SOME ISSUES IN THE COLLECTION AND ANALYSIS OF FIELD RELIABILITY DATA

J.F. Lawless and J.D. Kalbfleisch

SOME ISSUES IN THE COLLECTION AND ANALYSIS OF FIELD RELIABILITY DATA

J.F. Lawless and J.D. Kalbfleisch

Department of Statistics and Actuarial Science
University of Waterloo
Waterloo, Ontario, N2L 3G1

ABSTRACT

Sources of data on field reliability or performance of manufactured products include field tracking (follow-up) studies, warranty and failure reports, and surveys. We discuss these sources in relation to the estimation of failure time distributions and event rates. Important issues include missing information, reporting delays, the need to combine information from different sources, and the possibility of selection bias.

1 Introduction

The ultimate test of a manufactured product is how well it performs in the field, that is, in the hands of the customer. Accordingly, the collection and analysis of data on the field performance or reliability of products is important to manufacturers and consumers alike. Such data can be used in many ways by a manufacturer, including (i) to assess field reliability and make comparisons with engineering predictions, (ii) to provide information for product modification and improvement, (iii) to assess the effect of design changes, (iv) to estimate and explain warranty costs, and (v) to aid in the design of warranty, maintenance and parts replacement programs. Nevertheless, many manufacturers pay insufficient attention to the collection of field performance data. One reason is that comprehensive data are often seen as expensive to obtain; another may be a lack of familiarity with methods for response-selective observational schemes and for combining information from different sources. For some problems methodology indeed needs to be developed.

This paper deals with reliability, which is an important component of field performance. Our objective is to discuss some methodologic issues and to illustrate them via specific problems. Our illustrations involve automobiles, but the issues and methods are quite general. In keeping with a broad orientation, the emphasis will be on laying out feasible approaches rather than on their detailed examination. Section 2 considers some general issues. Section 3 deals with the use of warranty data, and Section 4 with surveys. Section 5 concludes the paper with brief remarks on some other topics.

2 Some General Issues

To focus discussion, yet still allow for a broad range of applications, we will set up a little terminology. We suppose that individual units of a product enter service over a calendar time period, taken without loss of generality to be $(0, T)$. By time T a total of N units have entered service, at calendar times $x_1, \ldots, x_N$; these are referred to as the entry times. The

reliability of a product is usually assessed in terms of events such as failures, repairs, and part replacements, and in terms of time averages such as availability. To limit the discussion we will consider only (i) lifetimes or times to failure for units (or parts thereof), and (ii) repeated events such as repairs. Another concept with which we shall deal only indirectly is deterioration, or wear; our discussion of wear will here be limited to its relationship to lifetime.

We have referred to the occurrence of failures and other events. An important point concerns the time scales on which such events are considered and, in particular, the distinction between calendar time and an operational or usage time for a product. Many products are used intermittently, and it is often natural to define a (nondecreasing) process $W(x)$ that represents the total "usage" time for a unit up to calendar time x. Probabilities or rates of occurrence of failures and other events usually depend both on calendar time and the usage time process. Depending on one's objectives and the process in question, analysis may focus on calendar time, usage time, or both. In many studies, however, the difficulty of obtaining accurate information about a unit's usage time process limits what can be done.

Automobiles provide examples of points just made. Typical lifetime variables in this case would be the time until initial front brake pad replacement or the time until initial head lamp replacement. Repairs on systems such as the emission control and electrical systems are examples of repeated events. The typical usage time variable would be mileage, and failure or repair rates are likely to depend both on the age of the car (calendar time) and its usage process (i.e. mileage accumulation process). Note that the complete usage process contains a great deal of information, for example on periods when the car is or is not in use, on speed and acceleration when in use, and so on. Of course, except in the rare circumstance where a car is monitored continuously, we usually have a quite incomplete record of its usage process.

A general theoretical treatment of failures or repeated events for unit i is best given in terms of joint processes $N_i(x)$, $W_i(x)$, $Z_i(x)$, where x represents calendar time, $N_i(x)$ is

a right-continuous counting process that indicates failure or counts repeated events, $W_i(x)$ is the usage time process, and $Z_i(x)$ represents covariates, some of which may be time-dependent. The intensity function $\lambda_i(x)$ for $N_i(x)$ at time x may depend on the history $H_x = \{N_i(s), W_i(s), Z_i(s) : 0 \le s < x\}$, i.e. we have

$$\lambda_i(x; H_x) = \lim_{\Delta x \to 0} \frac{Pr\{N_i(x + \Delta x) - N_i(x) = 1 \mid H_x\}}{\Delta x}. \tag{2.1}$$

For the most part, however, we will consider very special cases of (2.1) and will not rely on the general formulation.

Information on field reliability is obtained in various ways. Longitudinal field tracking or followup studies where a group of units is closely monitored over time are of course ideal, since they provide maximal information and may be analyzed with an array of well known methods (e.g. Kalbfleisch and Prentice 1980, Lawless 1982, Cox and Oakes 1984, Andersen and Borgan 1985). Studies of this type are sometimes conducted by companies who designate and track a group of units; they tend to be expensive on a per unit basis, and the groups are usually small. Users of large fleets of units (e.g. car rental agencies) sometimes obtain similar but less complete information from maintenance and repair records.

Warranty or failure record data are a much larger source of information for many products. Manufacturers who support a warranty plan usually collect details about repairs or claims under the warranty. For example, with automobiles the date of sale of the car, the date and type of repair, and the mileage at repair, are routinely obtained. There are, however, problems with warranty or failure record data; the exact number and the ages of units in service at any point in time are often unknown, and the values of covariates are missing entirely or known only for units with claims or repairs and not for other units. In addition, the data bases maintained by many companies are accounting-driven and not set up to facilitate statistical analysis. As a result, it is necessary to supplement warranty or failure record data with information from other sources. Kalbfleisch and Lawless (1988) discuss some aspects of this problem, and others are discussed in Section 3. See also Robinson and McDonald (1987).

Surveys and cross-sectional samples are another source of information. Some surveys are conducted on an industry-wide basis; for example, companies such as Rogers Research and J.D. Power and Associates conduct surveys and sell information on product performance and customer perceptions to automobile makers. Information about reliability from such studies is usually rather limited. Cross-sectional samples or surveys to assess reliability are also carried out by manufacturers, often in response to a perceived problem. For example, Kalbfleisch and Lawless (1991a) discuss a car study on brake pad wear wherein car owners were randomly sampled from those having purchased cars from selected dealers over a particular time period. Potential problems with such studies include the possibility that selection for the study is response-related, and that data may be heavily censored or truncated. Data that are essentially cross-sectional are also sometimes available from repair or maintenance shops; Erto (1989) provides an example.

It is a high priority for manufacturers and users of products that information be obtained economically. Timeliness is also important: manufacturers in particular want to know about reliability problems and general performance as early as possible. Warranty, failure record data and survey data are cheaper to obtain than longitudinal followup data, and it is of great interest to develop methods that utilize them. The remainder of the article deals with these areas. Because many different situations can arise in practice we will not attempt a general treatment, but will instead focus on a few specific problems. These do, however, illustrate the main issues.

3   Uses of Warranty Data

3.1   Cumulative Claims Analysis In Calendar Time

For some purposes analyses of warranty claims in calendar time are very useful. We consider a discrete time framework where units enter service on calendar days $x = 0, 1, \ldots, T$. (Analogous continuous time results can be given, but most data of this type are given in terms of

4   .

days or weeks.) Let N_x be the number of units entering service on day x, and assume that the expected number of warranty claims for a single unit t days after it enters service (i.e. at "age" t) is λ_t ($t = 0, 1, \ldots$); $\Lambda_t = \sum_{u=0}^{t} \lambda_u$ is the expected number of claims up to age t, per unit. In most applications λ_t is small and can be interpreted as the probability of a claim at age t.

Suppose that data are available up to day T and that the N_x's and the number n_{xt} of age t claims on units which entered service on day x are known; note that $0 \leq x + t \leq T$. Since $E(n_{xt}) = N_x \lambda_t$, we may estimate λ_t by

$$\hat{\lambda}_t = \sum_{x=0}^{T-t} n_{xt} / \sum_{x=0}^{T-t} N_x, \qquad (3.1)$$

giving $\hat{\Lambda}_t = \sum_{u=0}^{t} \hat{\lambda}_u$. Such estimates are very useful to manufacturers for comparing warranty claims experience for different groups of units, for assessing design changes, for predicting warranty claims, and so on. For example, Figure 1 shows estimates $\hat{\Lambda}_t$ for a system on a particular car model. The data were obtained over an 18 month period during which approximately 37,000 cars entered service (were sold), generating about 5800 claims. The cars have been stratified according to which of six 2 month periods they were produced in. Figure 1 shows the striking fact that for one period (mid-November to mid-January), the cars produced have substantially higher warranty claims.

We remark that since car warranties involve both a mileage and age limit, and because cars may be temporarily or permanently withdrawn from service for certain reasons, $\hat{\lambda}_t$ underestimates the **repair** rate for age t, which is of particular interest to reliability engineers. What it estimates is λ_t, the expected number of age t warranty claims. For ages t that are not too large, this will usually be close to the repair rate.

Confidence limits for Λ_t or prediction limits for total claims can be obtained by assuming a distribution for the n_{xt}'s and using maximum likelihood, or directly from (3.1) with minimal assumptions. Extensions to allow covariates or to relax the assumption that the λ_t's are constant over calendar time are also straightforward. In practice, unfortunately, the simple methods above can rarely be used. We note two main problems, and some remedies.

1. There are usually delays in the reporting of warranty claims. For example, with the car data associated with Figure 1 delays of 20-60 days were the norm. Reasons for such delays are that claims must be verified before entry into the data base used for analysis, and this takes time. In addition, information has to be passed from the party making the claim to the manufacturer, and once at the company the claims information is often passed from one information system to another. Such delays are similar to those encountered in other areas such as insurance and disease reporting (see Kalbfleisch and Lawless 1991b).

One way to deal with reporting delays is to discard recently reported cases; this is undesirable if information is wanted promptly. Another approach is to introduce reporting delay probabilities,

$$f_r = Pr(\text{a claim is reported } r \text{ days after it occurs}), \quad r = 0, 1, 2, \ldots$$

and to obtain the data $n_{xtr} =$ the number of age t claims that have a reporting delay of r days, for units which entered service on day x. The model previously used is now replaced by $E(n_{xtr}) = N_x \lambda_t f_r$ $(0 \le x + t + r \le T)$, and if the f_r's are known, λ_t is estimated by

$$\hat{\lambda}_t = \sum_{x+r \le T-t} \sum n_{xtr} / \sum_{x=0}^{T-t} N_x F_{T-t-x}, \tag{3.2}$$

where $F_r = \sum_{u=0}^{r} f_u$. The estimates (3.2) are obtained under a Poisson model for the n_{xtr}'s (see Kalbfleisch, Lawless and Robinson 1991, who discuss reporting delays at some length), but are valid more generally. The estimates $\hat{\Lambda}_t$ in Figure 1 are based on (3.2).

2. The N_x's are often unknown. With expensive products such as cars, manufacturers are often informed more or less immediately when units are sold. However, for many products neither retailers or buyers may be compelled to inform the manufacturer of

dates of sale. When a warranty claim is made, the date of sale usually must be verified, but the total numbers entering service in different time periods is unknown.

In this case, let $N = \sum_{x=0}^{T} N_x$, $r_x = N_x/N$ and $\theta_t = N\lambda_t$. Then (ignoring reporting delays for simplicity) $E(n_{xt}) = r_x\theta_t$ $(0 \leq x + t \leq T)$, where $\sum_{x=0}^{T} r_x = 1$, and the r_x's and θ_t's are estimable. If the n_{xt}'s are independent Poisson random variables this problem is in fact identical to one for triangular contingency tables (e.g. Bishop, Fienberg and Holland 1975, section 5.2) and to nonparametric estimation problems with truncated data (e.g. Kalbfleisch and Lawless 1991b). Closed forms exist for these estimates: if $R_x = \sum_{u=0}^{x} r_u$ then

$$\hat{R}_x = \prod_{l=x+1}^{T} (1 - \frac{d_l}{N_l}) \qquad x = 0, 1, \ldots, T-1 \tag{3.3}$$

$$\hat{\theta}_t = \sum_{x=0}^{T-t} n_{xt}/\hat{R}_{T-t}, \qquad t = 1, \ldots, T \tag{3.4}$$

where

$$d_l = \sum_{t=0}^{T-l} n_{lt}, \quad D_l = \sum_{x=0}^{l} \sum_{t=0}^{T-l} n_{xt}.$$

If the total number of units N entering service over $(0, T)$ is known, we can also estimate $\lambda_t = N^{-1}\theta_t$; in any case we can estimate λ_t/Λ_T. In most situations N can be estimated from other sources, and uncertainty about N can be taken into account when confidence limits for Λ_t or prediction limits are obtained. We remark in passing that a Poisson model for the n_{xt}'s is frequently not adequate (nor is it for the n_{xtr}'s discussed previously); although (3.3) and (3.4) are valid provided $E(n_{xt}) = r_x\theta_t$, a choice of model or variance function that allows extra-Poisson variation is usually needed. A discussion of this topic will be given elsewhere.

3.2 Usage Time Analysis

It is often important to understand the relationship of failures or other events to the usage time process, and we will comment on this problem in Section 5. Here we consider the

7

analysis of failures in terms of usage time; such analyses are especially important when accumulated usage time is a major determinant of failure. Furthermore, in-house testing and analysis of product reliability is usually in terms of usage time variables, as opposed to calendar time.

Warranty data provide information about failures or repairs in terms of usage time. We consider a simple but useful class of models to illustrate key ideas. We will work with continuous time and assume that the time to some specific event such as a first failure is of interest; covariates are ignored. Let T_i denote the elapsed usage time at which this event occurs for unit i. Thus, if the unit enters service at calendar time x_i and the failure occurs at calendar time x_i^*, then $T_i = W_i(x_i^*)$. We assume also that the observational scheme implied by the warranty plan and the calendar time x^* at which the data are obtained determine an upper limit τ_i which functions as a censoring time for T_i. In other words, T_i is known exactly only if $T_i \leq \tau_i$. For example, if the warranty plan were of unlimited duration, then we would have $\tau_i = W_i(x^*)$. If the warranty was as for most cars, say with a calendar time (age) limit of 2 years and a mileage (= usage time) limit of 20,000 miles, then we would have $\tau_i = \min[W_i(x^*), W_i(x_i + 2), 20000]$.

Our objective will be to estimate the marginal distribution function $F(t)$ of usage time at failure. Throughout most of the remaining discussion we make the rather strong assumption that T_i and τ_i are independent ($T_i \perp \tau_i$). This holds, for example, when the hazard function (2.1) for failure is of the form $g(W_i(x))W_i'(x)$ and is a reasonable assumption for certain types of products. (In this case T_i is independent of the unit's usage history process; for further discussion see Section 5.) Now, consider N units entering service over $(0, x^*)$; for each there is a pair (T_i, τ_i). If N is known and $\min(T_i, \tau_i)$ and $\delta_i = I(T_i \leq \tau_i)$ are observed for each $i = 1, \ldots, N$, then the usual censored data likelihood function applies, i.e.

$$L_1 = \prod_{\delta_i=1} f(t_i) \prod_{\delta_i=0} \bar{F}(\tau_i) \tag{3.5}$$

where f and $\bar{F}$ are the marginal density and survivor function for T. Parametric or non-parametric estimation can be based on (3.5), as is well known.

8

In practice, the τ_i's are not observed for units that do not fail under warranty (i.e. with $\delta_i = 0$), so that (3.5) cannot be used. We discuss some alternatives:

1. Use only the truncated data T_i, given $T_i \leq \tau_i$, for units observed to fail. This gives the likelihood

$$L_2 = \prod_{\delta_i=1} \frac{f(t_i)}{F(\tau_i)}. \qquad (3.6)$$

   This can be quite uninformative with parametric models unless a substantial number of $F(\tau_i)$'s are large (close to one); see Kalbfleisch and Lawless (1988). Nonparametrically, only the conditional distribution $F(t)/F(\tau_{\max})$ is estimable (e.g. Woodroffe 1985), where $\tau_{\max} = \max(\tau_i)$. If the τ_i's are not observed for items that fail, this approach is of course unavailable.

2. Suppose that the marginal density for the τ_i's is known, $f_\tau(\cdot)$. Then

$$Pr(T_i > \tau_i) = \int_0^\infty f_\tau(u)\bar{F}(u)du$$

   and inference may be based on the likelihood function

$$L_3 = \prod_{\delta_i=1} f(t_i) \prod_{\delta_i=0} Pr(T_i > \tau_i). \qquad (3.7)$$

   This assumes that N is known. In practice, $f_\tau(\cdot)$ has to be estimated; with products such as automobiles this may be done through surveys or followup studies. In some cases it may be possible to estimate $f_\tau(\cdot)$ from the warranty data. For example, it is a reasonable approximation for some products to assume that $W_i(t) = \alpha_i(t-x_i)I(t \geq x_i)$, i.e. that usage time accumulates linearly at rate α_i, where the α_i's $(i = 1, \ldots, N)$ are independent random variables. The distribution of the τ_i's then follows from that of the α_i's.

3. Supplement the failure times t_i for units that fail with τ_i's from a random sample of the unfailed units. Suzuki (1985ab) and Kalbfleisch and Lawless (1988) discuss this approach and propose pseudo-likelihood estimation methods. An alternative is the

case cohort approach of Prentice (1986), for which a cohort of units is followed up and their τ_i's determined. Both approaches need further study in the case where entry times $x_1, \ldots, x_N$ are not known for all units.

The approach adopted depends upon what type of information is conveniently available. Note that when covariates are included in the models for T_i or τ_i, similar problems arise due to the fact that covariate values are generally unavailable for units that do not fail. This requires that we model the covariate distribution or use supplementary sampling of unfailed units. Kalbfleisch and Lawless (1988) deal with one type of situation.

Finally, we can develop similar approaches in cases where T_i is not independent of the usage process, so that T_i and τ_i are not independent. For example, (3.7) could be replaced by

$$\prod_{\delta_i=1} f(t_i \mid \mathcal{W}_i) \prod_{\delta_i=0} Pr(T_i > \tau_i) \tag{3.8}$$

where $\mathcal{W}_i = \{W_i(x) : \; x \geq 0\}$ is the complete usage history for unit i. Note that

$$Pr\{T_i > \tau_i\} = E_{\mathcal{W}_i}\{\bar{F}(\tau_i \mid \mathcal{W}_i)\} \tag{3.9}$$

so that a usage process model is needed. Alternatively, we could model the joint distribution of T_i and τ_i in (3.8), τ_i being a function of $\mathcal{W}_i$.

4   Cross Sectional Samples and User Surveys

Surveys of units randomly selected from those in the field are another source of information. Mail or telephone surveys are often used to assess customers' reactions to a product. Although these may generate information about product performance, they are rarely designed to provide accurate quantitative information about reliability. We will therefore discuss only surveys in which response bias is a non-problem and where fairly accurate information can

be obtained about a unit's entry time and its repair or maintenance history prior to the time of sampling. We discuss a number of points rather than attempt a comprehensive treatment.

1. If the objective is to estimate a failure time distribution, we can usually assume that as a minimum we will observe the entry time x_i, the current elapsed time τ_i (calendar or usage time) being used as the primary failure time variable, and whether or not failure has occurred prior to the time unit i is sampled. This provides so-called current status data (e.g. Diamond and McDonald 1991), which has been studied a good deal in demography and epidemiology. If we also observe the failure time T_i whenever it occurs prior to sampling (i.e. $T_i \leq \tau_i$), then we have a standard censored sample. With either current status or censored data, the possibility that T_i and τ_i are not independent should be considered. Since the τ_i's are available for all units sampled, checks on independence are possible (e.g. see Tsai 1990, Kalbfleisch and Lawless 1991ab, Keiding 1991b). Keiding (1990, 1991a) gives a general discussion of cross sectional sampling in demography and epidemiology that is helpful.

2. If samples are selected from records indicating service entry times of all field units, then it may be advantageous to select a sample stratified on entry time. With many products the manufacturer does not have such records for more than a fraction of the units. In this case, the method of sample selection needs more care; units for which there are recorded entry times may not be representative of the general field population.

3. Sometimes the amount of deterioration or wear in a unit or system can be measured at sampling. If the wear process is well understood it may be possible to impute a time to failure. Kalbfleisch and Lawless (1991a) discuss a study on the lifetime of car brake pads where this was possible.

4. The possibility of selection bias requires careful consideration. For example, as units age there may be a selection effect created by units with poorer reliability records being withdrawn from service. Vardi (1988) considers a special instance of this in connection

11

with measured deterioration. When times between events are of interest, for example the time between repairs or replacements of a particular part, problems of length bias can arise (e.g. Cox 1969, Vardi 1982) under certain observation schemes.

5. For determining deterioration or wear patterns, cross-sectional surveys are considerably less informative, but cheaper, than studies which 'observe' a unit on two or more occasions. Costs and broad objectives usually determine the type of study, but a careful examination of some specific problems would be useful.

5 Additional Remarks

The reliability or performance of a unit usually depends on the usage pattern for the unit and on various other factors, as indicated in (2.1). We have not pursued general modelling issues here, but will make some brief comments in the context of failures.

Suppose that the calendar time of a failure on unit i is denoted by X_i and the usage time at failure by T_i; for simplicity we continue to ignore covariates. We note first that if the hazard function for X_i (see (2.1)) has the form

$$h_{X_i}(x; H_x) = g(W_i(x))W_i'(x), \tag{5.1}$$

where $g(\cdot)$ is a positive-valued function, then

$$Pr(T_i \geq t \mid \mathcal{W}_i) = \exp\{-G(t)\},$$

where $G(t) = \int_0^t g(u)du$. Thus $T_i \perp \mathcal{W}_i$ and usage time is fully informative about failure. More generally, we might search for a time parameter, defined in terms of x and $W_i(x)$, that has this property. If

$$h_{X_i}(x; H_x) = h(x, \mathcal{W}_i(x)), \tag{5.2}$$

where $\mathcal{W}_i(x) = \{W_i(s) : 0 \leq s < x\}$, then if

$$Y_x^* = \int_0^x h(s, \mathcal{W}(s))ds$$

12

is strictly increasing in x, $Y_i = Y^*_{X_i}$ is such a variable.

In practice, we can formulate parametric models (5.2) and use some criterion to select a model from the parametric family. For example, Oakes (1989) considers a family where (5.2) is of the general form

$$h_{X_i}(x; H_x) = (\cos\theta + W'_i(x)\sin\theta)h(x\cos\theta + W_i(x)\sin\theta),$$

and chooses θ by fitting parametric models $h(x; \beta)$ (e.g. Weibull) via maximum likelihood. Farewell and Cox (1979) consider another approach. The formulation of models of this type, and methods for choosing among them, deserves further study. It seems best to do this in the context of problems with quite clear objectives.

Finally, we have focussed here on failures and repeated events, and types of data that provide information about them. We have not discussed environmental factors or other covariates to any extent, but these can usually be incorporated in models (2.1) in straight-forward ways. Accelerated failure time and multiplicative or additive hazard models yield familiar approaches. Other issues not discussed are multivariate aspects of reliability or product performance; these include relationships among different performance measures for a specific unit, and relations between different components or subsystems of a unit. A treatment of these problems is beyond the scope of the present article.

Acknowledgement

References

Andersen, P.K. and Borgan, O. (1985) 'Counting process models for life history data: a review' (with discussion), *Scand. J. Statist.* **12**, 97-158.

Bishop, Y.M.M., Fienberg, S.E. and Holland, P.W. (1975) *Discrete Multivariate Analysis*, Cambridge, Mass: MIT Press.

Cox, D.R. (1969) 'Some sampling problems in technology', in *New Developments in Survey Sampling*, eds. N.L. Johnson and H. Smith, Jr., New York: John Wiley, pp. 506-527.

Cox, D.R. and Oakes, D. (1984) *Analysis of Survival Data*, New York: Chapman and Hall.

Diamond, I.A. and McDonald, J.W. (1991) 'Analysis of current status data', in *Demographic Applications of Event History Analysis* (eds. J. Trussel, R. Hankinson and J. Tilton), Oxford: Oxford University Press.

Erto, P. (1989) 'Reliability assessment by repair shops via maintenance data', *J. Appl. Statist.* **16**, 303-313.

Farewell, V.T. and Cox, D.R. (1979) 'A note on multiple time scales in life testing', *Applied Statistics* **28**, 73-75.

Kalbfleisch, J.D. and Lawless, J.F. (1988) 'Estimation of reliability from field performance studies' (with discussion), *Technometrics* **30**, 365-388.

_____ (1991a) 'Truncated data arising in warranty and field performance studies, and some useful statistical methods', Submitted for publication.

_____ (1991b) 'Regression models for right truncated data with application to AIDS incubation times and reporting lags', *Statistica Sinica* **1**, 19-32.

Kalbfleisch, J.D., Lawless, J.F. and Robinson, J.A. (1991) 'Methods for the analysis and prediction of warranty claims', *Technometrics* **33**, 273-285.

Kalbfleisch, J.D. and Prentice, R.L. (1980) *The Statistical Analysis of Failure Time Data*, New York: John Wiley.

Keiding, N. (1990) 'Statistical inference in the Lexis diagram', *Phil. Trans. Roy. Soc. London A*, **332**, 487-509.

_____ (1991a) 'Age-specific incidence and prevalence: a statistical perspective' (with discussion), *J. Roy. Statist. Soc. A*, **154**, in press.

_____ (1991b) 'Independent delayed entry', this volume.

Lawless, J.F. (1982) *Statistical Models and Methods for Lifetime Data*, New York: John Wiley.

Oakes, D. (1989) 'An equivalency model for multiple time scales in survival analysis', unpublished manuscript.

Prentice, R.L. (1986) 'A case-cohort design for epidemiologic cohort studies and disease prevention trials', *Biometrika* **73**, 1-11.

Robinson, J.A. and McDonald, G.C. (1987) 'Issues related to field reliability and warranty data', *General Motors Research Labs, Report GMR-5949*.

Suzuki, K. (1985a) 'Estimation of lifetime parameters from incomplete field data', *Technometrics* **27**, 263-272.

_____ (1985b) 'Nonparametric estimation of lifetime distributions from a record of failures and follow-ups', *J. American Statist. Assoc.* **80**, 68-72.

Tsai, W.-Y. (1990) 'Testing the assumption of independence between truncated time and failure time', *Biometrika* **77**, 169-178.

Vardi, Y. (1982) 'Nonparametric estimation in the presence of length bias', *Ann. Statist.* **10**, 616-620.

_____ (1988) 'Statistical models for intercepted data', *J. American Statist. Assoc.* **83**, 183-197.

Woodroffe, M. (1985) 'Estimating a distribution function with truncated data', *Ann. Statist.* **13**, 163-177.