RELIABILITY IMPROVEMENT VIA TAGUCHI'S ROBUST DESIGN

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ABSTRACT

Taguchi's robust design strategy, whose aim is to make processes and products insensitive to factors which are hard or impossible to control (termed noise factors), is an important paradigm for improving products and processes. We present an overview of the strategy and tactics for robust design and demonstrate its usefulness for reliability improvement. Two important components of robust design are a criterion for assessing the effect of the noise factors and experimentation according to specialized experimental plans. Recent criticism of Taguchi's criterion and his analysis of its estimates has led to an alternative approach of modeling the response directly. We give additional reasons for using this response-model approach in the context of reliability improvement. Using the model for the response, appropriate criteria for assessing the effect of the noise factors can then be evaluated. We consider an actual experiment and reanalyze its data to illustrate these ideas and methods.

Key words: Censoring, Control and noise factors, Designed experiments, Loss-model and response-model approaches, Maximum likelihood estimation, Parameter design, Product array, Signal-to-noise ratios.

1. INTRODUCTION

Genichi Taguchi's [10] strategy of designing a product or process so that its performance is insensitive to noise factors, i.e., manufacturing factors that cannot easily be controlled or factors with which one has little control over such as environmental conditions in which the product is used, has attracted much attention in recent years. Such products or processes are said to be *robust* to the noise factors. His robust design strategy appears to have received little attention in the reliability field, however. Hence, the motivation for this paper, whose objectives are to present these important ideas and show how they are useful for reliability improvement.

The paper is organized as follows. First, we present an overview of robust design in Section 2, which discusses its strategy and tactics. In particular, two important components of robust design are a criterion for assessing the effect of the noise factors and experimentation which use specialized experimental plans. Two approaches for analyzing the resulting experimental data are discussed: (i) estimating the criterion and then modeling it or (ii) modeling the response and then using it to evaluate the criterion. The latter approach is preferred and is referred to as the response-model approach.

In Section 3, robust design is discussed in the context of reliability improvement. Taguchi's criterion for assessing noise factor effects in the reliability context, the larger-the-better signal-to-noise ratio, is presented. Here, we note several criticisms of his criterion which provide additional reasons for using the response-model approach in this context. Alternative criteria are also considered.

In Section 4, a reanalysis of an experiment reported by Montmarquet [7] illustrates these ideas and methods. Note that experiments have been done in the past for studying how factors affect reliability (e.g., Zelen [12]); Taguchi appears to have been the first to address the issue of robustness to noise factors.

Notation

LTB	larger-the-better
S/N	signal-to-noise
$\mathbf{x}_{control}$	vector of control factors
$\mathbf{x}_{noise}$	vector of noise factors
$l(\cdot)$	loss function
$L(\mathbf{x}_{control})$	loss at $\mathbf{x}_{control}$
Y	response
$Y(\mathbf{x}_{control}, \mathbf{x}_{noise})$	response at $(\mathbf{x}_{control}, \mathbf{x}_{noise})$
Y_i	ith response from noise array
y_i	ith response
$f(\cdot)$	joint pdf of noise factors
τ	target
ML	maximum likelihood
MLE	ML estimate
C	control factor main effect
N	noise factor main effect
$N \times N$	noise factor by noise factor interaction
$C \times N$	control factor by noise factor interaction
$C \times C$	control factor by control factor interaction
$\log_{10}$	logarithm base 10
log	natural logarithm
$\mathbf{x}_i$	ith vector of covariate values
β	regression parameters or effects
σ	scale parameter
ϵ_i	ith error associated with Y_i
$\{i \in C\}$	set of censored observations
$\{i \in D\}$	set of actual failures
A_j	jth level of factor A
Std. Err.	standard error

2. AN OVERVIEW OF ROBUST DESIGN

Taguchi's robust design is also referred to as parameter design because its objective is to find levels of engineering parameters (called control factors) that yield a robust product or process, i.e., that minimize the effect of the noise factors. Robust design is therefore strikingly different than the traditional approach of handling sources of manufacturing variation by control which can be costly, e.g., purchasing expensive state-of-the-art equipment. Kackar [4] recounts the now famous story of the Ina Tile Company's first encounter with robust design. The company was faced with reducing an unacceptable amount of variation in their tiles' size caused by an uneven temperature distribution in the kiln. Rather than purchasing an expensive kiln which would have better controlled the temperature distribution, it was found through designed experiments that increasing the lime content in the tile formulation decreased the tile size variation by a factor of ten. That is, a tile formulation was found that was insensitive to the existing oven's uneven temperature distribution.

In the tile example, two important components of robust design were mentioned: a criterion that assesses the effect of the noise factors (i.e., variation) and experimentation (i.e., designed experiments). First, we discuss a criterion for assessing the effect of noise factors. Following the notation used in Welch, Yu, Kang and Sacks [11], a criterion for assessing the effect of the noise factors (termed the loss statistic or simply loss) at a particular combination of control factor levels $\mathbf{x}_{control}$ can be defined for a general loss function $l(\cdot)$ as:

$$L(\mathbf{x}_{control}) = \int l(Y(\mathbf{x}_{control}, \mathbf{x}_{noise}) f(\mathbf{x}_{noise}) \partial \mathbf{x}_{noise} \ , \tag{1}$$

where $Y(\mathbf{x}_{control}, \mathbf{x}_{noise})$ is the random quality characteristic observed at a particular combination of control and noise factor levels $(\mathbf{x}_{control}, \mathbf{x}_{noise})$ and $f(\cdot)$ is the joint pdf of the noise factors $\mathbf{x}_{noise}$. In other words, the loss statistic is the expected loss over the distribution of noise factors. In this formulation, the objective of robust or parameter design then is to find a product or process design $\mathbf{x}_{control}$ with minimum loss. As an example of a loss function, take the situation where the quality characteristic has an ideal value known as the target τ. Take the loss function to be squared error loss or squared distance from target. Then $l(Y) = (Y - \tau)^2$, with $L(\mathbf{x}_{control})$ in (1) being the average squared error (over the noise factors' joint distribution) at a given $\mathbf{x}_{control}$.

The other important component of robust design is experimentation; i.e., Taguchi's tactics for robust design involve estimating the loss (1) using data collected according to specialized experimental plans referred to as product (or crossed) arrays. A product array consist of two plans or arrays, where the control factors are varied according to one array termed the "control array" and the noise factors are varied according to the other termed the "noise array". That is, a row of an array determines the combination of factors levels to be experimented at. The name product or crossed array arises because all the noise factor combinations specified by the noise array are experimented at every combination of the control factors specified by the control array. The experiment discussed in later sections with 11 control factors and five noise factors used the control and noise arrays presented in Tables 1 and 2, respectively. The control array gives 16 combinations of the 11 control factors; at each of these control factor combinations, the noise factors were varied according to the eight combinations given in the noise array. The product array, therefore, specifies 128 ($= 16 \times 8$) control and noise factor combinations, at which the quality characteristic (e.g., lifetime) is observed. Note that while a noise factor is difficult or impractical to control in production or in use, for purposes of the experiment (i.e., to learn about the effect of the noise factors), the noise factors need to be controlled during the experiment. Recent work by Freeny and Nair [3] suggests that this is not always necessary but is not discussed here.

Tables 1 and 2 about here.

There are two approaches for analyzing the resulting product array data. Taguchi [10] originally proposed estimating $L(\mathbf{x}_{control})$ for each $\mathbf{x}_{control}$ specified by the control array. The estimated loss statistics, which he generically calls signal-to-noise ratios, are obtained using the data from varying the noise factors according to the noise array and then modeled as a function of the control factors. Shoemaker, Tsui and Wu [8] refer to this as the loss-model approach. Alternatively, Welch et al. [11] proposed modeling the response Y directly as a function of both the control and noise factors and then evaluating the loss using the estimated response model. Welch et al.'s [11] rationale for their approach, which Shoemaker et al. [8] refer to as the response-model approach, was that it would be more likely to find a simple model for the response than one for the much more complicated estimated loss. Examples in Welch et al. [11] and Shoemaker et al. [8] provide convincing evidence for preferring the response-model approach for this reason and show that the approach also provides more information. We will give additional reasons for using the response-model approach in the context of reliability improvement in Section 3.

Next, we elaborate on the response-model approach. The product array data is fit by a model consisting of all C main effects (possibly some $C \times C$ interactions), all $C \times N$ interactions and all N main effects (possibly some $N \times N$ interactions). The fact that the loss (1) changes for different control factor combinations means that interactions between the control and noise factors must exist. Thus, $C \times N$ interactions are necessary for there to be an opportunity for robustness. For example, see figure 1a which displays a simplified relationship between a response Y and one control factor (at two levels) and one noise factor (over an interval). The interaction is evident since the response over the noise factor interval depends on the control factor level; figure 1a shows that at control factor level 2, the effect of the noise factor is substantially smaller than at control factor level 1. Thus, robust design exploits the existence of interactions between control and noise factors. Note that having a $C \times N$ interaction is not sufficient for an opportunity for robustness as is shown in figure 1b; the magnitude of the change over the noise factor interval at both levels of the control factor is the same. Consequently, an N main effect is also needed which explains the inclusion of both $C \times N$ interactions and N main effects in the model. The C main effects and $C \times C$ interactions indicate the general response value about which the response varies as the noise factors vary according to their distribution; the amount of variation depends on the magnitudes of the N main effects and $C \times N$ interactions. See Box and Jones [2] for a mathematical derivation of this response model.

Finally, the response-model approach has led to the proposal that alternative experimental plans be used. For example, Welch et al. [11] proposed using a single plan or array for both the control and noise factors. Shoemaker et al. [8], who referred to the single array as a combined array, explored the economic advantages of combined arrays over product arrays.

3. APPLICATION TO RELIABILITY IMPROVEMENT

To illustrate the usefulness of robust design for reliability improvement, consider an experiment for improving the lifetime of drill bits (i.e., number of holes drilled before breakage) used in fabricating multilayer printed circuit boards as reported by Montmarquet [7]. In designing multilayer circuit boards, small diameter holes are desired because they allow more room for the circuitry. The strength of small diameter drill bits is greatly reduced, however, so that breakage becomes a serious problem; broken bits cannot be removed from the boards requiring the boards to be scrapped at a cost of $200-$600. Consequently, identification of significant factors affecting bit breakage is one of the experimental objectives. Also, factor levels need to be chosen that give a sufficiently long lifetime to make the fabrication of a circuit board design feasible.

A product array consisting of a 16 run control array (Table 1) and an eight run noise array (Table 2) was used to study 11 control factors (A at four levels and B–J and L at two levels) and five noise factors (M–Q at two levels). The control factors were selected material composition and geometric characteristics of drill bits such as the carbide cobalt percentage in a drill bit (factor A) and radial rake (factor F). The noise factors dealt with characteristics of different types of multilayer circuit boards that would be drilled such as board material (factor O) and number of layers in a board (factor P). See Montmarquet [7] for more details. Note that a run (i.e., a particular combination of control factors levels being used at a particular combination of noise factor levels) was stopped after 3,000 holes were drilled; 14 (out of 128 or 11%) of the tested drill bits did not fail resulting in censored data.

With the drill bit experiment in mind, we consider robust design in the context of reliability improvement, i.e., identifying control factor combinations whose reliability is insensitive to the noise factors. First, we discuss Taguchi's specific criterion for assessing the effect of the noise factors which he termed the LTB S/N ratio. For a response Y such as lifetime for which large values are desired, Taguchi proposed using the loss function $l(Y) = [(1/Y) - 0]^2 = 1/Y^2$ in (1). It computes a squared distance on the reciprocal scale between the response and the ideal but unattainable value ∞ (whose reciprocal is zero). The loss in (1) is then estimated for each control factor combination (Table 1) using the eight lifetimes corresponding to the noise factor combinations (Table 2). Denoting these eight lifetimes by $Y_1, \ldots, Y_8$, the loss in (1) is estimated by $\sum_{i=1}^{8}(1/Y_i^2)/8$. Taguchi typically applies $-10log_{10}$ which makes larger values better, thereby giving the LTB S/N ratio as $-10log_{10}(\sum_{i=1}^{8}(1/Y_i^2)/8)$. Once the LTB S/N ratios are computed, they are then modeled as a function of the control factors.

In the previous section, a rationale for the response-model approach was given. A complementary view by Box [1] is that by combining the observations from the noise array, the LTB S/N ratio hides important information. For example, take the case of a single noise factor which is experimented at low, middle and high values as displayed in figure 2. Note that the LTB S/N ratio is the same for all four graphs which contain very different information about the effect of the noise factors. (Figure 2a has no variability. Figure 2b has the same mean as figure 2a, but more variability. Figure 2c has a higher mean than figures 2a and 2b, but more variability. Figures 2a, 2b and 2c are monotonic while figure 2d is not.) A response-model approach using an appropriate experimental plan could distinguish between these different cases.

In the reliability context, there is a more compelling reason for using the response-model approach since censored data naturally arise; i.e., all the units may not fail by the end of the experiment. Consequently, the LTB S/N ratio cannot even be properly evaluated; to use the censoring time (i.e., the duration of the experiment) could be misleading. Next, we discuss how existing statistical methods which handle censored data can be used to implement the response-model approach.

In the following, we assume a Weibull regression model for the lifetimes (Chapter 6 of

Lawless [5]). A convenient representation for this model is:

$$\log(y_i) = \mathbf{x}_i{}^T \beta + \sigma \epsilon_i,\ i = 1,\ \ldots,\ n, \tag{2}$$

where the $\{y_i\}$ are the observed lifetimes, the $\{\mathbf{x}_i\}$ are the corresponding vectors of covariates values, β is the vector of location parameters, σ is the scale parameter and the $\{\epsilon_i\}$ are i.i.d. standard extreme-value r.v.'s, whose probability density function (pdf) and survivor function (Sf) are $exp(w - exp(w))$ and $exp(exp(-w))$, respectively. That is, the $\{y_i\}$ follow a Weibull distribution. In the robust design context, the covariates consist of an intercept, the C main effects, possibly some $C \times C$ interactions, the $C \times N$ interactions, the N main effects and possibly some $N \times N$ interactions.

Standard ML estimation methodology can easily handle both failure and censored data. The MLE's for (β, σ) are found by maximizing the following likelihood:

$$L(\beta,\sigma) = \prod_{i \in D} (1/\sigma) exp([(y_i - \mathbf{x}_i{}^T\beta)/\sigma] - exp((y_i - \mathbf{x}_i{}^T\beta)/\sigma)) \prod_{i \in C} exp(exp(-(y_i - \mathbf{x}_i{}^T\beta)/\sigma)) \tag{3}$$

Note that $\{i \in D\}$ denotes those observations which are failures and $\{i \in C\}$ denotes those observations which are censored. Standard errors for the MLE's can also be obtained (Lawless [5]). Various commercially available software perform these computations. For example, we used SURVIVAL, the SYSTAT survival analysis module (Steinberg and Colla [9]) to reanalyze the drill bit experiment.

Once the response has been modeled, recommendations for setting the important control factors need to be made. For a simple model with few noise factors, they may be apparent from inspection of the model directly; i.e., by observing what the significant effects are and their magnitudes. See Shoemaker et al. [8] for an example. For complicated models, as is the case for the drill bit experiment, however, this approach may be tedious if not difficult.

An alternative is to specify some meaningful criteria and use the model to evaluate them. For example, the loss in (1) can be calculated for a specified distribution of the noise factors. In practice, because it may be difficult to specify such a distribution, one might simply evaluate the criterion over the noise combinations specified by a noise array. Note that the noise array used in the experiment need not be used here; in fact, one can use a full factorial plan (i.e., all possible combinations). The noise combinations can also be weighted appropriately to reflect their probabilities of occurrence. Similarly, the loss in (1) can be calculated for all possible combinations of the control factors.

In Section 2, we noted Box's [1] criticism of the LTB S/N ratios; as he pointed out, they hide important information about the general level and amount of variation of the observations. Consequently, other appropriate criteria could be considered and then evaluated. For example, in the robust design framework, besides requiring high reliability on average, one also desires as little dependence as possible on the noise factors (i.e., small variation). Thus, the mean and variance (or standard deviation) of the response over the noise factor distribution might be evaluated. If there is no control factor combination that simultaneously maximizes the mean and minimizes the variance, then tradeoffs between the two need to be made. Based on a worst case approach, the minimum mean response over the noise factor distribution provides another criterion that could be evaluated. These additional criteria as well as the LTB S/N ratio are compared in the reanalysis of the drill bit experiment presented next.

4. REANALYSIS OF DRILL BIT EXPERIMENT

Based on the response-model approach, a Weibull regression model consisting of an intercept, C main effects, one $C \times C$ interaction ($D \times E$), N main effects, two $N \times N$ interactions ($M \times P$, $M \times Q$) and all the $C \times N$ interactions was fit by ML estimation as described in the previous section. Tables 3a and 3b present the MLE's and their respective standard errors with the significant effects in bold face. Note that factor A had four levels, so the main effect is represented by linear, quadratic and cubic components and denoted by A_L, A_Q and A_C, respectively. Also, the intercept is denoted by Int.

Tables 3a and 3b about here.

As can be seen from Tables 3a and 3b, the relationship between the response and the control and noise factors is too complicated to make control factor level recommendations simply by inspecting the model. Consequently, the mean, standard deviation, minimum mean and LTB S/N ratio over all possible combinations of noise factors ($32 = 2^5$) were evaluated at each of all possible combinations of control factor ($4096 = 4 \times 2^{10}$) and then ranked appropriately (out of 4096, with 1 being the best). Table 4 presents the best five control factor combinations for each criterion along with the other criteria and their ranks. Several observations can be made: (i) the combination least sensitive to the noise factors (i.e., smallest standard deviation) ranks rather poorly according to the other criteria, especially the mean reliability; (ii) the other three criteria identify many of the same combinations; in fact here, the LTB S/N ratio does quite well — the approximate relation between the LTB S/N ratio and the mean and standard deviation criteria given by Maghsoodloo [6] can be used to explain why the LTB S/N ratio tends to be driven by the mean criterion; this is especially true here, where values for the mean criterion are much larger than that for the standard deviation criterion; (iii) there is little difference between the top few combinations. From Table 4, a good choice of factor levels would then be $A_4D_2B_2C_2F_1G_2H_1I_2E_1J_2L_2$, where the subscripts denote the recommended level of their respective factors. Note that this combination is also rather robust to the noise factors.

Table 4 about here.

In the original analysis (Montmarquet [7]) which modeled the LTB S/N ratios (using the censoring times as actual failure times), factors A, D, C, F, H, E, J and L were identified as important, which resulted in the recommendation $A_4D_2C_2F_1H_1E_1J_2L_2$. Based on the estimated response model, Table 5 presents the criteria for this combination with the remaining factors B, G and I being allowed to vary. Note that the odd combinations which include the best combination recommended by the response-model approach are relatively better than the even ones; this can be explained by factor I which has a much larger main effect than factors B or G. Thus, the response-model approach provides additional important information about this experiment that the loss-model approach hid.

Table 5 about here.

ACKNOWLEDGEMENTS

I thank Will Welch for helpful comments on an earlier version. This research was supported by General Motors of Canada Limited, the Manufacturing Research Corporation of Ontario, and the Natural Sciences and Engineering Research Council of Canada.

REFERENCES

1. G. Box, "Signal-to-Noise Ratios, Performance Criteria, and Transformations (With Discussion)," *Technometrics,* vol 90, 1988 Feb, pp 1–40.

2. G. Box, S. Jones, " Designing Products that are Robust to the Environment," University of Wisconsin–Madison Center for Quality and Productivity Improvement Report 56, 1990.

3. A. E. Freeny, V. N. Nair, "Robust Parameter Design with Uncontrolled Noise Variables," AT&T Bell Laboratories Statistical Research Report No. 95, 1991.

4. R. N. Kackar, "Off-Line Quality Control, Parameter Design, and the Taguchi Method (With Discussion)," *Journal of Quality Technology,* vol 17, 1985 Apr, pp 176–209.

5. J. F. Lawless, *Statistical Models and Methods for Lifetime Data,* 1982; John Wiley & Sons, Inc.

6. S. Maghsoodloo, "The Exact Relation of Taguchi's Signal-to-Noise Ratio to His Quality Loss Function," *Journal of Quality Technology,* vol 22, 1990 Jan, pp 57–67.

7. F. Montmarquet, "Printed Circuit Drill Bit Design Optimization Using Taguchi's Methods – .013″ Diameter Bits" *Sixth Symposium on Taguchi Methods,* 1988; American Supplier Institute, Inc., pp 70–77.

8. A. C. Shoemaker, K. L. Tsui, C. F. J. Wu, "Economical Experimentation Methods for Robust Design," *Technometrics,* vol 33, 1991 Nov, pp 415–427.

9. D. Steinberg, P. Colla, *SURVIVAL: a Supplementary Module for SYSTAT,* 1988; SYSTAT Inc.

10. G. Taguchi, *Introduction to Quality Engineering,* 1986; Asian Productivity Organisation.

11. W. J. Welch, T. K. Yu, S. M. Kang, J. Sacks, "Computer Experiments for Quality Control by Parameter Design," *Journal of Quality Technology,* vol 22, 1990 Jan, pp 15–22.

12. M. Zelen, "Factorial Experiments in Life Testing," *Technometrics,* vol 1, 1959 Aug, pp 269–288.

Table 1: Control Array for Drill Bit Experiment

row	A	D	B	C	F	G	H	I	E	J	L
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	2	2	2	2	1	1	1	1	2	2
4	1	2	2	2	2	2	2	2	2	1	1
5	2	1	1	2	2	1	1	2	2	1	2
6	2	1	1	2	2	2	2	1	1	2	1
7	2	2	2	1	1	1	1	2	2	2	1
8	2	2	2	1	1	2	2	1	1	1	2
9	3	1	2	1	2	1	2	1	2	1	1
10	3	1	2	1	2	2	1	2	1	2	2
11	3	2	1	2	1	1	2	1	2	2	2
12	3	2	1	2	1	2	1	2	1	1	1
13	4	1	2	2	1	1	2	2	1	1	2
14	4	1	2	2	1	2	1	1	2	2	1
15	4	2	1	1	2	1	2	2	1	2	1
16	4	2	1	1	2	2	1	1	2	1	2

Table 2: Noise Array for Drill Bit Experiment

row	M	N	O	P	Q
1	1	1	1	1	1
2	1	1	1	2	2
3	1	2	2	1	2
4	1	2	2	2	1
5	2	1	2	1	2
6	2	1	2	2	1
7	2	2	1	1	1
8	2	2	1	2	2

Table 3a: MLEs and Standard Errors for Drill Bit Experiment

Effect	MLE	Std. Err.	Effect	MLE	Std. Err.
Int	6.182	0.047	E	0.051	0.043
$\mathbf{A_L}$	0.279	0.021	**J**	-0.231	0.043
$\mathbf{A_Q}$	-0.268	0.043	**L**	-0.272	0.043
$\mathbf{A_C}$	0.071	0.018	**DE**	-0.225	0.041
D	-0.265	0.043	**M**	0.179	0.058
B	-0.048	0.043	**N**	0.136	0.047
C	-0.194	0.043	**O**	0.898	0.059
F	0.154	0.042	**P**	0.862	0.057
G	0.132	0.048	**MP**	0.237	0.057
H	0.218	0.048	**Q**	0.548	0.057
I	-0.272	0.044	MQ	0.036	0.057

Table 3b: MLEs and Standard Errors for Drill Bit Experiment

Effect	MLE	Std. Err.	Effect	MLE	Std. Err.
A_LM	0.030	0.027	HO	0.107	0.061
A_QM	0.097	0.060	**IO**	-0.429	0.054
A_CM	-0.040	0.023	**EO**	-0.376	0.061
DM	0.086	0.059	JO	-0.039	0.059
BM	0.005	0.046	**LO**	0.294	0.059
CM	0.038	0.059	A_LP	-0.013	0.027
FM	-0.073	0.046	**A_QP**	-0.123	0.061
GM	0.236	0.054	A_CP	0.035	0.023
HM	0.011	0.061	**DP**	0.269	0.059
IM	-0.090	0.054	**BP**	0.213	0.048
EM	0.095	0.061	**CP**	-0.119	0.059
JM	0.149	0.059	FP	-0.070	0.048
LM	0.123	0.059	GP	0.022	0.054
A_LN	-0.047	0.021	HP	-0.080	0.061
A_QN	0.076	0.043	**IP**	0.195	0.054
A_CN	0.003	0.018	**EP**	0.143	0.061
DN	0.017	0.043	**JP**	0.156	0.060
BN	-0.094	0.042	**LP**	-0.194	0.060
CN	-0.046	0.043	A_LQ	0.007	0.027
FN	0.025	0.042	**A_QQ**	-0.174	0.061
GN	0.049	0.048	A_CQ	0.031	0.023
HN	-0.012	0.047	DQ	0.037	0.059
IN	-0.111	0.044	BQ	-0.037	0.048
EN	0.019	0.043	CQ	-0.060	0.059
JN	-0.065	0.043	FQ	-0.031	0.048
LN	-0.054	0.043	GQ	0.139	0.054
A_LO	0.024	0.027	**HQ**	0.079	0.061
A_QO	0.034	0.061	**IQ**	-0.117	0.054
A_CO	0.006	0.024	EQ	-0.184	0.061
DO	0.181	0.058	JQ	0.042	0.060
BO	-0.136	0.047	**LQ**	0.202	0.060
CO	0.026	0.058	σ	0.350	0.030
FO	-0.061	0.047			
GO	0.277	0.054			

Table 4: Best Combination Levels for Various Criteria

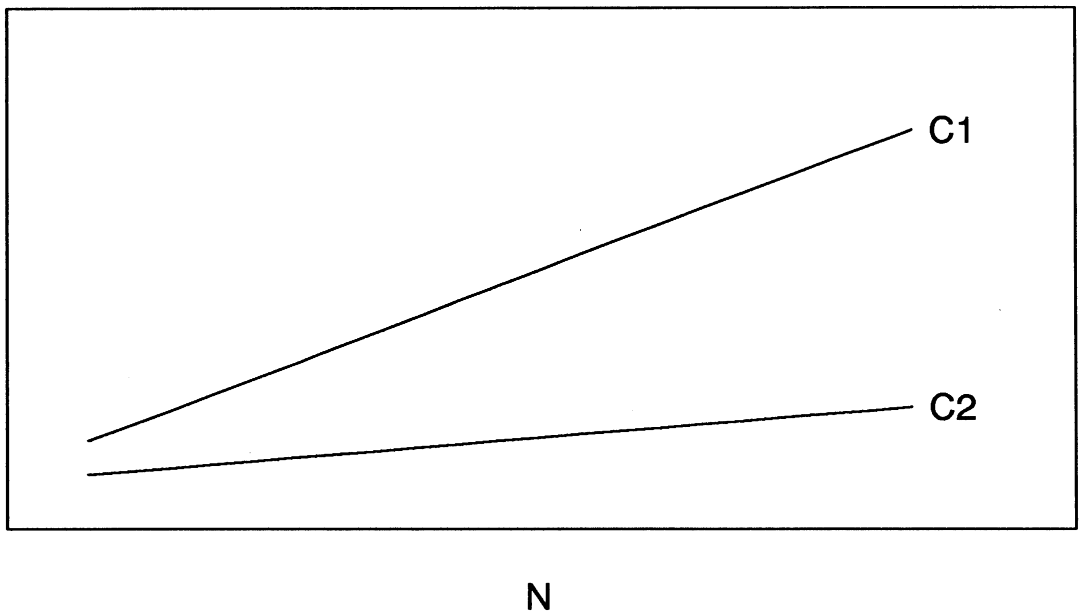
levels											mean		std dev		LTB S/N		min mean	
five largest means																		
A	D	B	C	F	G	H	I	E	J	L	value	rank	value	rank	value	rank	value	rank
4	2	1	2	1	1	1	2	1	2	2	8.877	1	1.084	497	18.776	2	7.122	4
4	2	2	2	1	1	1	2	1	2	2	8.877	2	1.064	469	18.781	1	7.088	7
4	2	1	2	1	2	1	2	1	2	2	8.613	3	0.903	205	18.552	4	7.150	3
4	2	2	2	1	2	1	2	1	2	2	8.613	4	0.681	66	18.618	3	7.116	5
3	2	1	2	1	1	1	2	1	2	2	8.571	5	1.396	1246	18.312	10	6.128	71
five smallest standard deviations																		
levels											mean		std dev		LTB S/N		min mean	
A	D	B	C	F	G	H	I	E	J	L	value	rank	value	rank	value	rank	value	rank
4	2	2	1	2	2	2	1	1	2	1	6.209	2171	0.325	1	15.825	1071	5.540	199
4	2	2	1	1	2	2	1	1	2	1	6.517	1629	0.325	2	16.249	735	5.848	117
4	2	2	1	2	2	1	1	1	2	1	6.829	1102	0.325	3	16.658	463	6.160	66
4	2	2	1	1	2	1	1	1	2	1	7.137	674	0.325	4	17.043	271	6.468	35
1	2	2	1	1	2	1	1	1	2	1	5.321	3306	0.418	5	14.439	2233	4.414	835
five largest LTB S/N ratios																		
levels											mean		std dev		LTB S/N		min mean	
A	D	B	C	F	G	H	I	E	J	L	value	rank	value	rank	value	rank	value	rank
4	2	2	2	1	1	1	2	1	2	2	8.877	2	1.064	469	18.781	1	7.088	7
4	2	1	2	1	1	1	2	1	2	2	8.877	1	1.084	497	18.776	2	7.122	4
4	2	2	2	1	2	1	2	1	2	2	8.613	4	0.681	66	18.618	3	7.116	5
4	2	1	2	1	2	1	2	1	2	2	8.613	3	0.903	205	18.552	4	7.150	3
4	2	2	2	2	1	1	2	1	2	2	8.569	8	1.064	470	18.461	5	6.780	17
five largest minimum means																		
levels											mean		std dev		LTB S/N		min mean	
A	D	B	C	F	G	H	I	E	J	L	value	rank	value	rank	value	rank	value	rank
4	2	2	2	1	1	1	1	1	2	2	8.333	20	0.815	120	18.299	11	7.424	1
4	2	2	1	1	1	1	1	1	2	2	7.945	88	0.594	32	17.934	41	7.274	2
4	2	1	2	1	2	1	2	1	2	2	8.613	3	0.903	205	18.552	4	7.150	3
4	2	1	2	1	1	1	2	1	2	2	8.877	1	1.084	497	18.776	2	7.122	4
4	2	2	2	1	2	1	2	1	2	2	8.613	4	0.681	66	18.618	3	7.116	5

Table 5: Performance of Original Recommendation for Various Criteria

levels											mean		std dev		LTB S/N		min mean	
A	D	B	C	F	G	H	I	E	J	L	value	rank	value	rank	value	rank	value	rank
4	2	1	2	1	1	1	1	1	2	2	8.333	18	1.272	938	18.124	21	6.600	25
4	2	1	2	1	1	1	2	1	2	2	8.877	1	1.084	497	18.776	2	7.122	4
4	2	1	2	1	2	1	1	1	2	2	8.069	53	1.541	1691	17.618	86	5.052	384
4	2	1	2	1	2	1	2	1	2	2	8.613	3	0.903	205	18.552	4	7.150	3
4	2	2	2	1	1	1	1	1	2	2	8.333	20	0.815	120	18.299	11	7.424	1
4	2	2	2	1	1	1	2	1	2	2	8.877	2	1.064	469	18.781	1	7.088	7
4	2	2	2	1	2	1	1	1	2	2	8.069	54	1.054	446	17.900	44	5.938	98
4	2	2	2	1	2	1	2	1	2	2	8.613	4	0.681	66	18.618	3	7.116	5

Figure 1: Example Response Functions and Opportunity for Robustness

(a) opportunity for robustness



(b) no opportunity for robustness

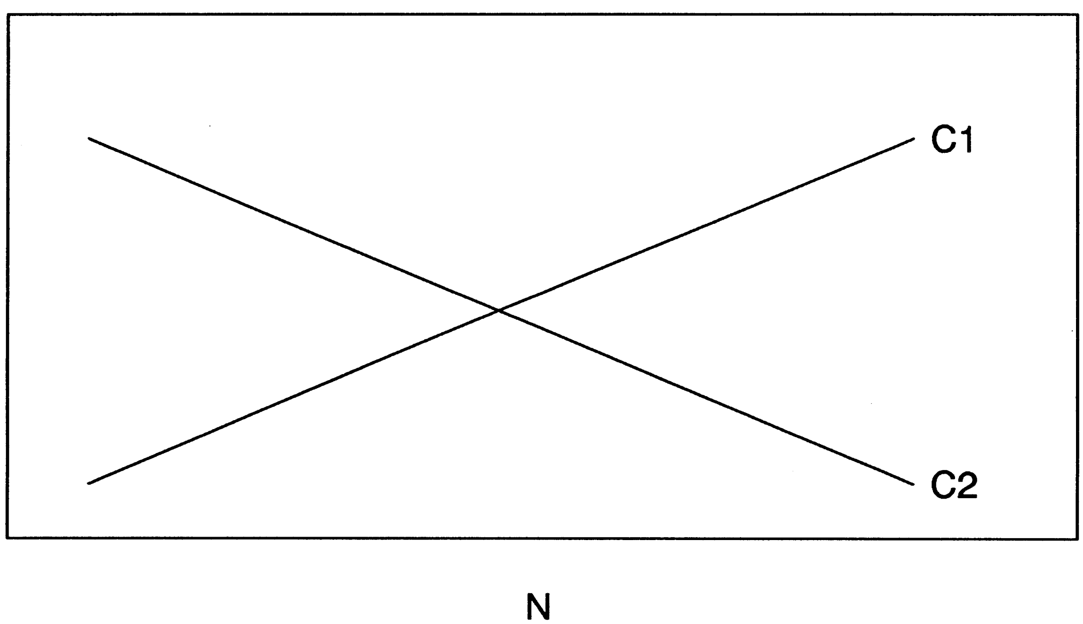


Figure 2: Hidden Information by Larger-The-Better Criterion