial experiments are still valid when the number of chips on a wafer is large. A potential problem arises when the wafer yield is near 0 or 100% for some of the runs, however. Since the join-count statistic behaves approximately like a Poisson random variable for these situations, the standard analysis may not provide a valid procedure for combining Poisson and normally distributed responses.

In the electronic engineering literature, yield and cluster are defined somewhat differently than that used in this article. A collection of definitions for yield based on the compound Poisson distribution of the number of faults on each chip can be found in Stapper (1989b) and numerous references cited there. The clustering parameter α defined in equation (5) of Stapper (1989a) measures clustering at a micro level, where clustering is treated as a deviation from a random (Poisson) point process. He addressed a completely different issue and did not consider the notions of neighboring chips and distance between chips. In contrast, the definition given in this article is neighborhood based and intuitively simpler.

If several replicates (wafers) are produced at each experimental setting and the observations are binary, then the replicates can be combined and fit by parametric spatial models to look for spatial dependence. In this way, the binary response case can be handled like the continuous measurement or point process case.

If the observations obtained from each wafer are continuous measurements, such as resistance, thickness, and gap between a specific link, then spatial models (Besag (1974, 1977), Tjostheim (1978), Moran (1973)) can be used to estimate the dependence between the chips on a wafer. Further work is currently underway.

Other investigations of IC fabrication procedures using exploratory statistics can be found in the engineering literature. Perloff, Mallory, Wahl and Mylroie (1981) discussed some issues in monitoring IC manufacturing. Pecen, Neukermans, Kren and Galbraith (1987) studied recent developments in detecting and sizing contaminants on manufactured wafers. Perloff, Smith, Lybeck and Lane (1987) explored the use of resistivity measurements to monitor and control the process. Perloff, Mallory, Smith and Kumagai (1985) examined data management procedures in IC fabrication process control. Lim and Ridley (1983) reviewed thickness monitoring procedures in process control. Perloff, Wahl and Reimer (1977) used contour maps of sheet resistance to demonstrate systematic non-uniformity introduced by various operational parameters. Finally, Ferris-Prabhu, Smith, Bonges and Paulsen (1987) conducted an empirical study of radial variation in yield. Other statistical tools to achieve these goals also need to be considered.

6 Acknowledgments

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