THE TREATMENT OF RELATED EXPERIMENTAL FACTORS BY SLIDING LEVELS

M. Hamada and C.F.J. Wu

IIQP Research Report
RR-93-07

August 1993

THE TREATMENT OF RELATED EXPERIMENTAL FACTORS BY SLIDING LEVELS

M. Hamada and C.F.J. Wu
Department of Statistics and Actuarial Science
and
The Institute for Improvement in Quality and Productivity
University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada

August 1993

ABSTRACT

Two factors are related when one factor's desirable range depends on the level of another factor. For example, in a chemical reaction, the desirable reaction-time range shifts downward as temperature increases. This is reflected in the experimental design by choosing levels of one factor that depend on the level of the other factor. Taguchi refers to this technique as sliding levels and justifies it by rationales of bad region avoidance and interaction elimination. We explore these rationales and their implications. In particular, we point out problems with Taguchi's analysis method and demonstrate it by reanalyzing three documented experiments; the reanalysis provides new insight about their respective processes. Finally, using the bad region avoidance rationale, we discuss alternative experimental designs.

Key words: Experimental Design, Interaction, Nested Factors, Regression

Introduction

Often experiments involving factors with sliding levels arise when a factor's desirable range depends on the level of another factor. Taguchi (1987, p. 150) introduces this idea with an example of temperature and reaction time factors in a chemical reaction in which the reaction-time range shifts downward as the temperature is increased. See Figure 1 which depicts a nine run experiment using "sliding" levels. Taguchi (1987, p. 151) then provides the following rationale: "It is necessary to choose levels so that the range which one wishes to learn about from the experiment will be included in the experiment, and any range which is known not to be actually usable is not included." For the chemical reaction example, the range is chosen so that the best reaction time is thought to be contained within it, thereby avoiding temperature–reaction time combinations which are known to be bad, such as a low temperature–short reaction time or high temperature–long reaction time. Note that a factorial design obtained by choosing the factor levels independently would have contained these undesirable combinations.

Taguchi (1987, p. 155) gives another reason for using sliding levels: "Forming levels of factors by considering interrelations among various factors ... not only has the excellent aspect that it enables one to avoid experiments in an unnecessary range and to zero in on an experiment in the necessary range, but most importantly it has the advantage that it cancels interactions." Figure 2 displays the interaction between temperature and reaction time which would be evident from a full factorial design (temperature at 500C, 600C and 700C and reaction time at 3, 4 and 5 hours). Figure 3 displays how the interaction is removed by using a design with sliding levels (temperature (reaction time) at 500C (5-7 hours), 600C (4-6 hours) and 700C (3-5 hours)); the interaction shown in Figure 3a is removed by coding the (low, middle, high) levels of both factors as (-1, 0, 1). See Figure 3b. This interaction elimination rationale has several implications. Taguchi (1987, p. 155) goes on to state, "If it is not necessary to analyze interactions, this will heighten the precision of data analysis rather considerably ..." That is, by eliminating interactions a simpler main effects model is needed. Note that another ramification of having no interactions is that another factor can be studied (i.e., whose alias is the interaction between the factors with sliding levels). We will consider this interaction elimination rationale later in more detail.

The strategy of sliding levels is not solely Taguchi's; it has been used in practice for a long time, although there seems to be little if any discussion in the statistical literature. In the chemical engineering literature, however, Hillyer and Roth (1972) proposed "shear" designs which are a special case of designs with sliding levels. Nevertheless, there are numerous documented experiments by Taguchi and his followers which use this strategy. See Taguchi (1955) which reports on one of his early experiments. Other examples of experiments with sliding levels are: aperture and exposure time in an integrated circuit (IC) fabrication experiment (Phadke, Kackar, Speeney and Grieco 1983), where exposure time was decreased as aperture increased; pulse rate and weld time in a welding process experiment (Chen, Ciscon and Ratkus 1984), where weld time was decreased as pulse rate increased; and gas and air quantity in a light bulb sealing process experiment (Taguchi 1987, p. 463), where the air quantity was increased as the gas quantity increased.

The purpose of this paper is to study the design and analysis of experiments with sliding levels. We consider the rationale of eliminating interactions and its impact on the design and analysis of such experiments. In particular, questionable original analyses of documented experiments lead us to propose a more suitable analysis strategy. Reanalyses of three experiments which provide further insight into their respective processes will be reported. Finally, the primary motivation for designs with sliding levels, i.e., experimentation in a constrained region, leads us to consider alternative designs.

The Interaction Elimination Rationale

From the second quote in the Introduction, Taguchi states that interactions are cancelled by using sliding levels. Phadke (1989, p. 144) qualifies this rather strong statement (and others elsewhere): "If two or more control factors affect the same aspect of the basic phenomenon, then the possibility of interaction among these factors becomes high. When such a situation is recognized, we can reduce or even eliminate the interaction through proper transformation of the control factor levels. We refer to this transformation as *sliding levels*." The transformation being referred to is the coding of the factor levels, e.g., (–1, +1) for two-level factors and (–1, 0, +1) for three-level factors.

Consider the choice of levels for two factors, A and B, in which there will be three sliding levels for factor B at each level of factor A. This amounts to deciding on the location (i.e., the middle level) and scale (i.e., how far apart the high and low levels are) of the levels. Thus, the use of sliding levels is a centering and scaling transformation, where the amount of centering and scaling is dependent on the other factor. It can be seen that an interaction in the original factors can be eliminated only if the relation between the mean response, $E(Y)$, and factors A and B satisfy

$$E(Y) = f_1[(X_A - c_A)/s_A] + f_2\{[X_B - c_B(X_A)]/s_B(X_A)\} , \tag{1}$$

where the c's and s's denote the centering and scaling constants (with those for factor B depending on factor A) and f_1 and f_2 are two arbitrary functions. Figure 3, as previously discussed, illustrates that a proper choice of sliding levels can eliminate an interaction provided the mean response satisfies the form in Equation 1. That is, if the nine run design (Design I) in Figure 3a is used and the three levels of each factor are coded as (-1, 0, +1), then the relation between yield and the coded factors becomes that as displayed in Figure 3b. Note that for this same example, improperly locating the sliding levels will not eliminate the interaction, however. See Figure 4a which uses a different nine run design (Design II) that does not eliminate the interaction as shown in Figure 4b. Similarly, the interaction will not be eliminated for an improper choice of scale, e.g., where the reaction time scale depends on the temperature level.

As the last example suggests, a potentially removable interaction will not be eliminated unless the sliding levels are chosen properly. In order that they be chosen properly, one needs to know the exact relationship between $E(Y)$ and the experimental factors. Because the relationship is not known and hence the need for conducting an experiment, we believe that the interaction elimination rationale is questionable and definitely not the most important reason for using sliding levels. Rather, we find the bad region avoidance rationale the compelling reason for using sliding levels.

The belief that interactions are eliminated by sliding levels has two ramifications. First, in the analysis of such factors, only main effects need to be considered (Taguchi 1987, Chapter 5.8). Since interactions may not be eliminated, we believe that interactions should still be considered. In fact, in analyzing only main effects as proposed by Taguchi, it is possible to declare no effect for the slid or nested factor B when an $A \times B$ interaction exists. See Figure 5 for such an example. Second, believing interactions need not to be estimated, the design matrix columns associated with them can be used to study other experimental factors. Consequently, these other factors' main effects will be aliased with the related factors' interactions. Suffice it to say, that misleading conclusions about these other factors could result. Consequently, we recommend using a design which allows related factor interactions to be estimated.

Finally, we comment that even when interactions are eliminated by proper centering and scaling, important information about robustness may be hidden. A simple example displayed in Figure 6 shows the same quadratic effect for recoded reaction time at each level of temperature. What is hidden in assessing the effects in terms of the recoded factors is that the process is more robust or insensitive to changes in reaction time at the lower temperature.

A Nested Model and Analysis Strategy

In analyzing related factors with sliding levels, we will entertain their interaction which proves to be a more prudent analysis strategy. For factor B's levels depending on A's, two cases need to be considered; either A is qualitative (or quantitative but is effectively qualitative if its levels represent the only choices possible) or quantitative.

When factor A is qualitative or effectively qualitative, we propose analyzing the effect of recoded factor B at each level of factor A, i.e., the effect of the two-level factor B at the ith level of A (denoted by $B|A_i$) or the linear, quadratic, etc. effects of B at the ith level of A when B has more than two levels (denoted by $B_l|A_i$, $B_q|A_i$, etc.). In this way, the potential interaction between the related factors is accounted for. The factor A effect can also be fitted in addition to $B_l|A_i$ and $B_q|A_i$. If A has three levels, we can entertain the two contrasts $A_{1,2}$ and $A_{1,3}$, where $A_{i,j}$ denotes the contrast between levels i and j of factor A. See Table 1 for the corresponding covariates, where $A_{1,3}$ is the same as A_l. The meaning of $A_{i,j}$ is different from the usual one in factorial design, however; the levels of B at level i of A are different from those at level j of A. That is, the factor A contrasts represent differences between "group means", the means for the different groups of B levels. When A is quantitative, we can replace $A_{1,2}$ and $A_{1,3}$ by the "linear" and "quadratic" effects of A, A_l and A_q, as given in Table 1. Again, A_l and A_q should be interpreted differently from the usual ones; they are the linear and quadratic contrasts of the three "group means". Figures 3b and 7b illustrate a "linear" and "linear plus quadratic" temperature effect, respectively. That is, the average level for each group of three reaction-time levels increases linearly in temperature in Figure 3b and linearly and quadratically in Figure 7b.

Note that the (coded) effects of the related factors need not be orthogonal to those of other unrelated factors in the design. For example, A_l and A_q in Table 1 are, but $B_l|A_i$ and $B_q|A_i$ are not. This suggests using multiple regression techniques to fit the model and assess these effects. See the next section for more details. The nonorthogonality of the design with sliding levels in the original factors suggests the possibility of entertaining interactions between other unrelated factors which are partially aliased with the related factor effects (as opposed to being completely aliased). Hamada and Wu (1992) proposed a strategy for analyzing experiments with partially aliased effects.

Table 1: Covariates for Factor B Nested Within Factor A

Factor		Covariates									
A	B	$A_{1,2}$	$A_{1,3}$	A_l	A_q	$B_l\|A_1$	$B_q\|A_1$	$B_l\|A_2$	$B_q\|A_2$	$B_l\|A_3$	$B_q\|A_3$
1	1	-1	-1	-1	1	-1	1	0	0	0	0
1	2	-1	-1	-1	1	0	-2	0	0	0	0
1	3	-1	-1	-1	1	1	1	0	0	0	0
2	1	1	0	0	-2	0	0	-1	1	0	0
2	2	1	0	0	-2	0	0	0	-2	0	0
2	3	1	0	0	-2	0	0	1	1	0	0
3	1	0	1	1	1	0	0	0	0	-1	1
3	2	0	1	1	1	0	0	0	0	0	-2
3	3	0	1	1	1	0	0	0	0	1	1

Examples

We present reanalyses of three experiments using the strategy proposed in the previous section. The additional insight gained by these reanalyses will be highlighted.

Welding Experiment

Chen et al. (1984) conducted an experiment on a spot welding process of a motor support bracket to improve weld strength. An 18 run experiment (L_{18}) with four replications was used to study two two-level factors (A,H) and six three-level factors (B-G). (The two-level factor H was assigned by collapsing one of the three-level factors in the L_{18} design.) Equally spaced sliding levels were used for weld time (B), whose location depended on pulse rate (A); i.e., at A=2, B=(32,36,40) and at A=4, B=(18,22,26). A particularly fortunate property of this design is that the $A \times B$ interaction is orthogonal to the other 3-level factors and can therefore be estimated. This allows the nested effects $B|A$ to be fitted. Consequently, by treating the pulse rate factor as being effectively qualitative, we entertained $B_l|A_1$, $B_q|A_1$, $B_l|A_2$ and $B_q|A_2$ in addition to the main effects for A and C-H. The least squares estimates (LSEs) of these effects and their corresponding standard errors (SEs) as given in Table 2

Table 2: LSEs and Standard Errors for Welding Experiment

Effect	LSE	SE
Int	669.2	13.9
A	-81.0	13.1
$B_l\|A_1$	181.7	22.6
$B_q\|A_1$	27.9	13.1
$B_l\|A_2$	-23.8	22.6
$B_q\|A_2$	41.7	13.1
C_l	-108.0	16.0
C_q	-79.1	9.2
D_l	126.7	16.0
D_q	-24.2	9.2
E_l	17.5	16.0
E_q	-35.0	9.2
F_l	-27.2	16.0
F_q	2.0	9.2
G_l	195.5	16.0
G_q	-45.7	9.2
H	145.8	13.9

(intercept denoted by Int) show that an interaction (in particular, the strong $B_l|A_1$ effect) remains despite the sliding levels being used; that is, $B_l|A_1$ and $B_l|A_2$ are significantly different.

Note that if only the main effects in (coded) A and B had been entertained based on the interaction elimination rationale, A and B_q would have been detected. That is, by averaging over the levels of A, the $B_l|A_i$ effects cancel out. In contrast, the proposed analysis shows that the interaction has not been removed; at A_1, B has both linear and quadratic components, whereas at A_2, B has only a quadratic component. Besides providing additional insight, the proposed analysis gives a different recommended setting for B, A_1B_3 (pulse rate 2, weld time 40). The original analysis of Chen et al. (1984) recommended A_1B_2 (pulse rate 2, weld time 36), which gives a smaller weld strength.

Light Bulb Experiment

Taguchi (1987, Chapter 17.6) presented an experiment on a light bulb sealing process to improve a cosmetic problem that was frequently occurring (15%). A 16 run fractional factorial design (2^{10-6}) with 10 replicates was used to study 10 factors (A-J) of which four pairs of factors, (C,D), (E,F), (G,H) and (I,J) were related. See Table 3 for the design that was used. The pairs of factors consisted of the gas quantity and air quantity used at the four sealing stages in the process. Sliding levels were used with air quantity being increased as gas quantity was decreased and displayed in Table 4. In addition, to the main effects for the 10 factors, Taguchi (1987) indicates that $G \times H$ and $I \times J$ should also be considered since the third and the fourth stages are the most critical in the sealing process. It can be shown for the design used that $C \times D = G$ and $E \times F = G \times H$ so that $C \times D$ and $E \times F$ should be assumed negligible. (It turns out that a different 16 run fraction exists for which all four interactions and the eight main effects can be estimated as well as the main effects for A and B! See Table 3 for this alternative design.) The experimental response was light bulb appearance, which was classified into one of three ordered categories: defective, slight cosmetic problem (but passed inspection) and perfect. In the reanalysis, we scored the categories as 1, 2 and 3 and analyzed the scores as in Hamada and Wu (1990). The least squares estimates of these effects and their corresponding standard errors as presented in Table 5 indicate significant $H|G_1$, $J|I_1$ and I effects. Using the signs and magnitudes of the significant estimated effects, $G_1H_1I_1J_2$ is recommended. The original analysis considered G, H, $G \times H$, I, J and $I \times J$ in the coded factors and identified H, I and $I \times J$ as significant, yielding the recommendation, $H_1I_1J_2$. No recommendation was given for G although G_1 is the standard setting.

Taguchi's original analysis missed important information on how and why factors H, I and J are significant. The proposed analysis given in the previous paragraph suggests that H is significant only at level 1 of G and J is significant only at level 1 of I.

Table 3: Original Design with Aliases and Alternative for Light Bulb Experiment

original design														
G	H	GH	I		A	B	J		C	D	IJ	E	F	
CD		EF												
alternative design														
G	H	GH	I	C	A	E	J	EF		D	IJ	B	F	CD
-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
	+	+	-	-	-	-	-	-	-	-	+	+	+	+
+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 4: Gas and Air Levels for Light Bulb Experiment

(S_g and S_a denote standard gas and air settings)

		Air	
Gas		1	2
1	S_g	S_a	$S_a - 1$
2	$S_g - 1$	$S_a + 1$	S_a

Table 5: LSEs and Standard Errors for Light Bulb Experiment

Effect	LSE	SE
Int	1.92	.06
A	.07	.06
B	-.03	.06
C	-.09	.06
D	.01	.06
E	-.04	.06
F	-.07	.06
G	-.01	.06
$H\|G_1$	-.24	.08
$H\|G_2$	-.13	.08
I	-.23	.06
$J\|I_1$	.29	.08
$J\|I_2$	-.08	.08

Window Forming Experiment in IC Fabrication

Phadke et al. (1983) presented an experiment to optimize contact window formation in an integrated circuit fabrication process, one of the first experiments performed in the U.S. that Taguchi was involved with. An 18 run experiment (L_{18}) was used to study nine factors (A-I), which consisted of two pairs of related factors, (photoresist viscosity (B), spin speed (C)) and (aperture (F), exposure time (G)). An added complication is that two-level factors B and D were combined into a single three-level factor. See their paper for more details. For purposes of illustration the ten observations taken at each run will be treated as replicates. These "replicates" arise from noise factors using an outer array and have been analyzed elsewhere. However, to the best of our knowledge, this is the first analysis which accounts for the relationship between the pairs of factors. Here, we present a reanalysis of the post-etch window size response, which was classified into five ordered categories and scored 1 to 5.

Note that the L_{18} design has complex aliasing patterns, i.e., the factor main effects are

Table 6: Forward Selection Results for Integrated Circuit Experiment
(R^2 is for the model containing the current and previous selected terms.)

Proposed Analysis		Original Analysis	
Effect	R^2	Effect	R^2
$C_l\|B_1$	0.41	C_l	0.29
B	0.62	B	0.46
$G_l\|F_3$	0.68	H_l	0.53
		A_l	0.60
		G_l	0.64

partially aliased with many of the interactions between the other factors. Hamada and Wu (1992) observed that this partial aliasing could be exploited so that potentially a few important interactions might be detected. A design-theoretic justification for this empirical observation was later provided by Wang and Wu (1993). Thus, the $B \times C$ and $F \times G$ interactions can be considered. The added complication of B being combined with D into a three-level factor presents no problem since a main effect due to B alone can be obtained. For the pair (B, C) which were studied at two and three levels, respectively, $C_l|B_1$, $C_q|B_1$, $C_l|B_2$ and $C_q|B_2$ can be entertained. Similarly, for the pair (F, G) which were each studied at three levels, G_l, G_q, $F_l|G_1$, $F_q|G_1$, $F_l|G_2$, $F_q|G_2$, $F_l|G_3$ and $F_q|G_3$ can be entertained. Because these effects along with the main effects of the other factors are too many to be estimated simultaneously, a forward selection procedure was used to identify the important effects. Effects B, $C_l|B_1$ and $G_l|F_3$ were identified as the most significant with R^2=0.68, which is higher than the R^2=0.64 for the five effects identified by the original analysis (see Table 6). Note that the original analysis also identified B, C_l and G_l as being important but the proposed approach provides more insight; namely, C has a linear effect only at B_1 and G has a linear effect at F_3. This additional information is necessary in recommending optimum factor settings for B, C, F and G.

Symmetric and Asymmetric Factor Relationships

The relationship betwen two related factors can be *symmetric* or *asymmetric*. The window forming experiment provides good examples for both types. In the experiment, exposure time and aperture have a symmetric relationship because their product is related to total light energy. Consequently, one can slide the levels of aperture as the exposure time varies, which was done in the experiment; alternatively, one can slide the levels of exposure time as the aperture varies. Although they result in different factor level combinations, they both avoid the undesirable combinations of (high,high) and (low,low) aperture and exposure time. On the other hand, photoresist viscosity and spin speed have an asymmetric relationship. For high viscosities, the spin speed should be increased so that the photoresist can be spread uniformly on a wafer. It would be unnatural to first specify spin speed and then slide the viscosity levels. Asymmetric relationships also occur if one factor is qualitative and the other quantitative. Since the qualitative factor cannot be slid because it does not have enough levels, the quantitative factor is often nested within the qualitative factor, assuming it is physically feasible. Also, some quantitative factors may be practically qualitative; take for example, thickness in an experiment using commercially available materials which are only made in a few different thicknesses. Consequently, the relationship between factors, whether it is symmetric or asymmetric, has an impact on experimental design. For symmetric relationships, the choice of what factor to be nested is not crucial whereas for asymmetric relationships, there is only one way to slide the factor levels. Some further aspects of design will be discussed next.

Some Aspects of Design

As discussed above, the compelling reason for considering designs with sliding levels is the rationale of bad region avoidance. Motivated by the same concerns, Hillyer and Roth (1972) proposed two classes of designs for quantitative factors when the bad region is on the two sides of a parallel strip. One class of designs is constructed by twisting or "shearing" an initial design, a design with a regular grid (e.g., a factorial design), and fitting it into the region of interest. These sheared designs are obtained by linear transformation. The other class of designs is obtained by rotating the initial design, using a different linear transformation. See Figure 8 (b,c) for examples of sheared and sheared-rotated designs, respectively, where the latter design was obtained by rotating and then shearing the original factorial design. Note how they are superior to the factorial design in Figure 8a in being located within the strip, the desired experimental region. Similarly, these transformation can be applied to other designs such as central composite designs. While their methods are useful for generating a variety of alternative designs, they are limited in two ways. First, they can only be applied to quantitative factors. Second, the sheared designs are actually special cases of designs with sliding levels. That is, in Equation (1) the centering $c_B(X_A)$ is linear and the spread $s_B(X_A)$ is constant. Contrast Figure 8b with Figure 8d, a design with sliding levels in which the spread of the sliding levels increases in the other factor. The latter design in Figure 8d would be especially suitable if the desired experimental region is between the dashed lines rather than parallel (solid) lines.

Acknowledgements

This research was supported by General Motors of Canada Limited, the Manufacturing Research Corporation of Ontario, and the Natural Sciences and Engineering Research Council of Canada.

References

Chen, T.Y.W.; Ciscon, K.J.; and Ratkus, A.C. (1984). "ITT Bell and Gossett Taguchi Experiment: Motor Support Bracket Spot Welding" in *Second Symposium on Taguchi Methods,* American Supplier Institute, Inc., Romulus, MI, pp. 70–77.

Hamada, M. and Wu, C.F.J. (1990). "A Critical Look at Accumulation Analysis and Related Methods (with discussion)." *Technometrics* 32, pp. 119-162.

Hamada, M. and Wu, C.F.J. (1992). "Analysis of Designed Experiments With Complex Aliasing." *Journal of Quality Technology* 24, pp. 130-137.

Hillyer, M. J. and Roth, P. M. (1972). "Planning of Experiments When the Experimental Region is Constrained: Application of Linear Transformations to Factorial Design." *Chemical Engineering Science* 27, pp. 187-197.

Phadke, M.S., Kackar, R.N., Speeney, D.V. and Grieco, M.J. (1983). "Off-Line Quality Control in Integrated Circuit Fabrication Using Experimental Design." *The Bell System Technical Journal* 62, pp. 1273-1309.

Phadke, M.S. (1989). *Quality Engineering Using Robust Design,* Prentice Hall, Englewood Cliffs, NJ.

Taguchi, G. (1955). "An Experiment to Find an Optimum Operational Standard for Anodising Aluminium Alloy Parts." Unpublished report submitted to the Indian Statistical Institute.

Taguchi, G. (1987). *System of Experimental Design,* Unipub/Kraus International Publications, White Plains, NY.

Wang, J.C. and Wu, C.F.J. (1993). "A Hidden Projection Property of Plackett-Burman and Related Designs." University of Waterloo Institute for Improvement in Quality and Productivity Research Report RR-93-01. Revised for *Statistica Sinica.*

Figure 1: Design for Factors with Sliding Levels

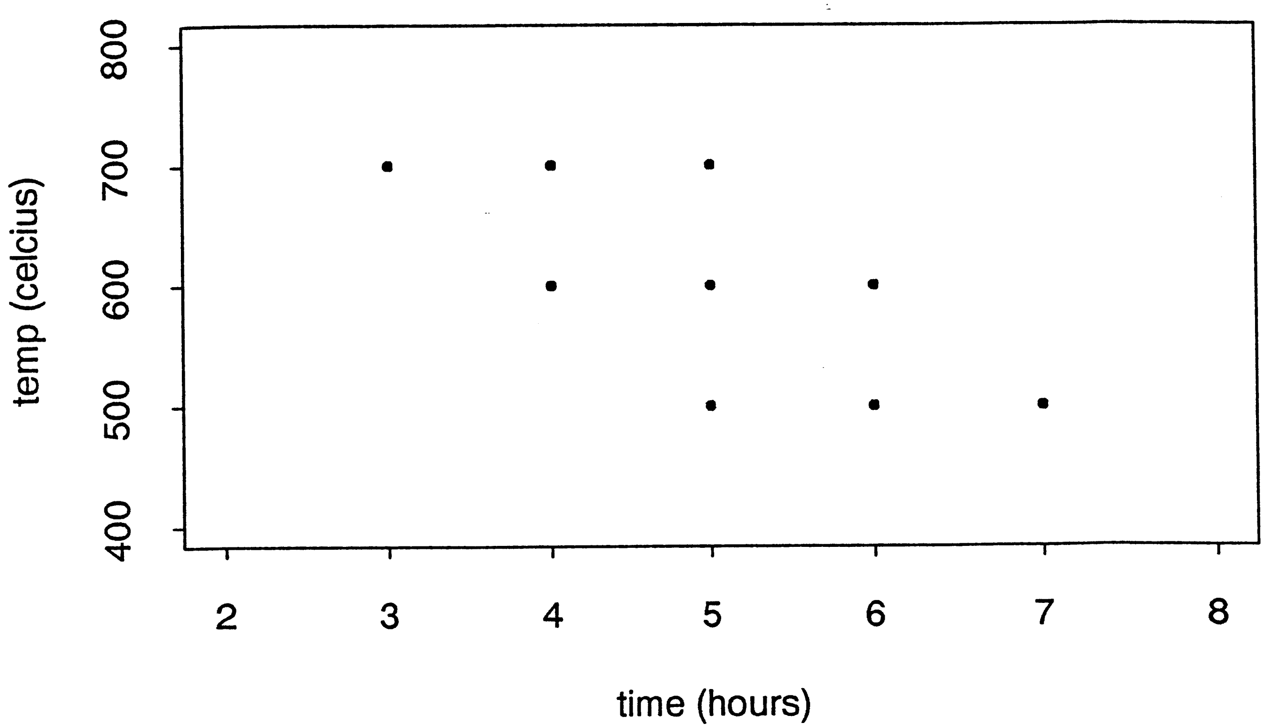


Figure 2: Interaction from Design without Sliding Levels

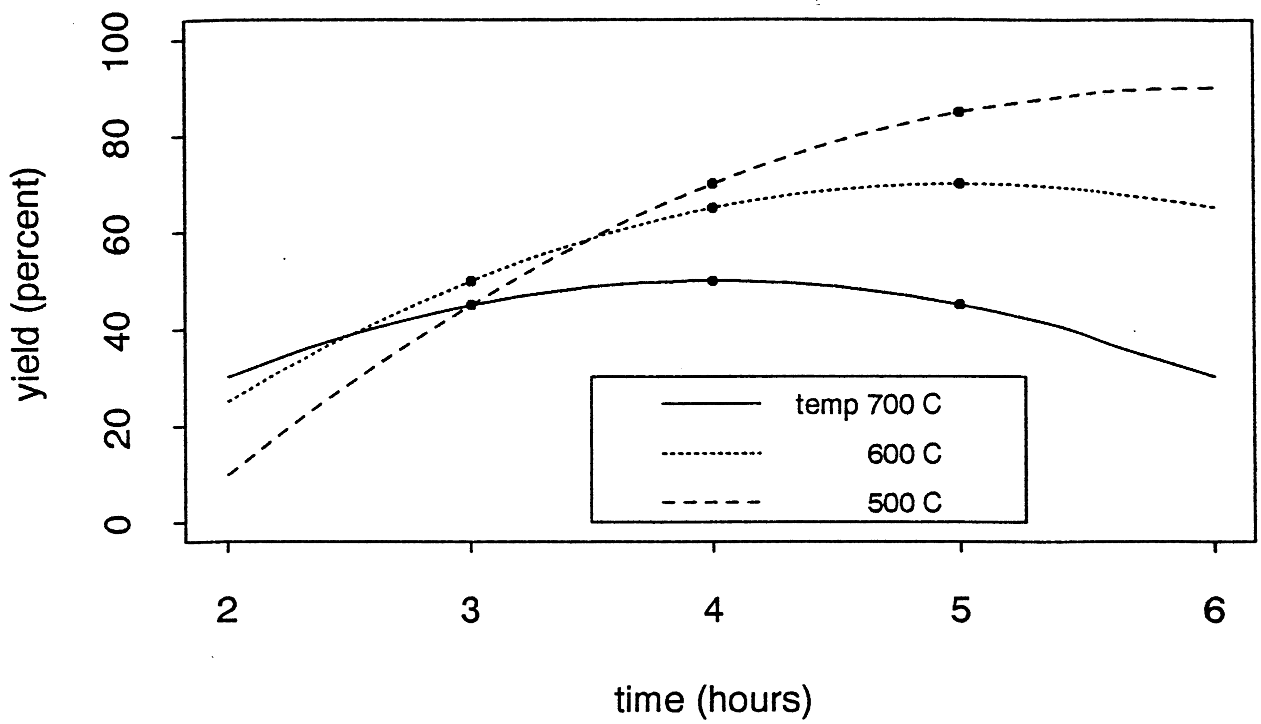
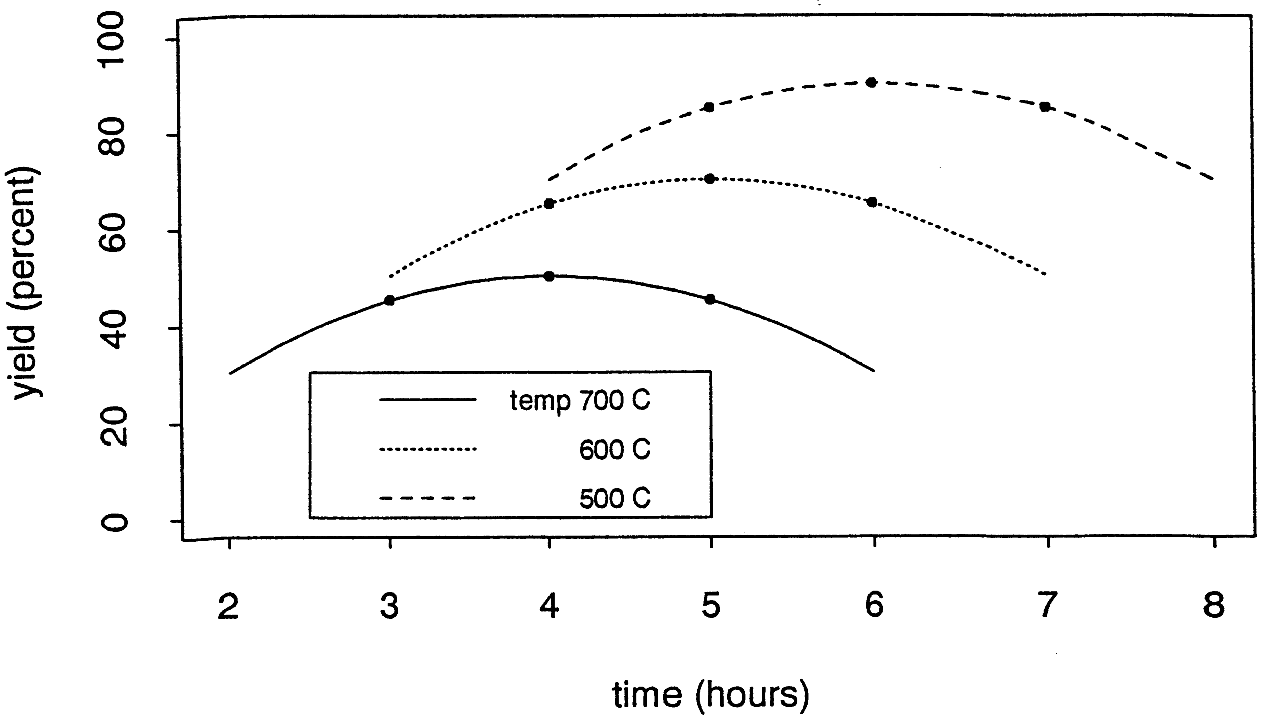


Figure 3: Design I for Factors with Sliding Levels

(a) Response and Design in Original Factors



(b) Response and Design in Centered/Scaled Factors

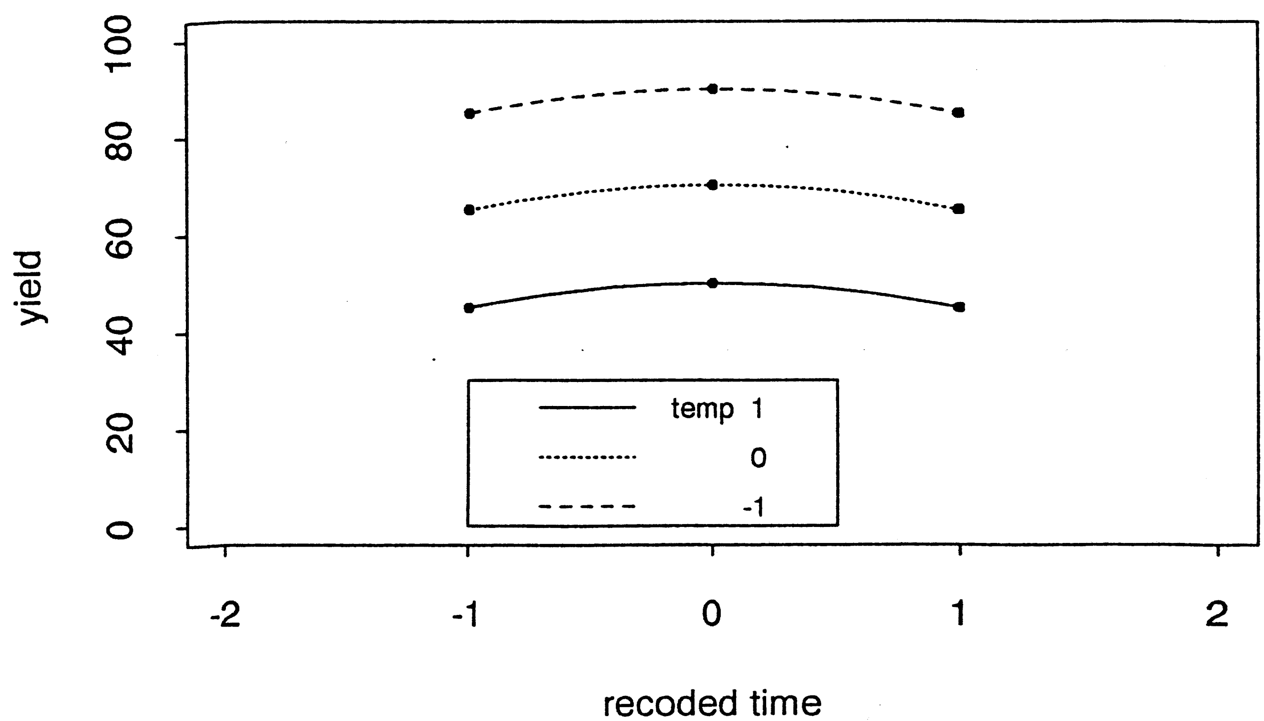


Figure 4: Design II for Factors with Sliding Levels

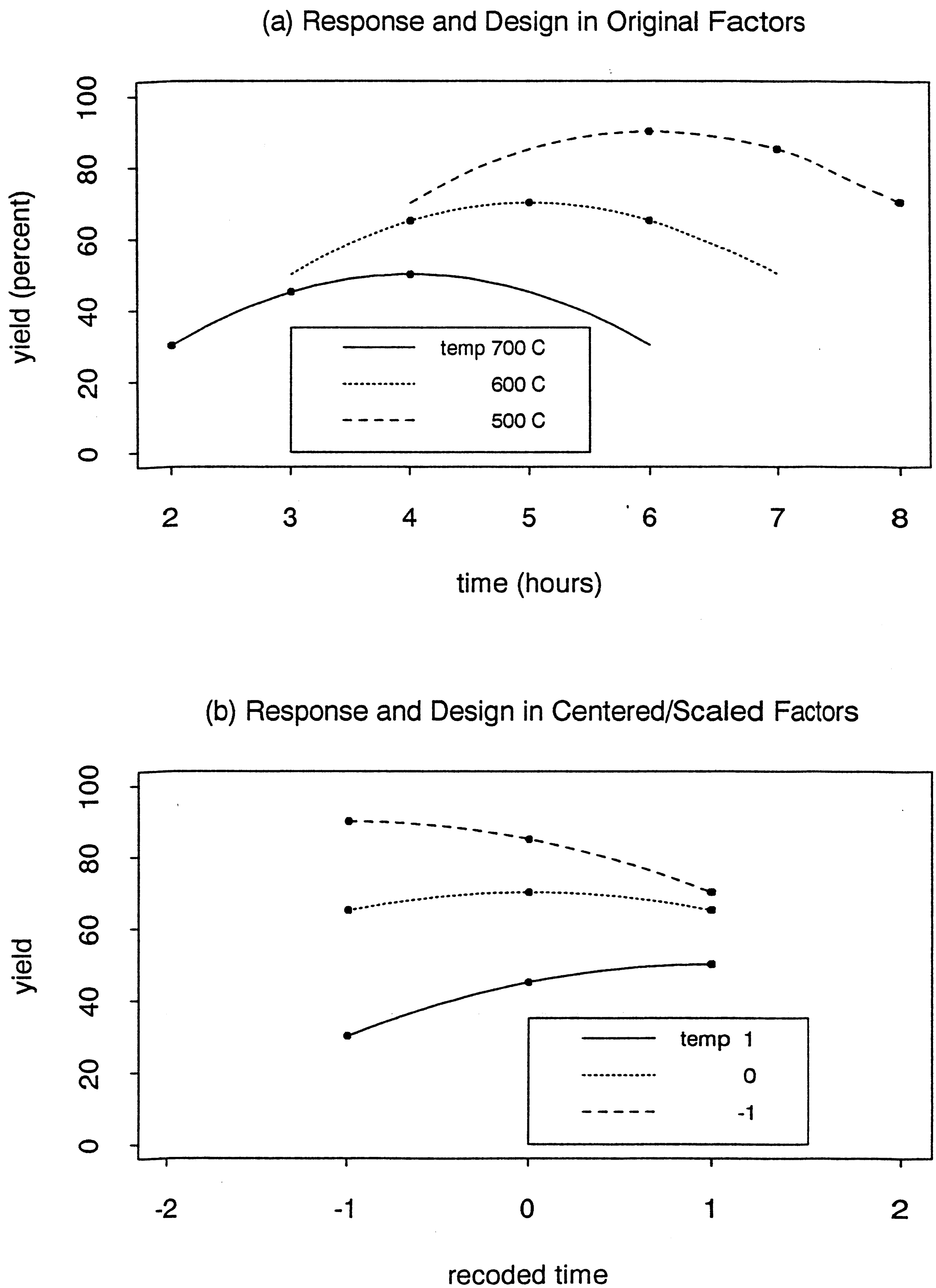
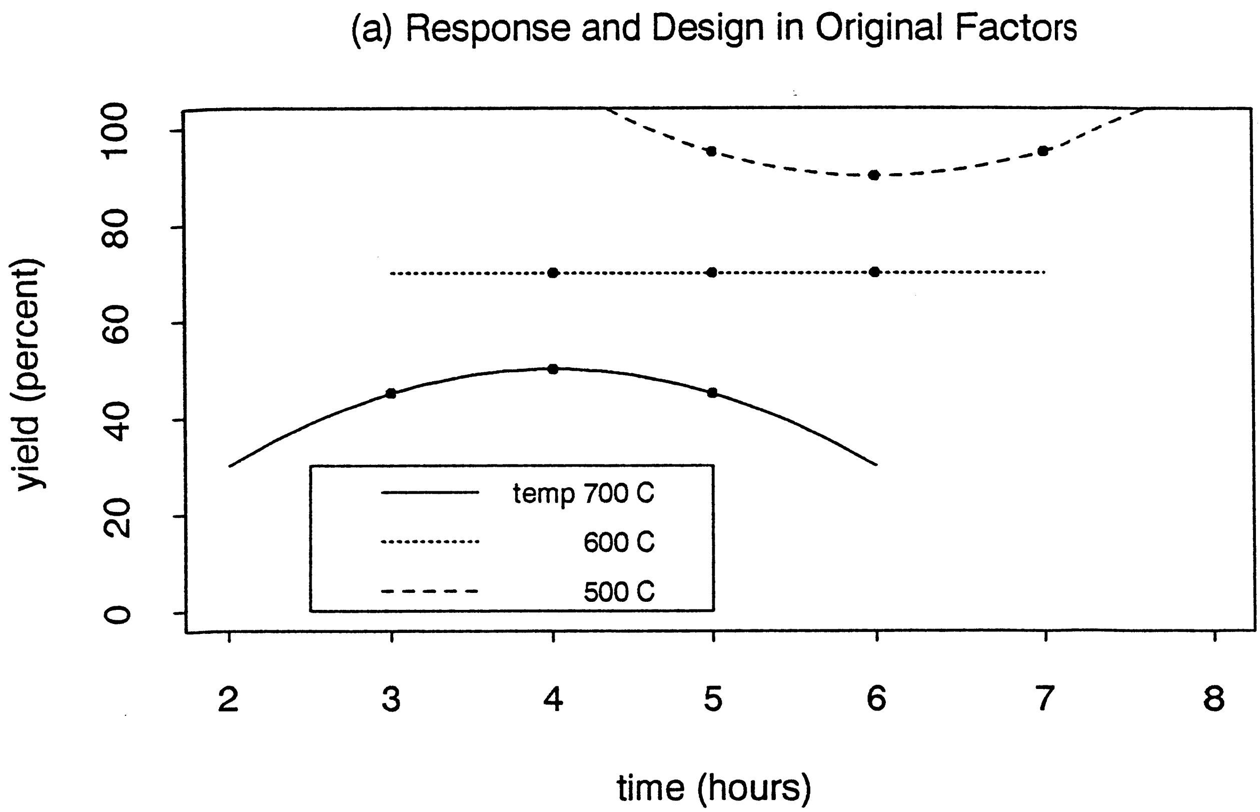


Figure 5: Missed Time Effect From Main Effects Only Analysis



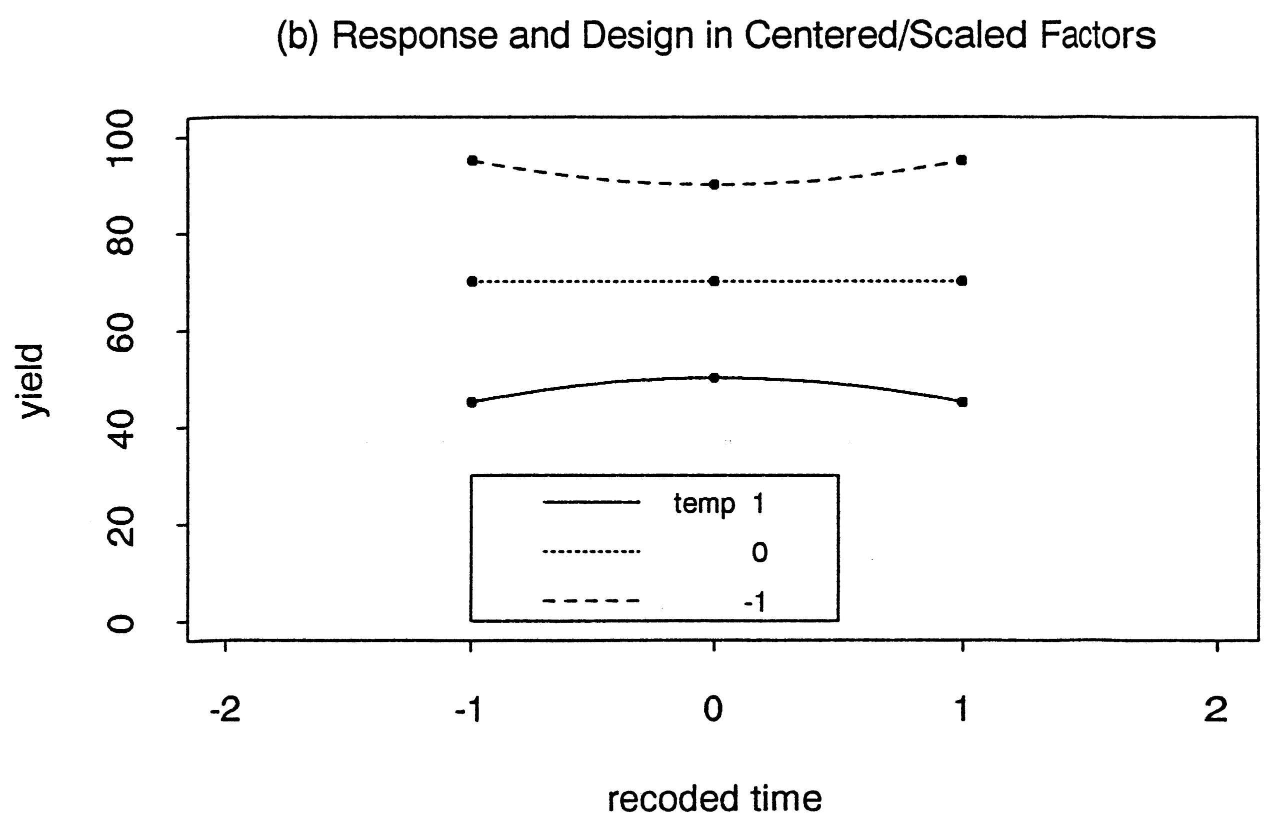
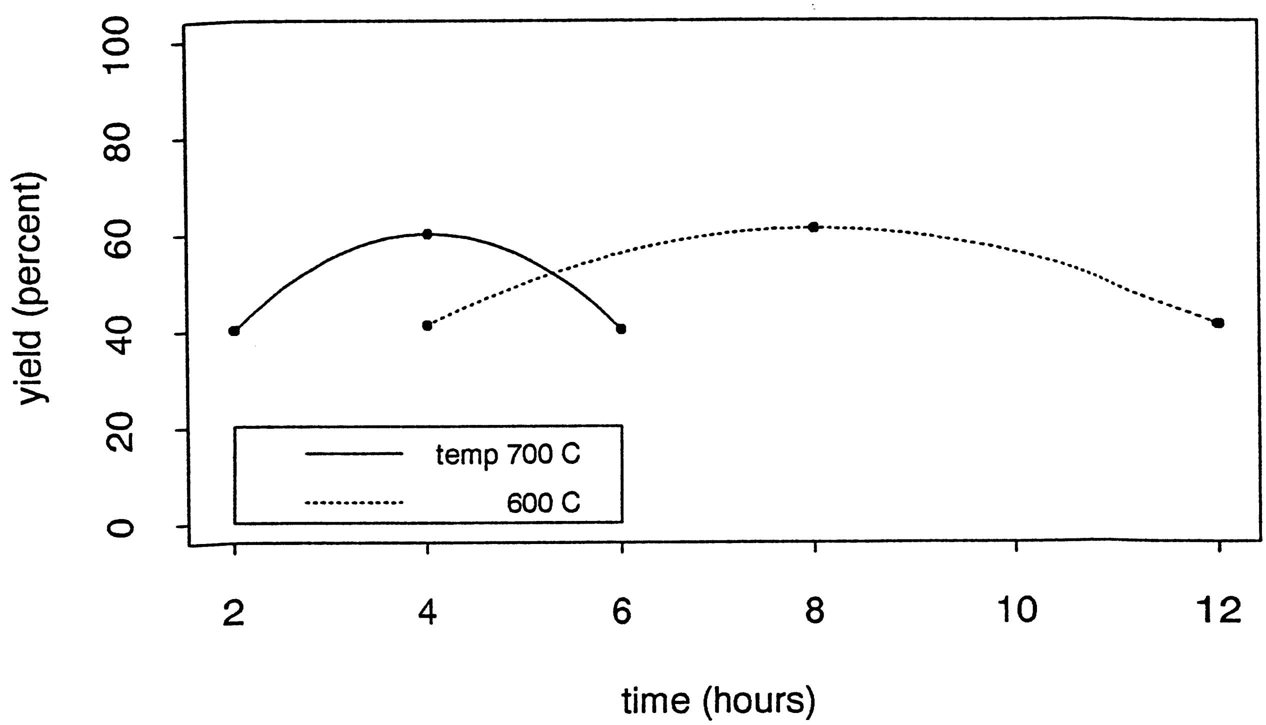


Figure 6: Hidden Information on Robustness

(a) Response and Design in Original Factors



(b) Response and Design in Centered/Scaled Factors

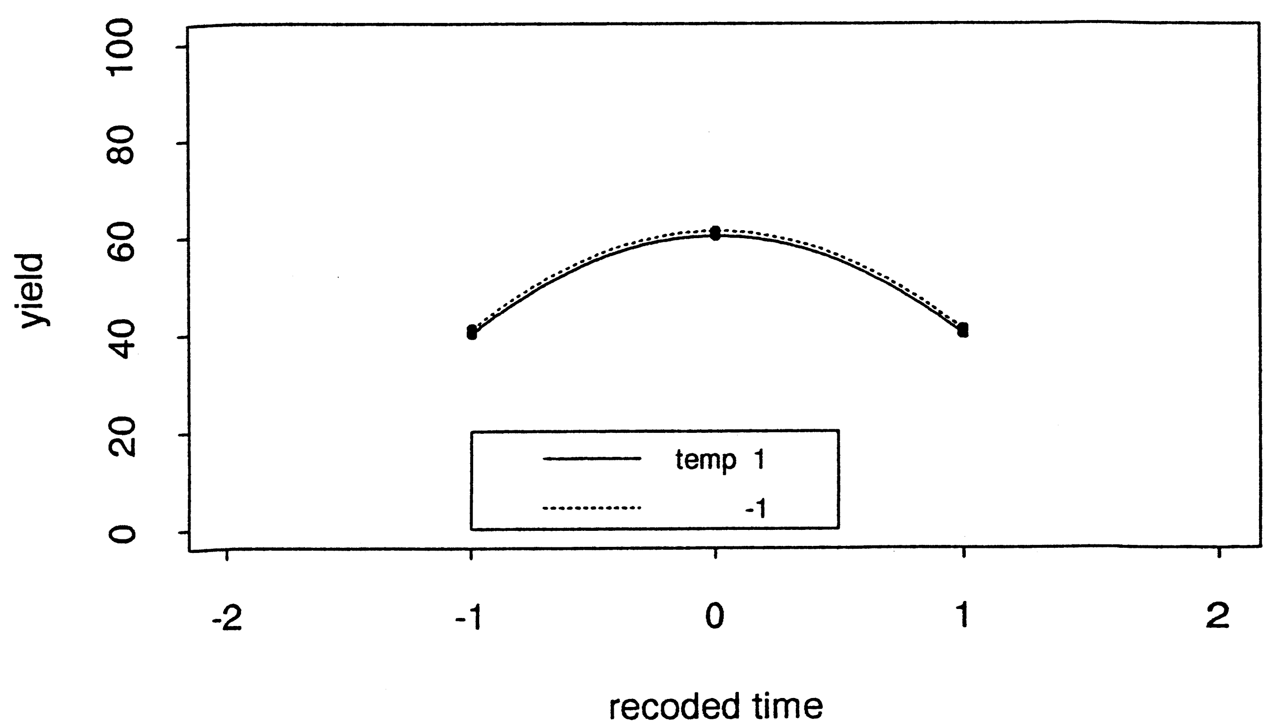
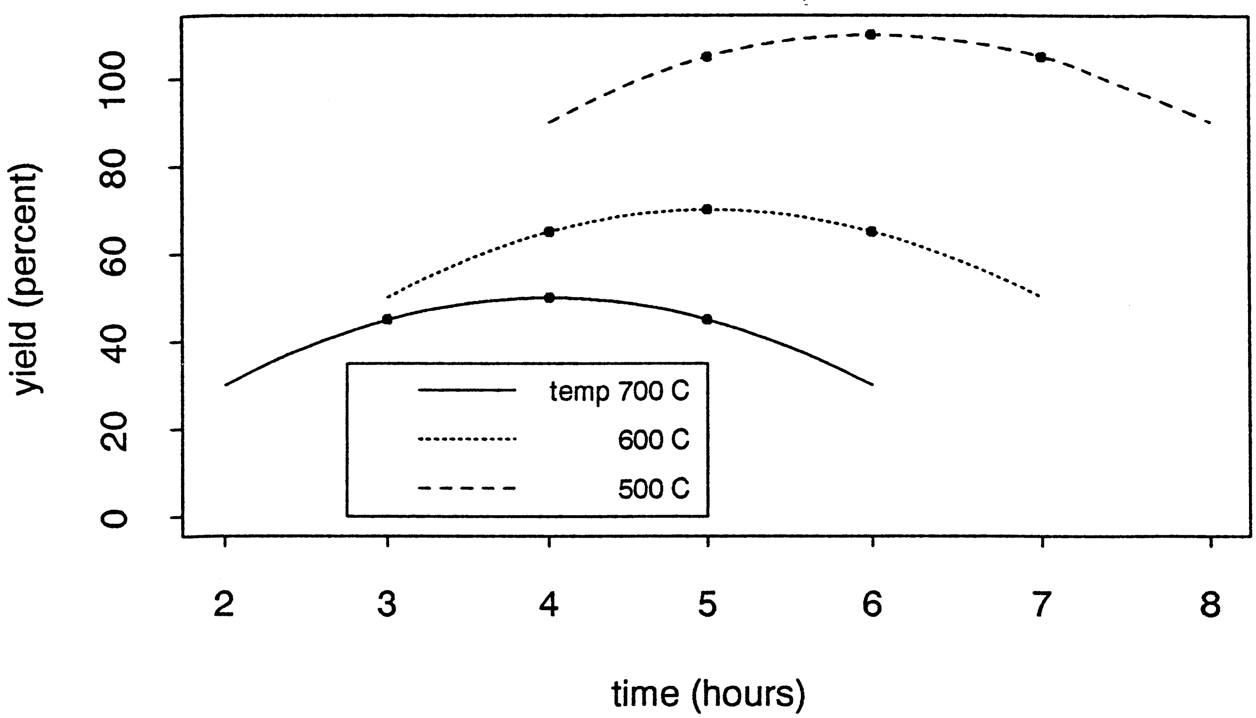


Figure 7: Design I Showing
Linear and Quadratic Temperature Effects

(a) Response and Design in Original Factors



(b) Response and Design in Centered/Scaled Factors

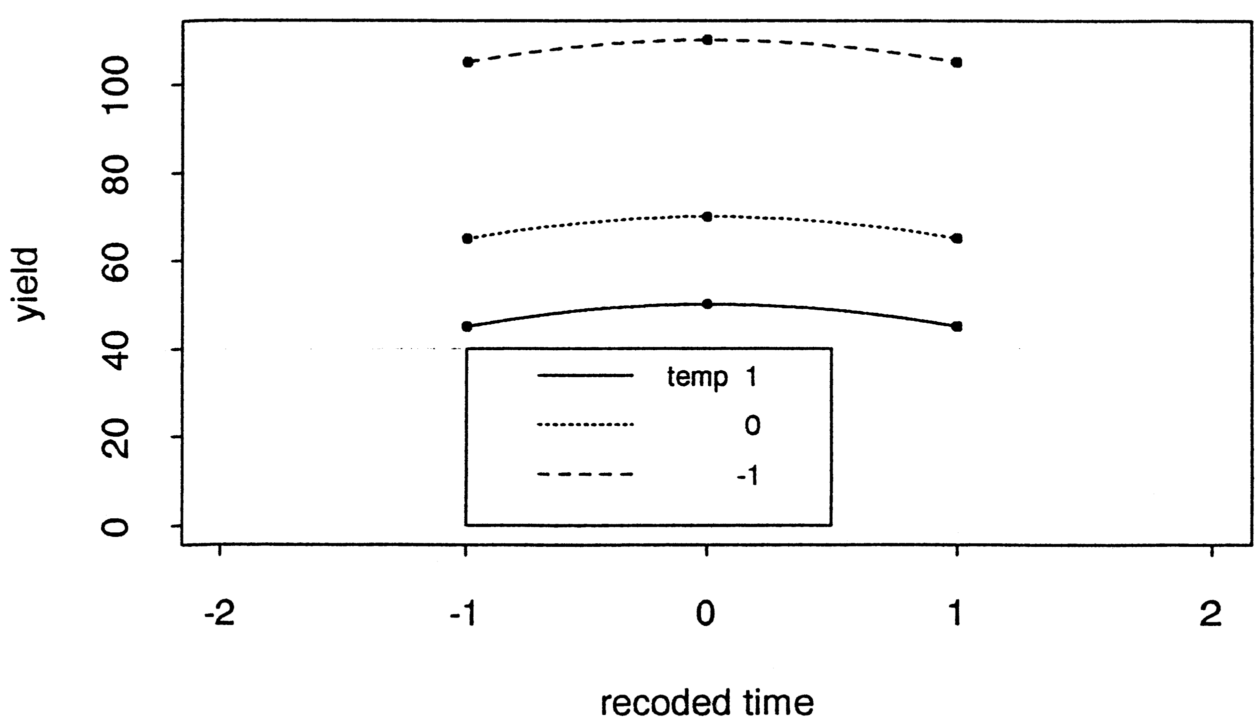


Figure 8: Some Alternative Designs