DESIGNED EXPERIMENTS FOR REDUCTION OF VARIATION

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ABSTRACT

This paper considers two strategies for reducing variation caused by an identified source. The strategies involve controlling the source of the variation, exploiting curvature in the response-source relationship or finding a special type of interaction between the source and a process control factor. For the final approach, we show by example how this can be achieved using either traditional designed experiments or the product arrays proposed by Taguchi.

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1. Introduction

Variation occurs in every process. Two machined shafts produced consecutively will have different diameters; the concentration of glycerine will be different in two batches of product; two measurements of the compatibility of sand taken from the same hopper will vary; the time to failure of two electric motors built to the same specifications will differ. Variation in the output of a process is caused by variation in one or more of the inputs. Different process set-ups by operators on different shifts, raw material variation, changes in the measurement system and so on, all transmit variation to the process output.

Meaning of Variation

There is considerable confusion over the meaning of variation. This is apparent in examples described by Taguchi [Taguchi and Wu (1979)] as "the larger the better" and "the smaller the better".

One aspect of variation is deviation from target. For example, if a shaft is turned, a measure of quality is the run-out (a measure of out of roundness). The ideal value is 0. A single measurement on one shaft may show a deviation or variation from this target value. Part to part consistency is another aspect of variation. Large part to part variation in run-out may cause difficulties in set-up of subsequent operations or assembly. A third aspect is within part variation. For example, it may be important that the run-out measured at each end of a shaft is close to the same. A single run-out reading may include all three aspects of variation as well as measurement error.

In simple situations, a statistical model for a response y is

$$y = f\ (process\ inputs) + e$$

where f is deterministic and the random error e has mean 0 and standard deviation which may depend on the process inputs. Confusion may arise because each aspect of variation can contribute to one or both components in the model. This seems particularly true in situations like the shaft run-out. Is the problem of excessive variation captured by the deterministic or random (or both) components of the model?

With appropriate data, measures can be defined to estimate the different aspects of variation. These measures of variation can be combined into a single performance measure or loss function. However, the causes of the different aspects of variation may be very different and it is easiest

to search for these causes using separate experiments or at least separate analyses. This point was well made by Box (1988) and several of the discussants to his paper.

Causes of Variation

Causes of variation can be classified in several ways. If the causes are ordered by the size of their contributions (largest to smallest), there is usually a Pareto effect. For example, in a study of an automatic weighing process, it was found that changes in ambient temperature and position of the material on the weighing pan were major sources of variation and dominated all others. The electronics of the process and the weighing pan were redesigned. Overall variability was reduced and the remaining variability was largely due to other causes.

Another classification is in terms of control. Process parameters such as tooling, raw material types and amount, and operating practice are under the control of the process designer and supervisor. These causes and the factors associated with them are called controllable. Many others are uncontrollable factors, or in Taguchi's terminology, noise factors [Taguchi and Wu (1979)]. Variation in incoming raw materials, tool wear, in-plant environmental factors such as humidity are all examples of noise factors. Once a product is shipped, characteristics such as time to failure and performance are affected by factors outside of the manufacturer's control such as the environment and methods of use. These are also noise factors.

Need for Variation Reduction

One dimension of quality is defined by the deviation of a characteristic from the nominal design specification. The closer the characteristic is to the nominal, the higher is the quality. Many efforts at quality improvement, especially in manufacturing, have been directed at reducing variation around the nominal value. The reasons for these efforts include reduction of scrap and rework, better fits and finishes, easier set-ups of downstream processes and so on.

For example, in the manufacture of pistons, the diameter of the piston is a critical final product characteristic. At one point, pistons were selectively fitted because of the variation in both piston and bore size. This meant that every piston produced had to be carefully sized and marked. Bores were also measured and a supply of pistons for each size of bore had to be kept available. This added tremendous complexity and cost to the process.

Another dimension of quality is the field performance of the product. A customer expects that a product will work as promised under a wide variety of conditions, some of which may not have been envisioned by the product designer. A high quality product, in this sense, is one which is robust. Its performance is insensitive to these changing conditions or noise factors.

For example, frost-free refrigerators were designed and manufactured for use in a temperate climate. The market was expanded to include many tropical countries where both the climate and culture were very different from the original market. Some customers in the tropics opened and closed their fridge very frequently, others removed all of the food and then returned a large warm mass all at once. This behaviour was very different from that of the original market. It soon became apparent that the product was not frost-free under these conditions of use. The product did not perform as promised and there were complaints of poor quality.

Most experiments described in case study books such as those published by the American Supplier Institute (1989), General Motors (1990), etc. are directed at centering a process on nominal or at increasing a characteristic, such as pull-off force, which has a one-sided specification. Taguchi (1987) in his two volumes gives some examples using experiments to reduce variation. Shoemaker et al. (1991) discuss some alternate approaches.

The goal of this paper is to discuss how experiments can be used to address problems like those discussed above in which a known source of variation has been identified. We focus on the planning of the experiment and not on the analysis of data from the experiment. If the experiment is well planned, then the analysis will be usually simple and can be approached in several ways. On the other hand if it is not well planned, no analysis can really fix the damage done.

2. Strategies for Variation Reduction

There are two basic strategies to mitigate the effects of variation transmitted by a noise factor. Suppose for example, the operating temperature of a process varies about its set point inducing variation in a quality characteristic as shown in the following figure 1(a). Here the set point is a control factor and the deviation from the set point is a noise factor.

The figure is an idealization because as the temperature varies so do many other factors which affect the process. This means that the actual variation in the quality characteristic due to temperature variation will be as shown only if there are no other sources of variation.

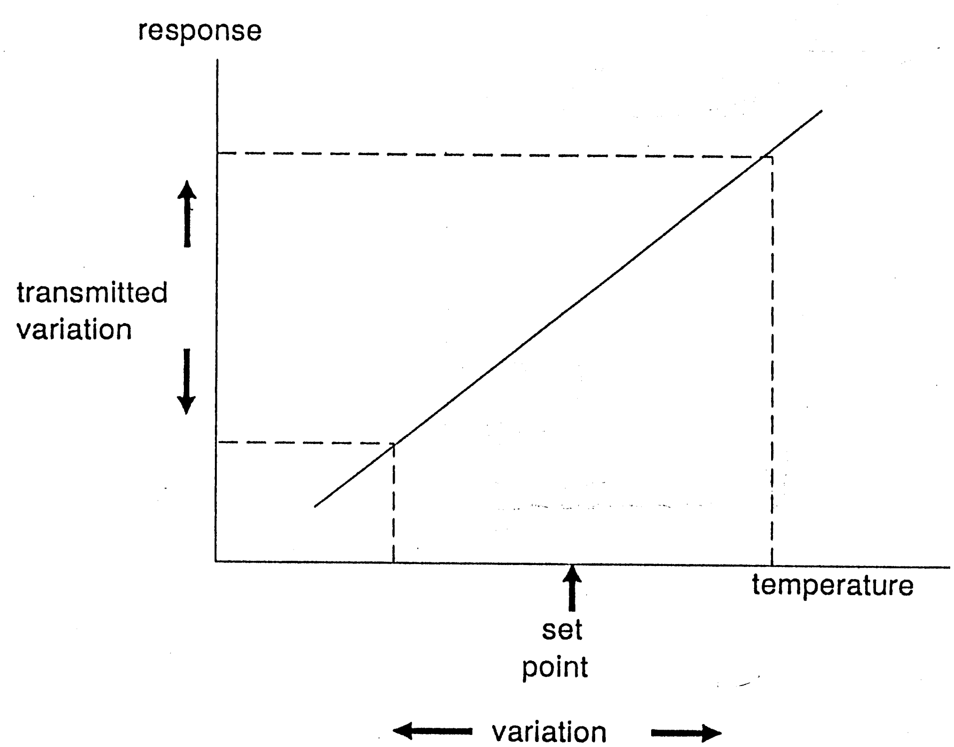

Figure 1(a) Variation in the noise factor

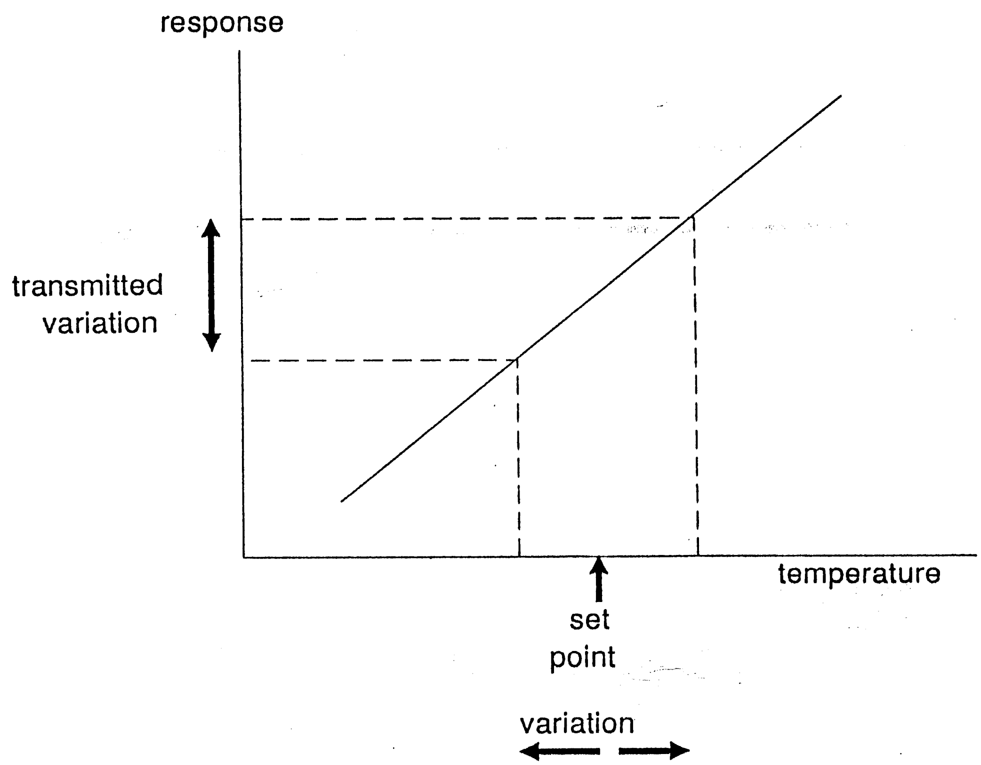

Figure 1(b) Variation in the noise reduced

The first strategy is to reduce the variation in the noise factor. If the temperature range was reduced by half without changing the set point as in figure 1(b), the transmitted variation is also reduced. If the size of an incoming part to a machining operation affects the outgoing critical dimension, then process variation can be lessened by reducing variation in the incoming size. This strategy may be difficult and expensive to implement. It corresponds to the maxim "move quality improvement upstream".

The second strategy is to ignore the noise factor and change some other more easily controllable factors. The goal is to find settings for the control factors so that the response graph is as shown in figure 2.

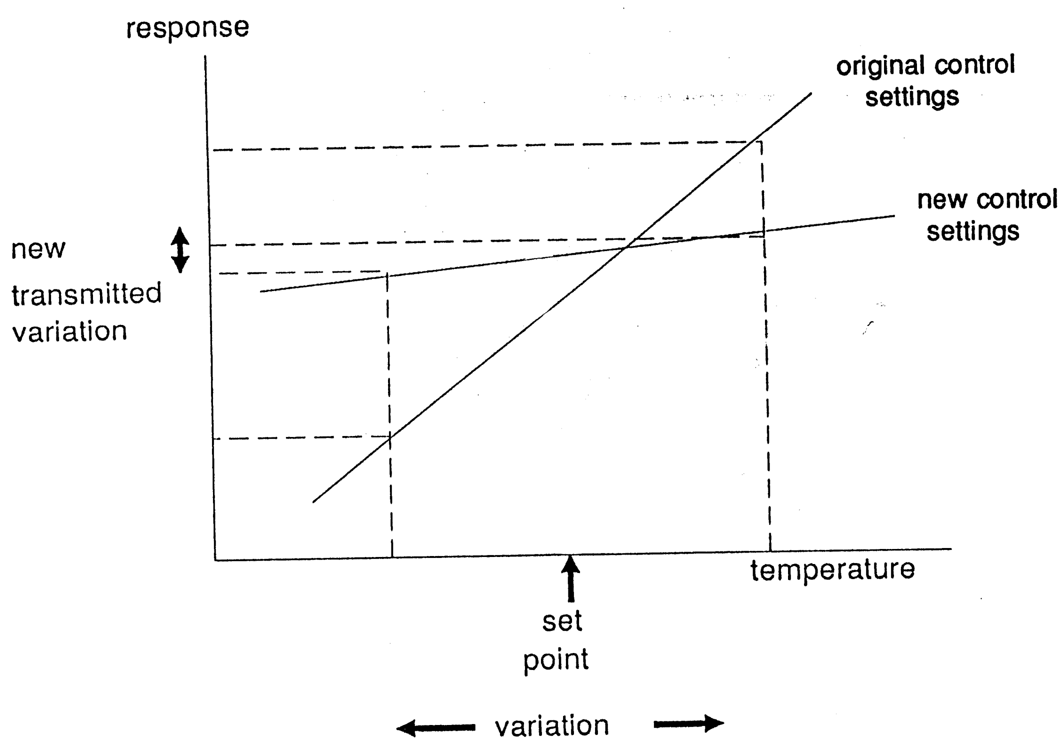

Figure 2 Variation reduction using interactions

This approach takes advantage of interactions between the noise and control factors. That is, the effect on the response of changing the noise factor is very different at different settings of the control factors. The interaction must be of the special form that flattens the response graph. Note that no action is taken with respect to the noise factor, only its effect is diminished. An example of the successful use of this strategy is given in Shoemaker et al (1991) in which the effect of facet on thickness of silicon on wafers was diminished by exploiting an interaction with the control factor, rotation method.

Problems may occur with this approach. It may be impossible to flatten the response graph. In some cases, the response curve may be flattened but shifted at the same time, so that another factor is required to move the response onto target. Changing control factors may change some other quality characteristics in an undesirable way, so that one problem is really replaced by another.

An apparent third strategy is to exploit curvature in the response graph. See, for example, Taguchi and Wu (1979, page 30). Suppose, in the example considered here, the response is related to temperature as shown in figure 3(a). Changing the set point as shown results in reduced variation due to the noise factor. However, the figure can be re-plotted to demonstrate that, in fact, we are really using the second strategy here. This becomes apparent if the variable on the horizontal axis is the noise factor, namely the deviation from the set point. See figure 3(b).

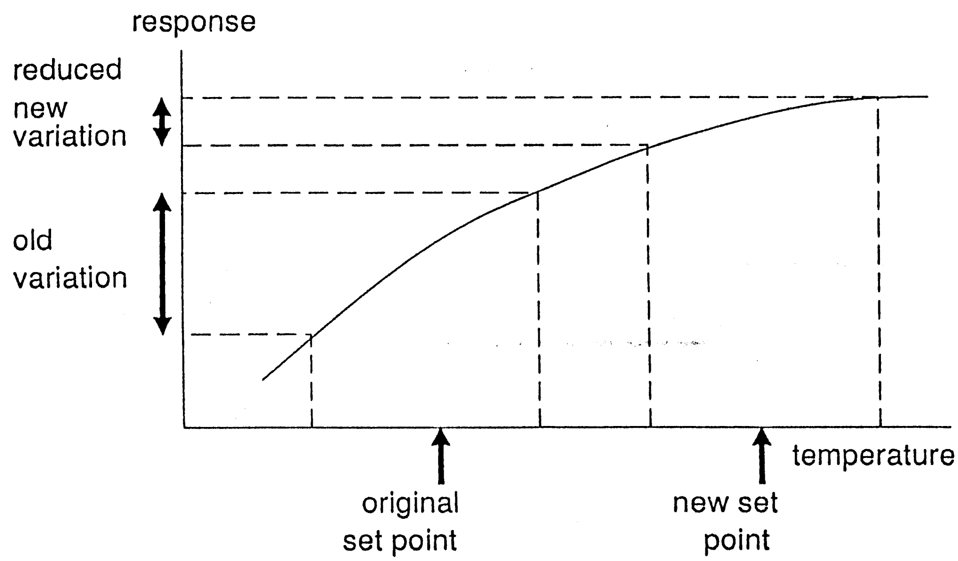

Figure 3(a). Exploiting curvature to reduce variation

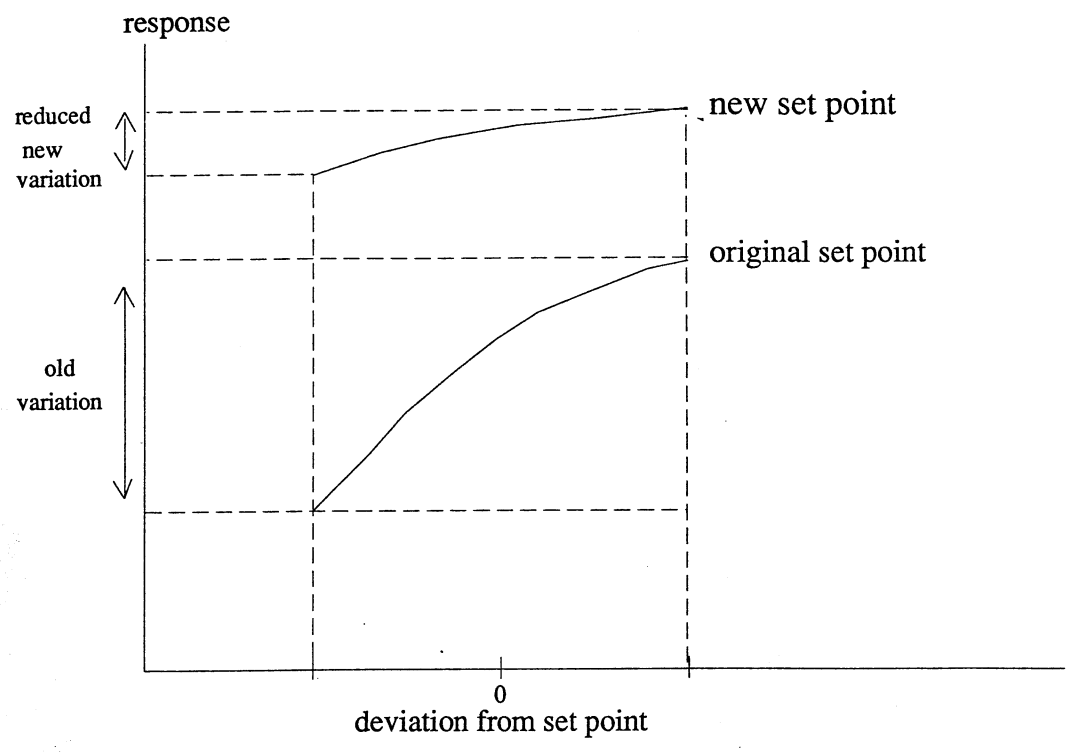

Figure 3 (b). Curvature and Interactions

A real example of this situation is given by Terasek (1985) to reduce the effect of deviation from the set point of barel temperature on a dimension of an injection molded part.

In this paper, we concentrate on some issues regarding experiments designed to exploit the second strategy to eliminate or reduce the variation transmitted by noise factors.

3. Role of Experiments

Some of the critical decisions to be made in the planning of an experiment are about the choices of responses, factors, levels of factors and experimental set-up. We consider the choice of responses, factors and experimental set-up here.

Choice of Responses

In choosing the response, the interest of the customer must be foremost. In the refrigerator example, the customers wanted a frost-free fridge. The response would seem to be the occurrence or not of a frost build-up over time. This choice of response was impractical for an experiment because it took too long to determine. Instead a surrogate response, the temperature on the cooling plate once the fridge had stabilized was used. Engineering considerations suggested that frost build-up would be eliminated if this temperature were kept within a specified range. The selection of a useful surrogate response is often critical. If the quality characteristic is an attribute and the process is currently producing a low defect rate, then a surrogate must be found unless experimental runs can use thousands of pieces.

Most experiments involve the measurement of many responses. The major problem is usually defined in terms of one response only. The other responses correspond to other important characteristics. These are monitored and analyzed as part of the experiment to ensure that changing the factors does not solve the problem of interest and create a new one.

Choice of Factors

There are different considerations for the noise and control factors included in an experiment.

Initially let us consider the noise factors. **It should be known that the noise factor is transmitting a significant amount of variation to the response.** If this is not known, then sometimes data can be collected to clarify the situation. In the piston example discussed above, the size of the piston incoming to the final grind operation was suspected to be a major noise factor. A sample of pistons was sized before and after the final grind and a scatter plot demonstrated that variation in incoming size was transmitted to the final size. **Another criteria is that the noise factor can be temporarily controlled during the course of the experiment.** In the piston example, the incoming size was a noise factor. For the experiment, pistons were sized and sorted into two categories, large and small, on the basis of incoming size.

Sometimes this second criterion cannot be met. For example, in many processes, ambient temperature is a noise factor which transmits variation but cannot be controlled even for an experimental period. One possibility is to make each run of the experiment, in which control factors are held fixed, long enough so that there will be considerable temperature variation within the run. Variation of the characteristic within the run can then be analyzed to study the effects of the control factors. For this to be effective, each run should experience approximately the same distribution of temperatures.

The levels of the noise factors should be far enough apart to capture variation of the factors during normal production or product use. The incoming piston size levels corresponded to the upper 20% and lower 20% of production sizes.

In the refrigerator example, the number of times the door was opened per hour was a noise factor. Since little information was available on actual user behaviour, the levels chosen were 4 and 8 times per hour to give a reasonable difference. Other noise factors were ambient temperature, relative humidity, food load, thermostat setting and the presence of a foreign body on the cooling plate. The temperature and humidity were controlled during the experiment by constructing a special laboratory.

Control factors correspond to product and process design specifications. There are several criteria for choosing the control factors and their levels. First, the factors must be controllable during production within a specified range. Levels for experimentation then correspond to two or more such ranges. Second, some of the control factors must interact with the noise factors or the experiment is doomed to failure. Engineering and process knowledge may suggest the existence of these interactions. Finally, the proposed levels of the control factors must be feasible for long term production.

For the piston experiment, six factors corresponding to process parameters of the final grind operation were used. One involved the use of a new coolant system that was installed temporarily for the experiment.

In the refrigerator example, there were four control factors corresponding to product design parameters that could be changed easily and relatively cheaply. The two levels for each factor corresponded to the original specification and a new proposal. The factors were chosen because it was thought that they played a major role in frost build-up when external conditions were extreme. For proprietary reasons, the factors will not be identified.

Choice of Experimental Plan

Taguchi (1987) recommends using one experimental plan for the control factors (the inner array) and one for the noise factors (the outer array). These two designs are then crossed to give the complete plan. The design is called a product array.

The plan for the refrigerator example is shown below along with the data collected. (the response is the temperature on the cooling plate - a surrogate response)

Environmental (Noise) Factors

T : Ambiant Temp. C					26	32	26	32	26	32	26	32
H : Relative Humidity %					70	70	90	90	70	70	90	90
D : Door Openings/hr.					4	4	4	4	8	8	8	8
L : Food Load					HI	LO	LO	HI	HI	LO	LO	HI
F : For'n Bdy on Clg.Plt.					Y	N	Y	N	N	Y	N	Y
S : Thermostat Setting					6	6	2	2	2	2	6	6
	1	2	3	4								
1	N	O	O	N	3.6	3.9	4.6	1.0	4.4	.1	4.4	.7
2	N	O	N	O	5.1	4.7	4.3	2.9	4.2	4.1	7.1	5.1
3	N	N	O	O	4.6	4.6	4.3	4.9	2.4	5.0	5.5	NR*
4	N	N	N	N	3.8	12.8	6.9	6.9	7.1	6.7	3.0	15.7
5	O	O	O	O	2.9	.2	-.2	-.2	-.2	-.2	-.2	NR*
6	O	O	N	N	.1	1.9	.8	1.3	5.9	5.1	0	14.7
7	O	N	O	N	.7	.8	.1	.1	.4	.2	.1	-.1
8	O	N	N	0	.2	3.4	.3	1.0	4.0	.2	5.2	NR*

* Compressor did not shut off : No Response

The four control factors were arranged in an 8 run design, shown on the left. The levels are the original design, denoted by O, and the proposed modification, given by N. The 6 noise factors were also arranged in an 8 run design, across the top. One run of the experiment consisted of building a fridge according to the recipe given in the control factor design and then testing that fridge under the conditions specified by the noise factor design. In total, there were 8 X 8 = 64 runs. The number given in the table is the response measured with the control factor determined by the row and the noise factors determined by the column.

This experimental plan has a major strong point. It is possible to measure separately the interaction between every control factor and noise factor. This is important because the object of the experiment is to find noise by control interactions which will decrease the effect of the noise factor and hence reduce transmitted variation. One drawback to this design is its large size. Another is that various control factor effects cannot be estimated separately. This is more serious in the next example.

For the piston experiment, the experimental plan is shown below. The six control factors are labelled A to F and the noise factor S. The two levels for the control factors are denoted by L and H. No data are given.

	Control factors						S	
Run	A	B	C	D	E	F	small	large
1	L	L	H	L	H	H		
2	H	L	L	L	L	H		
3	L	H	L	L	H	L		
4	H	H	H	L	L	L	data	
5	L	L	H	H	L	L		
6	H	L	L	H	H	L		
7	L	H	L	H	L	H		
8	H	H	H	H	H	H		

The control factors are arranged in an 8 run design and the noise factor in a 2 run design. The crossed design has 16 runs in total. For each set-up of the grinding operation given by the control factor design, small and large pistons were processed and then measured.

Since the 6 control factors are arranged in an 8 run design, some of the control by control factor interactions are confounded with control factor main effects. This means, for example, that if the experimental data indicates that factor A has a large effect on the process mean, this effect can equally well be attributed to a combined effect of B and C or D and E. Using A to adjust the process may be completely ineffective; there is no way to tell using the experimental results. This confusion of effects is called confounding. With this design, the main effect of factor A is confounded with both the BC and DE interactions.

An alternate design for the piston example is to arrange the 7 factors in a single 16 run design as show below. [For more discussion of such designs, see Box, Hunter and Hunter (1978)]

Run	A	B	C	D	E	F	S
1	L	L	L	L	L	L	L
2	H	L	L	H	H	L	L
3	L	H	L	H	L	H	L
4	H	H	L	L	H	H	L
5	L	L	H	H	H	H	L
6	H	L	H	L	L	H	L
7	L	H	H	L	H	L	L
8	H	H	H	H	L	L	L
9	L	L	L	L	H	H	H
10	H	L	L	H	L	H	H
11	L	H	L	H	H	L	H
12	H	H	L	L	L	L	H
13	L	L	H	H	L	L	H
14	H	L	H	L	H	L	H
15	L	H	H	L	L	H	H
16	H	H	H	H	H	H	H

The second design is different from the first. It cannot be rearranged to get a product array. For example, the first run with small incoming pistons has all the control factors at the low level. There is no run with large incoming pistons having the same combination of control factors. This design confounds control by control factor interactions with noise by control factor interactions. For example, FS is confounded with AD and BC.

Choosing between these designs may be difficult. Since the original purpose of the experiment is to reduce variation by exploiting noise by control factor interactions, the product array is desirable. However, if it is suspected that one of the control factors will be needed to adjust the level of the process, then the single array is attractive. To satisfy both requirements, an experiment with more runs is the only solution.

In general, a single array may be able to meet both criteria. Suppose that there are c control factors and n noise factors, all at two levels. A design is required in which all cn noise by control factor interactions can be estimated separately. Furthermore, the control by control factor interactions should not be confounded with control factor main effects. The following table shows the minimum size design required. A bracketed number gives the smallest product array which will meet the same goal.

	n		
c	1	2	3
1	4		
2	8	16	
3	8(16)	16(32)	32
4	16	32	32(64)
5	16(32)	32(64)	64
6	16(32)	64	64
7	16(32)	64	64

There are many realistic cases where a single array can meet the requirements and save runs. For example, if there are 3 control and 2 noise factors, a single array with 16 runs can provide estimates of all main effects and interactions. The smallest product array to meet the same specifications is a 32 run design.

Another consideration is the cost of making the runs, which are typically not made in random order. In the piston example, a large number of pistons were sorted and then, for each process set-up, small and large pistons were run. Changing the noise factor was relatively cheap within each control factor combination. With the refrigerators, the 8 model fridges were built and then tested simultaneously under the different noise conditions. Here the major cost was changing the noise factor.

The standard recommendation is to carry out the experimental runs in a random order. This is often not feasible because of the cost. However, the price is that the lack of randomization can cause major problems in interpreting the experimental results if the overall process is instable.

4. Concluding Remarks

This paper discusses two strategies for reducing transmitted variation due to an identified cause. Each strategy is important in particular situations, although for economic reasons it is usually appropriate to try the experimental methods before spending capital to control the noise factor corresponding to the cause. In the refrigerator example, the second strategy exploiting interactions between control and noise factors led to a new design which was frost-free under a wide variety of operating conditions. The same strategy was used in the piston example but was ineffective. No interactions of the required type were found. It did not seem possible to operate the final grind operation so that it was insensitive to incoming part size.

The paper has emphasized the set-up and design of experiments for variation reduction and spent no time on the analysis of the results. There has been considerable discussion over the analysis of experiments using a product array. See for example, Box (1988). Our feeling is that the choices of responses, factors, levels and experimental plan are the critical decisions. If these decisions are not correctly made, no analysis can repair the mistake. If the experiment is well planned and executed, then two or more analyses can be conducted on the data produced. In many instances, different analyses will lead to the same recommendations.

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