# EXPRESSING VARIABILITY AND YIELD WITH A FOCUS ON THE CUSTOMER

Kenneth K. Ng and Kwok-L. Tsui

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# EXPRESSING VARIABILITY AND YIELD WITH A FOCUS ON THE CUSTOMER

*Kenneth K. Ng*
AT&T Bell Laboratories

*Kwok-L. Tsui*
School of Industrial and Systems Engineering
Georgia Institute of Technology

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*ABSTRACT*

Yield is currently defined as the percentage of inspected units which meet specification limits. Yield focuses attention on meeting the tolerance of the next-in-line customer. High yield may not imply reduced variation in the passed units, and low yield may not imply large variation. The conclusions drawn from single measures such as yield or total defects per unit are incomplete at best and often misleading. As a result, customers are likely to receive units with product characteristics which are not tightly centered around design target values. Therefore, a more accurate, complete, and customer-oriented measure of yield, called *neoyield*, is proposed, defined, and mathematically derived in this memorandum. Sample calculations of neoyield for typical factory problems are given. Measurements of a small number of units are needed to convert yield to neoyield. Neoyield and yield complement each other and unite to indicate a direction for quality improvement.

## 1. MOTIVATION

Yield is currently defined as the percentage of units which pass inspection. Units are inspected according to specification limits placed on various key product characteristics and sorted into two categories: passed and rejected. Use of yield as a single measure implies that each rejected unit costs the factory an additional amount (for scrap or repair) while each passed unit costs the factory nothing additional. By inference, all passed units are equally acceptable to the next-in-line customer. Customer in this sense refers to any user of goods such as materials, components, subassemblies, assemblies, or systems.

This premise, that all passed units are equal, is not justified when product characteristics have target design values. Customers do notice unit-to-unit differences in these characteristics, especially if the variance is large and/or the mean is offset from the target. A single unit which deviates an amount $\delta$ from the target may cause no actual quality loss in a particular instance, but quality loss on average is incurred by units with deviation of an amount $\delta$ from target when many different customer usage conditions are considered. In general, quality loss can be expressed as a continuous function of a product characteristic.

In order to gauge quality loss as defined above, a more accurate, complete, and customer-oriented measure of yield is needed. Hereafter, we shall refer to the new yield measure as *neoyield* ($y_{neo}$ as a percentage; $Y_{neo}$ as a fraction) and the current yield measure simply as *yield* ($y$ in percentage; $Y$ in fractions). Neoyield accounts for both the amount of nonconforming units and variation from target for the passed units. It does this by penalizing yield commensurate with the amount of variation.

Our predominant motivation is to have a simple measure that complements yield and encourages designers and factory personnel to:

1. focus on quality from the view of the customer
2. reduce manufacturing variation in the product
3. pay attention to specifications.

Field returns encourage us to focus on the customer. Process capability indices encourage us to widen the design tolerance range and reduce processing variation. Yield encourages us to keep product within customer specifications. Neoyield, on the other hand, is explicitly defined to encourage all the above actions.

## 2. A GENERAL DERIVATION

Neoyield is defined in terms of:

1. the customer's quality loss as a function of the product characteristic $x$: $L(x)$ such that $L(T) = 0$ and is minimum at the target $x = T$.

2. the probability density function of the manufactured parts: $f(x)$ such that $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)\, dx = 1$.

3. the lower and upper specification limits for $x$: $LSL$ and $USL$.

We define neoyield as:

$$Y_{neo} \equiv \int_{LSL}^{USL} \left[ 1 - \frac{L(x)}{A} \right] f(x)\, dx \qquad (2.1)$$

where $A$ is the average quality loss at the specification limits.

The probability density function $f(x)$ and the customer quality loss function $L(x)$ are evaluated only in the specification interval $LSL \leq x \leq USL$. We will make use of two common probability density functions in Section 5. We now proceed to define $L(x)$ for the common situation in which the target $T$ is centered within the specification interval. Other situations will be covered in Section 7.

A general expression for $L(x)$ is:

$$L(x) = A \left| \frac{x-T}{\Delta} \right|^p \tag{2.2}$$

with $T = \frac{USL + LSL}{2}$, $\Delta = \frac{USL - LSL}{2}$, and $p > 0$. The reader can verify that $L(T) = 0$ and is minimum at $x = T$. Notice that $L(LSL) = L(USL) = A$, which agrees with the earlier definition of $A$.

When $L(x)$ is given by expression (2.2), definition (2.1) reduces to:

$$Y_{neo}^{[G]} = \int_{LSL}^{USL} \left[ 1 - \left| \frac{x-T}{\Delta} \right|^p \right] f(x)\, dx \tag{2.3}$$

The superscript $[G]$ refers to the general loss function given by expression (2.2).

## 3. TWO SPECIAL CASES

There are two special cases of the customer quality loss function as defined by equation (2.2):

- the step-wise loss function in which $p \to \infty$
- the quadratic loss function in which $p = 2$.

The $p = 2$ case has a unique property. It approximates any $L(x)$ that can be expanded about $x = T$ by a Taylor series and has $L(T) = L'(T) = 0$, where $L'(x)$ is the first derivative. Subsequent sections will be based on this quadratic loss function because the exact function of $L(x)$ is typically unknown.

### 3.1 STEP-WISE LOSS

As $p \to \infty$, $L(x) \to 0$ for all values of $x$ such that $|x - T| < \Delta$. Equation (2.3) becomes:

$$Y_{neo} = \int_{LSL}^{USL} f(x)\, dx \tag{3.1}$$

Note that the current yield measure is defined as:

$$Y \equiv \int_{LSL}^{USL} f(x)\, dx \tag{3.2}$$

Therefore, when $p \to \infty$, $Y_{neo} = Y$, which is to be expected since the yield definition implicitly assigns zero loss within the specification interval.

### 3.2 QUADRATIC LOSS

With $p = 2$ and use of definition (3.2), equation (2.3) becomes:

$$Y_{neo}^{[G]} = Y - \frac{1}{\Delta^2} \int_{LSL}^{USL} (x - T)^2 f(x)\, dx \tag{3.3}$$

To simplify for later usage, we define a term to represent the mean squared error of the passed units from the target value:

$$MSE_{pass} = \frac{\int_{LSL}^{USL} (x - T)^2 f(x)\, dx}{\int_{LSL}^{USL} f(x)\, dx} \tag{3.4}$$

Then equation (3.3) can be rewritten as:

$$Y_{neo}^{[G]} = Y \left[ 1 - \frac{MSE_{pass}}{\Delta^2} \right] \tag{3.5}$$

The multiplier of $Y$ on the right hand side of equation (3.5) simply penalizes yield for variation within the specification interval. If all passed units are on target, then $Y_{neo} = Y$ which is as expected. If all passed units are at the lower or upper limit, then $Y_{neo} = 0$ which indicates that the customer will, on average, incur the loss that the factory would have incurred if all parts had been rejected. $Y_{neo}^{[G]}$ is bounded by 0 and 1.

## 4. WHEN LOSS IS A FUNCTION OF YIELD

So far, we have assumed that $L(x)$ of equation (2.2) does not depend on the yield. This is valid in many practical cases, where there are high yields or one-time adjustment/repair operations for rejected units. In the case where yield is low and rejected units are valueless, average loss in the passed units becomes a function of yield.

### 4.1 MODIFIED LOSS FUNCTION

If $y$=50% and a unit is rejected, there is only a 50% chance that its replacement unit will be passed, and so on. Each passed part really costs the customer $M$ times the cost of producing a single part. The factor $M$ is the geometric progression $(1-q)^{-1}$, where $q$ is the fraction defective, also known as $(1-Y)$. Equation (2.2), when multiplied by $M = Y^{-1}$, becomes:

$$L(x) = \frac{A}{Y} \left| \frac{x-T}{\Delta} \right|^p \tag{4.1}$$

Notice that $L(LSL) = L(USL) = \frac{A}{Y}$. Average quality loss of a borderline passed unit is $A$ only when $Y \rightarrow 1$; otherwise, it is greater than $A$. We always define $A$ assuming a very small fraction defective.

### 4.2 MODIFIED NEOYIELD

Let us examine this modified loss function when it is quadratic, i.e., $p = 2$. By substituting equation (4.1) with $p = 2$ into equation (2.1):

$$Y_{neo}^{[M]} = Y - \frac{MSE_{pass}}{\Delta^2} \tag{4.2}$$

The superscript $[M]$ refers to the modified loss function given by expression (4.1).

If all passed units are on target, then $Y_{neo}^{[M]} = Y$ which is as expected. If all passed units are at the lower or upper limit, $Y_{neo}^{[M]} = 0$ when $Y = 1$, and $Y_{neo}^{[M]} \rightarrow -1$ when $Y \rightarrow 0$. Whereas the penalty factor in equation (3.5) is a multiplier of yield, the penalty factor in equation (4.2) is a subtrahend of yield.

Theoretically, $-1 < Y_{neo}^{[M]} \leq 1$ for equation (4.2). It is rather academic when $Y_{neo}^{[M]}$ is negative since such situations, regardless of the neoyield value, must be drastically remedied. Appendix A further examines what a negative neoyield implies.

## 5. USES OF NEOYIELD

Neoyield combines the advantages of yield and the customer quality loss function. It is more informative than yield alone since it provides a more accurate and customer-oriented measure of product quality. It is more informative than the loss function alone since it (i) is an absolute measure invariant to different characteristics, (ii) includes the cost but ignores the quality of rejected parts, and (iii) indicates a direction for quality improvement when contrasted with yield. These points will be illustrated in the following sections.

### 5.1 NEOYIELD VERSUS YIELD

We assume the mean is right on target $T$ in all four scenarios of this section. Discussion of scenarios when the mean is off target will be deferred to Sections 5.2 and 5.3. In scenarios (1) and (2) we assume the population distribution is normal with standard deviation equal to $\Delta$ and $\Delta/3$, respectively. In scenario (3) we assume the population distribution is continuously uniform within the range from $LSL$ to $USL$. In scenario (4) we assume the population distribution is a bimodal distribution which contains a mixture of two normal distributions, each with a mean located at a distance of $0.75\Delta$ from the target and a standard deviation equal to $0.1\Delta$.

Table 5.1 summarizes yield ($y$), mean squared error of passed units ($MSE_{pass}$), general neoyield ($y_{neo}^{[G]}$), and modified neoyield ($y_{neo}^{[M]}$) for these four scenarios.

| Table 5.1 | | | | |
|---|---|---|---|---|
| SCENARIO | $y$ | $MSE_{pass}$ | $y_{neo}^{[G]}$ | $y_{neo}^{[M]}$ |
| (1) | 68.27% | $0.291\Delta^2$ | 48.4% | 39.2% |
| (2) | 99.73% | $0.108\Delta^2$ | 89.0% | 88.9% |
| (3) | 100.00% | $0.333\Delta^2$ | 66.7% | 66.7% |
| (4) | 99.38% | $0.569\Delta^2$ | 42.8% | 42.4% |

In comparing scenarios (1) and (2), both yield and neoyield indicate that scenario (2) has better customer quality than scenario (1), which is reasonable and expected. In comparing scenarios (2) and (3), it is interesting to see that scenario (3) has a higher yield but a smaller neoyield than scenario (2). Clearly scenario (2) has better customer quality than scenario (3) because it has a much tighter distribution around the target. Thus neoyield indicates quality more accurately than yield does. The same point is also illustrated in the comparison of scenarios (2) and (4). Although they both have similar yield measures, neoyield reveals that scenario (2) is much better than scenario (4).

To illustrate how neoyield is different from yield in practice, we have computed both measures for a real circuit pack product. The background of this example and the relevant information are given in Appendix B. Although the yield of each different product characteristic in this example is 100%, as determined by measurements of approximately 275 circuit packs, the corresponding neoyield of each is very different. The higher that neoyield is, the better quality that the characteristic has. Thus neoyield can help us distinguish the different quality levels among many product characteristics of the same product.

### 5.2 NEOYIELD: ROADMAP FOR QUALITY IMPROVEMENT

We now present a sequence of events at a hypothetical widget factory to illustrate how neoyield can be used as a roadmap for quality improvement activities.

The widgets being produced by the factory come off the assembly line with a large variance and a mean which is far from the target. After a first-pass inspection, it was found that:

1. $y = 50\%$, $\quad MSE_{pass} = 0.8\Delta^2$, $\quad y_{neo}^{[G]} = 10\%$, $\quad y_{neo}^{[M]} = -30\%$.

The factory decides to fix all rejected units. The fixes are marginal so that the mean squared error of passed parts remains nearly the same as before. At the end of the second inspection (units that passed the first inspection are not re-inspected but included in the distribution), it was found that:

2. $y = 100\%$, $\quad MSE_{pass} = 0.8\Delta^2$, $\quad y_{neo}^{[G]} = 20\%$, $\quad y_{neo}^{[M]} = 20\%$.

The factory decides to improve its process by using statistical quality control, i.e., AT&T (1956). After a first-pass inspection, it was found that the widget mean $\mu$ is now on target and the widgets are normally distributed ($f(x) = [1/(\sigma\sqrt{2\pi})]\, e^{-(x-\mu)^2/(2\sigma)}$ *in* $-\infty < x < \infty$) with standard deviation $\sigma$ equal to $\Delta$:

3. $y = 68.27\%$, $\quad MSE_{pass} = 0.291\,\Delta^2$, $\quad y_{neo}^{[G]} = 48.4\%$, $\quad y_{neo}^{[M]} = 39.2\%$.

Widgets that do not pass first inspection continue to be fixed. After the second inspection, it was found that the widgets are more or less distributed uniformly within the specification limits ($f(x) = 1/(USL - LSL)$ *in* $LSL \le x \le USL$ and $f(x) = 0$ *otherwise*), so that:

4. $y = 100\%$, $\quad MSE_{pass} = 0.333\,\Delta^2$, $\quad y_{neo}^{[G]} = 66.7\%$, $\quad y_{neo}^{[M]} = 66.7\%$.

In time, the process improves so that the widgets continue to have their mean $\mu$ on target and a normal distribution with a standard deviation $\sigma$ equal to $\Delta/3$. Inspection continues and rejected units are analyzed for causes of failure:

5. $y = 99.73\%$, $MSE_{pass} = 0.108\ \Delta^2$, $y_{neo}^{[G]} = 89.0\%$, $y_{neo}^{[M]} = 88.9\%$.

The factory finds 100% inspection no longer necessary. Lots are sampled and the mean squared deviation for each lot is calculated. The possibility now exists that units passed to the customer may be outside of specification limits, but the customer can tolerate a minute number of defectives. Neoyield does not change noticeably when inspection is eliminated, which confirms that inspection adds little value:

6. $y \approx 100\%$, $MSE_{pass} = 0.111\ \Delta^2$, $y_{neo}^{[G]} = 88.9\%$, $y_{neo}^{[M]} = 88.9\%$.

There is still much room for improvement. The process is reaching its capability, but better design techniques, i.e., Phadke (1989), will make the next generation widget more robust to manufacturing variations. The ultimate goal is of course:

7. $y = 100\%$, $MSE_{pass} = 0$, $y_{neo}^{[G]} = 100\%$, $y_{neo}^{[M]} = 100\%$.

From this illustration, it is clear that neoyield can be used to:

1. baseline a product and measure improvements in terms of the customer
2. compare product produced with different processes
3. determine whether inspection per design specifications is worthwhile.

On Appendix C.1, a useful graph of $Y_{neo}$ versus $Y$ is shown. These two dimensions of product quality are powerful when used together because one dimension represents customer satisfaction while the other represents factory fulfilment. The triangle with vertices (0, 0), (100, 100), and (100, 0) contains the set of all $(Y, Y_{neo}^{[G]})$. The parallelogram with vertices (0, 0), (100, 100), (100, 0), and (0, -100) contains the set of all $(Y, Y_{neo}^{[M]})$. The objective of quality improvement is to move towards the apex of (100,100). The paths of improvement for the widget company are shown in Appendix C.1, one via $Y_{neo}^{[G]}$ and the other via $Y_{neo}^{[M]}$.

### 5.3 NEOYIELD: RELATIONSHIP TO SIX-SIGMA CONCEPT

To further illustrate how neoyield can be used for measuring and improving the progress of quality improvement, we compare yield and neoyield of five normally distributed populations and relate them to the Motorola Six-Sigma concept, i.e., Harry (1989). The mean $\mu$ and standard deviation $\sigma$ of these five populations are:

(a) $|\mu - T| = 1.5\sigma$ and $\Delta = 3\sigma$ (3-sigma, 1.5-sigma shift pop.)

(b) $|\mu - T| = 0$ and $\Delta = 3\sigma$ (3-sigma, on-target pop.)

(c) $|\mu - T| = 1.5\sigma$ and $\Delta = 6\sigma$ (6-sigma, 1.5-sigma shift pop.)

(d) $|\mu - T| = 0$ and $\Delta = 6\sigma$ (6-sigma, on-target pop.)

(e) $|\mu - T| = 0$ and $\Delta = 12\sigma$ (12-sigma, on-target pop.)

Appendix C.2 graphically depicts yield and neoyield for all five populations. Motorola's Six-Sigma program contrasts the quality difference in terms of ppm (parts per million, equal to $(100 - y) \times 10{,}000$) for the first four populations above. Motorola points out that over a very large number of lots, a change in the mean of about $1.5\sigma$ from lot to lot could be expected because of "typical" shifts and drifts.

By accounting for "typical" shifts and drifts, and assuming that the inspected units come from a normally distributed population, Motorola equates a standard deviation of $\Delta/6$ to a defect rate of 3.4 ppm. Population (c) above is the conservative representation of a product population which achieves the Six-Sigma quality standard.

This Six-Sigma concept is practical when the product characteristic tends to be normally distributed but is difficult to measure on a continuous scale. On the contrary, neoyield does not make assumptions about a normal distribution nor "typical" shifts and drifts. We state once more that yield (or the ppm measure in the Six-Sigma program) should not be the only measure of quality in a quality improvement program.

The following lessons are learned by observing the neoyield versus yield graph on Appendix C.2.

- By comparing populations (a) and (b) or (c) and (d), we find that a $1.5\sigma$ shift of the population mean from the design target substantially degrades product quality as perceived by the customer. In addition, populations (b) and (c) show that a 6-sigma population with a $1.5\sigma$ shift is not much better than a 3-sigma population with no shift of the mean from the target. *Lesson:* Strive to reduce lot-to-lot variation as well as within-lot variation.

- By comparing populations (d) and (e), we find that the impact of quality improvement on the customer diminishes as quality levels increase from $6\sigma$ ($Y_{neo} \approx 97.2$) to $12\sigma$ ($Y_{neo} \approx 99.3$). This points out the key advantage of yield and neoyield over mean and variance or $C_p$ and $C_{pk}$ capability indices. Neoyield economically justifies quality improvement because it is based on a customer quality loss function. *Lesson:* Aim for perfection but beware of diminishing returns as perfection is neared.

- The triangle defined by (60, 60), (100, 100), and (100, 60) is where most typical quality improvements are made, measured and compared against baselined values of $Y_{neo}$. *Lesson:* Locate the current quality of the product and incrementally advance toward the goal of perfection.

## 6. ESTIMATE OF NEOYIELD

As mentioned earlier, Appendix B shows estimates of neoyield for a finite number of samples. Further, some of the measured characteristics are targeted on one side rather than symmetrically between specification limits. This section covers computing neoyield from a finite population and estimating neoyield from a random sample. The next section describes the handling of asymmetric targeted characteristics.

In Sections 2, 3, and 4, neoyield is defined for a general product population with probability density function $f(x)$. In practice, we often have a finite population of products. Suppose $x_1, x_2, \ldots, x_N$ denote the product characteristics of $N$ units in a finite population; the definition of $Y_{neo}^{[G]}$ in equation (3.3) reduces to:

$$Y_{neo}^{[G]} = Y - \frac{1}{N\Delta^2} \sum_{LSL \le x_i \le USL} (x_i - T)^2 \tag{6.1}$$

Using this formula, 100% inspection is needed to measure the characteristic of each unit and to determine which $x_i$ is within the specification. When 100% inspection is impractical or the population is infinite, a sample of $n$ units is often measured to make inference about the population. Let $X_1, X_2, \ldots, X_n$ denote the characteristics of $n$ randomly selected units; the estimate of the population $Y_{neo}^{[G]}$ is defined to be:

$$\hat{Y}_{neo}^{[G]} = \hat{Y} - \frac{1}{n\Delta^2} \sum_{LSL \le X_i \le USL} (X_i - T)^2 \tag{6.2}$$

where $n\hat{Y}$ is the number of selected units that are within the interval $[LSL, USL]$.

The expected value and variance of this estimator are derived in Appendix D and given as follows:

$$E(\hat{Y}_{neo}^{[G]}) = Y_{neo}^{[G]} \tag{6.3}$$

$$nVar(\hat{Y}_{neo}^{[G]}) = Y - Y^2 + \frac{1}{\Delta^4} \left\{ \int_{LSL}^{USL} (x - T)^4 f(x)\, dx - \left[ \int_{LSL}^{USL} (x - T)^2 f(x)\, dx \right]^2 \right\} \tag{6.4}$$

Thus $\hat{Y}_{neo}^{[G]}$ is an unbiased estimate of $Y_{neo}^{[G]}$ and has a finite variance given by equation (6.4). The finite sample statistical properties of $\hat{Y}_{neo}^{[G]}$ are intractable because of its complex distribution. However, the large sample properties can be easily derived since $\hat{Y}_{neo}^{[G]}$ can be expressed as the average of an independently identically distributed sample. Appendix D shows that $\hat{Y}_{neo}^{[G]}$ converges to $Y_{neo}^{[G]}$ almost surely, and has an asymptotically normal distribution with mean $Y_{neo}^{[G]}$ and variance given by equation (6.4). Therefore, when the sample size is large enough, the usual normal statistical inference techniques, such as testing hypotheses and computing confidence intervals, can be used to make inferences about the population.

Analogously, definition (3.4), which defines the mean squared error of the passed units from the target value, can be estimated by measuring a sample of $n$ units:

$$M\hat{S}E_{pass} = \frac{1}{n\hat{Y}} \sum_{LSL \le X_i \le USL} (X_i - T)^2 \tag{6.5}$$

Equation (6.5) is useful when implementing equation (3.5) or (4.2).

## 7. EXTENSIONS

So far, we have dealt only with the situation in which the target value $T$ of the continuous product characteristic is midway between the upper (USL) and lower (LSL) specification limits. Let us now consider the following situations where:

1. $T$ is asymmetrically located between $LSL$ and $USL$ (see section 7.1)
2. $T$ is a physical limitation and only a $LSL$ or $USL$ exists (see section 7.2)
3. $T$ is at zero and only a positive $USL$ or negative $LSL$ exists (see section 7.3)
4. $T$ is at + or - infinity and only a $LSL$ or $USL$ exists (see section 7.4).

### 7.1 ASYMMETRIC TARGET

In an asymmetric target problem, the characteristic is in the range $-\infty < x < \infty$. The target $T$ can be any value of $x$. $(T - LSL) = \Delta_l$ is not equal to $(USL - T) = \Delta_u$. Any term defined hereafter with subscripts $l$ and $u$ implies that term is defined in the interval $LSL \le x < T$ and $T < x \le USL$ respectively. Units at either of the limits have equal loss $A$.

By splitting definition (2.1) into two integrals and substituting in two forms of equation (2.2) or (4.1) with $p = 2$, one with $\Delta_l$ and one with $\Delta_u$, we obtain:

$$Y_{neo}^{[G]} = Y \left[ 1 - \frac{\gamma_l \, MSE_{pass-l}}{\Delta_l^2} - \frac{\gamma_u \, MSE_{pass-u}}{\Delta_u^2} \right] \tag{7.1}$$

$$Y_{neo}^{[M]} = Y - \frac{\gamma_l \, MSE_{pass-l}}{\Delta_l^2} - \frac{\gamma_u \, MSE_{pass-u}}{\Delta_u^2} \tag{7.2}$$

where $\gamma_l + \gamma_u = 1$ and $MSE_{pass}$ is given by equation (3.4) with the limits changed according to subscript $l$ or $u$. The term $\gamma$ indicates the fraction of total passed units within the lower or upper specification limit, as indicated by the subscript.

In practice, we measure the characteristic of a finite number of sample units. We then use equation (6.5) to estimate $MSE_{pass-l}$ for $\alpha$ units with $x < T$ and $MSE_{pass-u}$ for $\beta$ units with $x > T$. Since $\gamma_l = \alpha/(\alpha + \beta)$ and $\gamma_u = \beta/(\alpha + \beta)$, equation (7.1) or (7.2) can be used to calculate $Y_{neo}$.

**7.2 ONE-SIDED TARGET**

In a one-sided target, $T$ defines one limit of the characteristic $x$. Typically, $T$ is a physical limitation such as the strength of a pure material. When $T$ is the *USL* and a lower specification limit *LSL* is defined, the situation can be considered to be an asymmetric target problem. Use equation (7.1) or (7.2) and set $\gamma_l = 1$ and $\gamma_u = 0$.

The opposite situation in which there is a lower physical limitation and an upper specification limit can be handled in an analogous manner. That is, when $T$ is the *LSL* and an *USL* is defined, use equation (7.1) or (7.2) and set $\gamma_l = 0$ and $\gamma_u = 1$.

**7.3 ONE-SIDED TARGET $= 0$**

In the case where $|x| \geq 0$ and the goal is to have $|x|$ as small as possible, it is the same as a one-sided target problem (Section 7.2) where $T = 0$ and the appropriately subscripted $MSE_{pass}$ reduces to the average of all $x_i^2$.

**7.4 ONE-SIDED TARGET $\rightarrow \infty$**

In the case where $|x|$ has a minimum and the goal is to have $|x|$ as large as possible, a convenient way to handle this situation is to transform $|x|$ into another characteristic $|x|^{-1}$. The problem is then simply the one-sided target problem with the target equal to zero (Section 7.3). Caution: first consider whether a physical limitation exists and use a one-sided target solution (Section 7.2) if possible.

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**APPENDIX A**

Taguchi has developed his methods based on the quadratic loss function, i.e., Taguchi (1986), Japanese Industrial Standard (1986), and Taguchi, et. al. (1989). We will use a small part of his work to further discuss loss and $Y_{neo}^{[M]}$. Taguchi (1986) defines averaged total loss when 100% inspection is performed as:

$$L_{total} = L_{inspect} + L_{defect} \, ( 1 - Y ) + \left[ \frac{A_{cust}}{\Delta_{cust}^2} \right] MSE_{pass} \tag{A.1}$$

where all terms are expressed per passed unit, $L_{inspect}$ is the average inspection cost, $L_{defect}$ (also known as $A$) is the cost of scrapping a defective unit, and $A_{cust}$ is the average loss to the customer if a passed unit is at the customer specification limit $\Delta_{cust}$.

It is inherent in Taguchi's approach that the pairs ( $\Delta_{cust}$ , $A_{cust}$ ) and ( $\Delta$ , $A$ ) are related by the same quadratic loss function. The ideal procedure is to define the quadratic loss function of a product by estimating the customer-oriented pair ( $\Delta_{cust}$ , $A_{cust}$ ). Then the cost to the product supplier of a failed unit $A$ is found. Lastly, the manufacturing or inspection tolerance $\Delta$ is determined. Therefore, we can state:

$$\frac{A_{cust}}{\Delta_{cust}^2} = \frac{A}{\Delta^2} \tag{A.2}$$

When $A_{cust}$ is not estimable, a practical solution is to use the pair ( $\Delta$ , $A$ ) to define the quadratic loss function.

Equation (A.1) can be reduced to the following form by substituting in equation (A.2), dividing all terms by $A$, and rearranging terms:

$$Y - \frac{MSE_{pass}}{\Delta^2} = 1 - \frac{L_{total} - L_{inspect}}{A} \tag{A.3}$$

The left hand side of equation (A.3) is simply $Y_{neo}^{[M]}$ as shown in equation (4.2).

According to equation (A.3), $Y_{neo}^{[M]} < 0$ if $L_{total} > A + L_{inspect}$. The implication is that if passed units to the customer have an average total loss that is greater than the average inspection cost per passed unit plus the cost of the factory in fixing a unit, then the units should not have been produced in the first place. Manufacture of the units should be halted until the process or product is corrected.

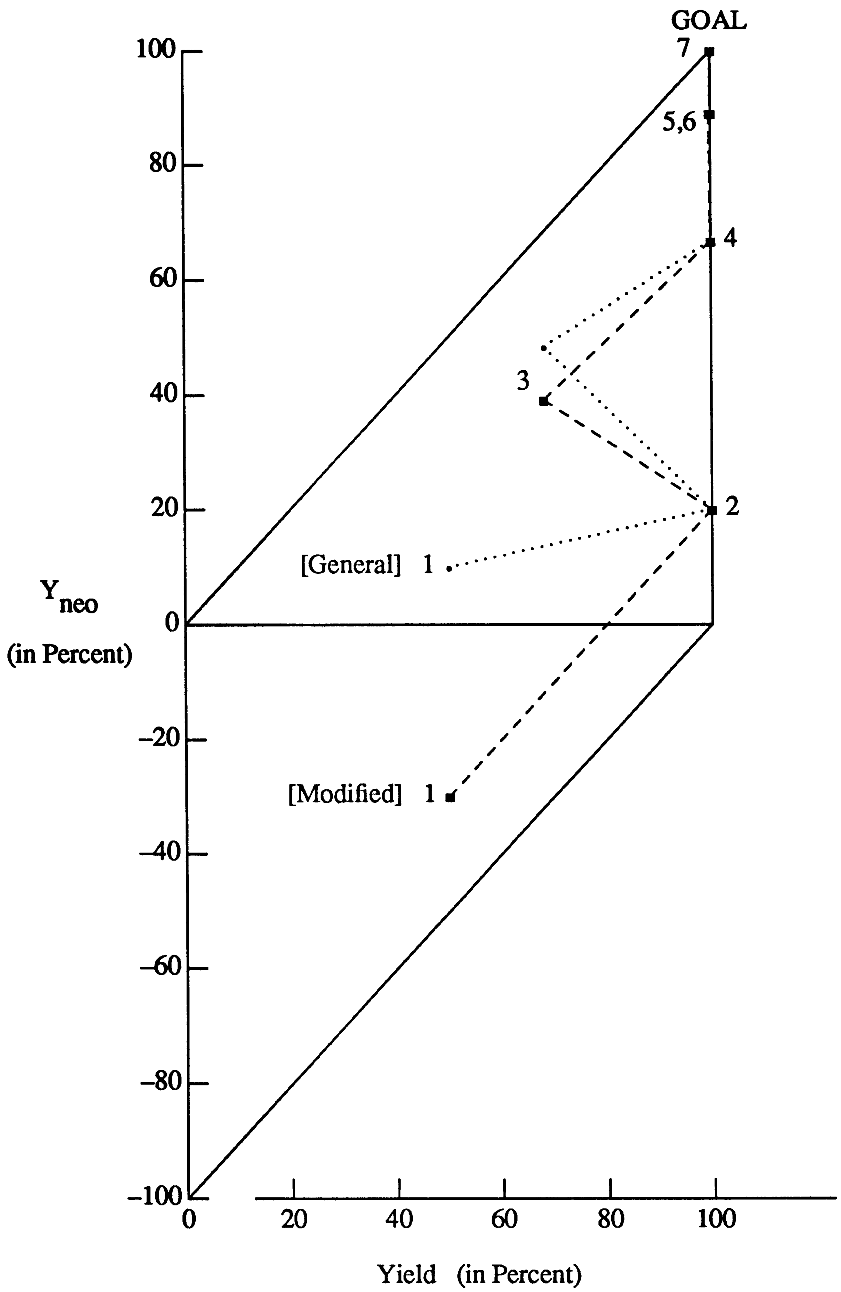
**APPENDIX B**

Factory data from a number of critical electrical tests run on a lot of circuit packs were made available by A. Seghatoleslami and S. Mcardle. Neoyield values were calculated for each test. Some results are tabulated below. The columns are self-explanatory, except the last which indicates the degree to which improvement is needed.

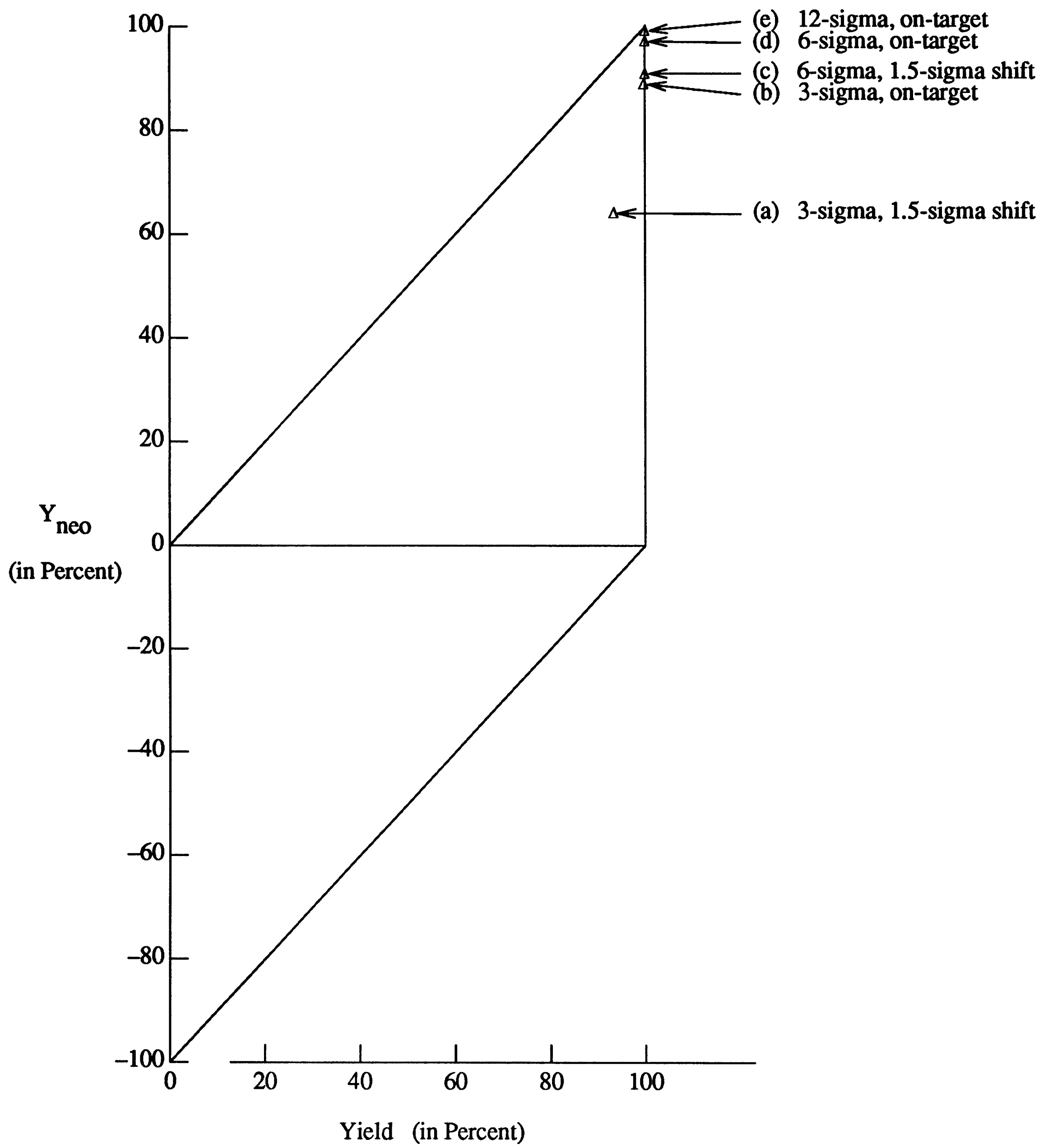
Note: 3-sigma (on-target normal) quality = 89.0% neoyield
6-sigma (on-target normal) quality = 97.2% neoyield
12-sigma (on-target normal) quality = 99.3% neoyield

| TEST | DESCRIPTION OF TEST | TARGET | | # UNITS | YIELD | YNEO | FIX? |
|---|---|---|---|---|---|---|---|
| 7201 | forward disconnect (R) | -37.25 | Volts | 277 | 100% | 98.60% | |
| 7202 | forward disconnect (T) | -10.5 | Volts | 277 | 100% | 97.72% | |
| 6201 | on-hook rcv. gain (A) | -1.25 | dbm | 275 | 100% | 99.43% | |
| 6202 | on-hook rcv. gain (A) | -0.70 | dbm | 275 | 100% | 89.56% | ** |
| 6203 | on-hook rcv. gain (A) | 0.0 | dbm | 275 | 100% | 87.46% | ** |
| 6301 | off-hook rcv. gain (A) | 0.0 | dbm | 275 | 100% | 82.26% | ** |
| 6302 | off-hook rcv. gain (A) | -0.175 | dbm | 275 | 100% | 91.40% | * |
| 6303 | off-hook rcv. gain (A) | 0.01 | dbm | 275 | 100% | 66.95% | **** |
| 6401 | rcv. disable (A) | -inf | dbm | 275 | 100% | 78.48% | *** |
| 6402 | rcv. disable (B) | -inf | dbm | 275 | 100% | 68.65% | **** |
| 6601 | on-hook xmt. gain (A) | -1.25 | dbm | 275 | 100% | 97.90% | |
| 6602 | on-hook xmt. gain (A) | 0.0 | dbm | 275 | 100% | 86.02% | ** |
| 6603 | on-hook xmt. gain (A) | 0.0 | dbm | 275 | 100% | 94.23% | * |
| 6701 | off-hook xmt. gain (A) | 0.0 | dbm | 275 | 100% | 82.27% | ** |
| 6702 | off-hook xmt. gain (A) | -0.025 | dbm | 275 | 100% | 75.72% | *** |
| 6703 | off-hook xmt. gain (A) | 0.025 | dbm | 275 | 100% | 95.01% | * |
| 6901 | transhybrid loss (A) | -inf | db | 275 | 100% | 50.31% | ***** |
| 6902 | transhybrid loss (A) | -inf | db | 275 | 100% | 63.71% | **** |
| 6903 | transhybrid loss (A) | -inf | db | 275 | 100% | 30.42% | ****** |
| 6801 | transmit noise (A) | 0.0 | dbmc | 274 | 100% | 65.00% | **** |
| 6501 | receive noise (A) | 0.0 | dbmc | 274 | 100% | 97.63% | |
| 8000 | crosstalk, A into B | 0.0 | dbmc | 273 | 100% | 69.42% | **** |
| 8001 | crosstalk, A into B | 0.0 | dbmc | 273 | 100% | 71.73% | *** |
| 8002 | crosstalk, A into B | 0.0 | dbmc | 273 | 100% | 65.73% | **** |
| 8003 | crosstalk, B into A | 0.0 | dbmc | 273 | 100% | 80.91% | ** |
| 8004 | crosstalk, B into A | 0.0 | dbmc | 273 | 100% | 84.19% | ** |
| 8005 | crosstalk, B into A | 0.0 | dbmc | 273 | 100% | 71.36% | *** |

**APPENDIX C.1**


GOAL
7
5,6
4
3
2
[General] 1
[Modified] 1
100
80
60
40
20
0
–20
–40
–60
–80
–100
$Y_{neo}$
(in Percent)
0
20
40
60
80
100
Yield (in Percent)

**APPENDIX C.2**



100
80
60
40
20
0
-20
-40
-60
-80
-100
$Y_{neo}$
(in Percent)
0
20
40
60
80
100
Yield (in Percent)
(e) 12-sigma, on-target
(d) 6-sigma, on-target
(c) 6-sigma, 1.5-sigma shift
(b) 3-sigma, on-target
(a) 3-sigma, 1.5-sigma shift

**APPENDIX D**

Suppose $X_1, X_2, \ldots, X_n$ are identically and independently distributed (IID) with continuous probability density function $f(x)$ and $-\infty < x < \infty$.

Define:

$$Y_{neo}^{[G]} = Y - \frac{1}{\Delta^2} \int_{LSL}^{USL} (x - T)^2 f(x)\, dx \tag{D.1}$$

with $Y = \int_{LSL}^{USL} f(x)\, dx$ where *LSL*, *USL*, and $\Delta$ are known constants.

From equation (6.2), we have:

$$\hat{Y}_{neo}^{[G]} = \frac{1}{n}\sum_{i=1}^{n} I_{[LSL \le X_i \le USL]} - \frac{1}{n\Delta^2}\sum_{i=1}^{n} I_{[LSL \le X_i \le USL]} (X_i - T)^2 \tag{D.2}$$

$$= \frac{1}{n}\sum_{i=1}^{n} I_{[LSL \le X_i \le USL]} \left\{ 1 - \frac{(X_i - T)^2}{\Delta^2} \right\}$$

$$= \frac{1}{n}\sum_{i=1}^{n} Z_i$$

where $Z_i = I_{[LSL \le X_i \le USL]} \left\{ 1 - \frac{(X_i - T)^2}{\Delta^2} \right\}$

It follows that:

$$E(\hat{Y}_{neo}^{[G]}) = E(Z_i) = E\left[ I_{[LSL \le X_i \le USL]} \left\{ 1 - \frac{(X_i - T)^2}{\Delta^2} \right\} \right] \tag{D.3}$$

$$= E\left[ I_{[LSL \le X_i \le USL]} \, E\left\{ \left[ 1 - \frac{(X_i - T)^2}{\Delta^2} \right] \Big| \, I_{[LSL \le X_i \le USL]} \right\} \right]$$

$$= E\left[ I_{[LSL \le X_i \le USL]} \left\{ 1 - \int_{LSL}^{USL} \frac{(x - T)^2}{\Delta^2} \frac{f(x)}{Y} \, dx \right\} \right]$$

$$= Y \left\{ 1 - \int_{LSL}^{USL} \frac{(x - T)^2}{\Delta^2} \frac{f(x)}{Y} \, dx \right\}$$

$$= Y_{neo}^{[G]}$$

$$nVar(\hat{Y}_{neo}^{[G]}) = Var(Z_i) = E\left[ I^2_{[LSL \le X_i \le USL]} \left\{ 1 - \frac{(X_i - T)^2}{\Delta^2} \right\}^2 \right] - Y_{neo}^{[G]2} \tag{D.4}$$

$$= E\left[ I_{[LSL \le X_i \le USL]} \, E\left\{ \left[ 1 - \frac{(X_i - T)^2}{\Delta^2} \right]^2 \Big| \, I_{[LSL \le X_i \le USL]} \right\} \right] - Y_{neo}^{[G]2}$$

$$= E\left[ I_{[LSL \le X_i \le USL]} \left\{ 1 - 2\int_{LSL}^{USL} \frac{(x - T)^2}{\Delta^2} \frac{f(x)}{Y} \, dx + \int_{LSL}^{USL} \frac{(x - T)^4}{\Delta^4} \frac{f(x)}{Y} \, dx \right\} \right] - Y_{neo}^{[G]2}$$

$$= Y - \frac{2}{\Delta^2} \int_{LSL}^{USL} (x - T)^2 f(x) \, dx + \frac{1}{\Delta^4} \int_{LSL}^{USL} (x - T)^4 f(x) \, dx - Y_{neo}^{[G]2}$$

$$= Y - Y^2 + \frac{1}{\Delta^4} \left\{ \int_{LSL}^{USL} (x - T)^4 f(x) \, dx - \left[ \int_{LSL}^{USL} (x - T)^2 f(x) \, dx \right]^2 \right\}$$

Since $LSL$ and $USL$ are finite constants and $Z_i$ is a function of $X_i$ truncated within $[LSL, USL]$, all finite moments of $Z_i$ are finite. As $\hat{Y}_{neo}^{[G]}$ is the mean of an IID sample, $Z_1, \ldots, Z_n$, it follows from the Strong Law of Large Numbers and the Central Limit Theorem that:

$$\hat{Y}_{neo}^{[G]} \rightarrow Y_{neo}^{[G]} \quad \text{almost surely, and} \quad \sqrt{n}\,(\hat{Y}_{neo}^{[G]} - Y_{neo}^{[G]}) \rightarrow N(0, Var(Z_i)) \quad \text{as} \quad n \rightarrow \infty.$$