A CATALOGUE OF CONFOUNDING SCHEMES FOR EIGHT-, SIXTEEN-, AND THIRTY TWO-RUN FRACTIONAL FACTORIAL DESIGNS

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A CATALOGUE OF COUNFOUNDING SCHEMES FOR EIGHT-, SIXTEEN-, AND THIRTY TWO-RUN FRACTIONAL FACTORIAL DESIGNS

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ABSTRACT

A very compact set of tables is provided showing the maximum resolution designs and the alias structure for all main effects and two-factor interactions for fractional factorial designs with eight, sixteen, and thirty two-runs. There is one table showing both the design structure and the relevant two-factor alias structure for each of the three sizes of experiment. An example of the use of the tables is included.

Key words: Fractional factorial designs, Confounding schemes, Alias structure, Planning of experiments, Experimental design, Interactions.

INTRODUCTION

Engineers involved in industrial process improvement are often confronted with many potentially influential factors and, because of pressure to meet production schedules, a severely limited amount of line time available for experimentation. In order to screen out those few factors with the greatest influence, fractions of 2^n factorial experiments are frequently used. Eight-, sixteen-, and occasionally thirty two-run fractions are most common.

A drawback of these designs is that the differences in means (contrasts) that measure the effects of interest may be influenced by two or more effects at the same time. Such effects are said to be confounded with each other (Box, Hunter and Hunter, 1978, Chapter 12). The tables at the end of this note are intended to save time in the planning of fractional factorial experiments by providing a very compact catalogue of the confounding patterns (alias structure) for the main effects and two-factor interactions for the most common designs. Higher order interactions are not included since they are less common and their inclusion would make the tables far too cumbersome.

The added complexity of confounding should not discourage people from using fractional factorial experiments. Their systematic or balanced approach to the study of several factors simultaneously makes them one of the most efficient and effective methods yet devised for the study of complex industrial processes when many factors are of interest and time for experimentation limited.

Unfortunately, as had already happened with control charts, factorial designs have often been promoted as being the solution to all of industries' problems. This has lead to designed experiments being attempted where they should not have been used. The result has been disappointment in the results of factorial experimentation and a corresponding decline in interest in their application. In fact, strict adherence to standard operating procedure and simple data analysis using stratification by shift, raw material lot, line, time of day and so on will solve many of the problems of excess variation in industry.

Even in situations where designs should have been effective, they have often failed. The reason is usually a lack of careful initial planning: People miss the significance of the word "design". Such basic requirements as the careful choice of factors and levels are often missing. Fundamental requirements of good design such as blocking, randomization and replication are rarely incorporated because they are rarely covered in adequate detail in industrial short courses and are very poorly understood by many industrial practitioners. An outstanding source of information on these and other fundamentals of the planning of experiments is the book by Cox (1958). Case studies illustrating their impact on industrial experimentation can be found in Young, Abraham and Whitney (1991) and Young (1994). The importance of these techniques cannot be over-emphasized. It is unfortunate to see this powerful tool being abandoned because of a lack of understanding of these fundamentals.

THE TABLES AND THEIR USE

The designs in the tables are essentially an extension of the designs in table 12.15 in Box, Hunter and Hunter (1978). In the tables, factor names and the corresponding contrasts for estimating their effects are indicated by numerals; thus 1, 2,...9, $\overline{10}$, $\overline{11}$, $\overline{12}$, and so on represent the specific factors to be studied. Interactions are then represented by number combinations; thus 235 represents the interaction (and corresponding contrast) for the three-factor interaction between factors 2, 3, and 5. Note that $\overline{12}$ is used to distinguish factor twelve and its contrast from the two-factor interaction 12. Thus 3 $\overline{12}$ represents the two-factor interaction between factors 3 and $\overline{12}$.

For convenience, Table 1 provides contrasts for all effects that can be measured in fractions of the 2^n series of designs in up to 32 runs. If preferred, the "+" and "-" signs can be replaced with the numerals "1" and "2" to provide designs equivalent to the L_4, L_8, L_{16} and L_{32} series of designs discussed in, for instance, Chapter 6 of Taguchi (1987). Note that, when the signs are used, interaction contrast signs are obtained as products of main effect signs. Thus the signs of the 235 interaction constrast are given by the product of the signs of the 2, 3, and 5 main effect contrasts. Many people find this fact useful in motivating intuitively the nature of the various interactions and the corresponding alias structure.

The designs in the tables are chosen to ensure that main effects and two-factor interactions are confounded with other effects of as high an order as possible. They thus maximize the *resolution* (Box, Hunter and Hunter, 1978, Section 12.4) of the design. The designs with resolution V in the tables ensure that main effects are confounded with four-factor interactions or higher and two-factor interactions with three-factor interactions or higher. At worst, resolution IV designs confound main effects with three-factor interactions and two-factor interactions with each other. In resolution III designs at least some main effects are confounded with two-factor interactions (but not each other). Normally, designs of maximum possible resolution are to be preferred.

For example, we see in Table 3 that a resolution V design for studying five factors in sixteen runs is possible by setting factor 5 using the signs of the four-factor interaction 1234. However, to achieve the highest possible resolution (IV) for six factors in sixteen runs we set factors 5 and 6 using the 123 and 234 interactions. Had we set $5=1234$ and $6=123$ then the 56 interaction would have been confounded with the 4 main effect and the resolution would have been III.

Although, in Table 3, levels of 5 and 6 have been set using the signs of 123 and 234, any pair of three-factor interactions could have been used. The designs shown are, for the most part, based on Table 12.15 of Box, Hunter and Hunter. There are, for the sake of consistency, a few slight departures. For instance, with six or more factors in a sixteen-run experiment, the fifth factor is always set using with the same three-factor interaction (234).

The actual settings of the additional factors in any design can be set using either the same signs as the interaction contrast used or exactly the opposite. For example, in order to study eight factors in thirty two runs, the extra factors can be set using

$$6 = \pm 2345$$

$$7 = \pm 1345$$

$$8 = \pm 1245$$

Further details on the choice of signs and the ramifications of the choice are found in Box, Hunter and Hunter.

AN EXAMPLE

Following the introduction of a new design of the control arm in the suspension of a motor vehicle, an undesirable level of cracks and foldbacks were found in a rubber bushing that was part of the assembly. After much discussion, management agreed to free up the line time necessary to perform a sixteen-run experiment. Brainstorming sessions resulted in the choice of the following seven factors and levels:

	Factor	Levels	
		-	+
A:	Position of Stop	3/8"	5/8"
B:	Bushing Radius	Current	Modified
C:	Insertion Speed	Low	High
D:	Alignment	No Shim	Shim
E:	Cone Design	Current	New
F:	Air Pressure	30 psi	45 psi
G:	Bushing Lubrication	Standard	High

In order to ensure that the conclusions would be reasonably reliable (accurate) it was decided to carry out the experiment in four blocks each consisting of a different batch of bushings. To further ensure that the experiment was truly representative of normal operating conditions the four blocks were spread out over two weeks. The four runs within each block were to be run under as nearly identical process conditions as possible to ensure high precision. The inclusion of the four blocks requires the use of two more contrasts which we will label H and I. Thus the design effectively involves nine factors. As well, the HI contrast will be affected by block differences, so a total of ten contrasts are used up by the basic design.

Since a sixteen-run design results in information on fifteen separate (orthogonal) contrasts, and since the above requirements have used up ten of them, only 5 are left to estimate interactions that are unconfounded with each other. After much discussion it was decided that the AE, AC, CE, CF and CG interactions were the most likely candidates, BE and BD interactions were also thought to be remotely possible. Although it was rather optimistic to hope that we could determine the most likely five of the twenty one possible interactions, it was agreed that the balanced factorial approach was the most reliable way to narrow down the possibilities.

The design was set up using Table 3 and assigning the blocking factors H and I to contrasts 1 and 2 and thus losing the 12 (HI) contrast to blocks as well. It was then noted that if factor C was set using the 7=134 contrast and A, E, F and G set using contrasts 3, 4, 5=123 and 8=124 then the required interactions AE, AC, CE, CF and CG would be estimated by the five unconfounded (with each other) contrasts 34, 14, 13, 24 and 23 respectively. The design thus has resolution IV in the main effects and interactions of interest. Factors B and D were then set using contrasts 6=234 and 9=1234 with the result that BD and BE were confounded with H (one of the blocking factors) and CG. The final design is shown in Table 6. The runs within each block should be carried out in randomized order.

TABLE 6: The Final Design for the Sixteen-Run Experiment on Seven Factors in Four Blocks

	Factor Names								
	H	I	A	E	F	B	C	G	D
	1	2	3	4	5=123	6=234	7=134	8=124	9=1234
Block 1	-	-	-	-	-	-	-	-	+
	-	-	+	-	+	+	+	-	-
	-	-	-	+	-	+	+	+	-
	-	-	+	+	+	-	-	+	+
Block 2	+	-	-	-	+	-	+	+	-
	+	-	+	-	-	+	-	+	+
	+	-	-	+	+	+	-	-	+
	+	-	+	+	-	-	+	-	-
Block 3	-	+	-	-	+	+	-	+	-
	-	+	+	-	-	-	+	+	+
	-	+	-	+	+	-	+	-	+
	-	+	+	+	-	+	-	-	-
Block 4	+	+	-	-	-	+	+	-	+
	+	+	+	-	+	-	-	-	-
	+	+	-	+	-	-	-	+	-
	+	+	+	+	+	+	+	+	+

It should be noted that it took time and juggling to come up with the final design.

The experiment was run and factor F was found to be much more influential than any of the other factors. The result was a substantial reduction in the problem.

Although it was not needed in this example, the complete listing of the two factor interaction aliases of all the required main effects and interactions is given in Table 7. These were found using Table 3. If the predicted yield using settings recommended on the basis of significant two-factor interactions that the design was set up to measure is not obtained in follow-up work, further small experiments can be designed using the alias relationships in Table 7 and the methods described in Appendix 12B of Box, Hunter and Hunter (1978).

TABLE 7: Alias Relationships Between the Two-Factor Interactions for the Example

	Contrast Name	Effects Measured
	1	H (Block Differences), BD
	2	I (Block Differences), CD
	12	HI (Block Differences), AF, EG, CB
A	3	A, DG
	13	CE, BG
	23	CG, BE
F	5=123	F, DE
E	4	E, DF
	14	AC, BF
	24	CF, AB
G	6=124	G, AD
	34	AE, FG
C	7=134	C
B	8=234	B
D	9=1234	D AG, EF

It can be seen in the table that, with the exception of the aliases of factor D, the only aliases of main effects involve interactions with factor D. This was done on purpose since interactions with factor D were expected to be negligible except for the remote possibility of an interaction with factor B. The BD interaction is aliased with blocks (contrast H).

ACKNOWLEDGEMENT

I would like to thank Alison Burnham for her help in preparing the tables.

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TABLE 1: Signs Indicating Factor Settings for Four-, Eight-, Sixteen-, and Thirty Two-Run (Fractional) Factorial Designs

	EFFECT (CONTRAST) NAMES																													
Run	1	2	12	3	13	23	123	4	14	24	124	34	134	234	1234	5	15	25	125	35	135	235	1235	45	145	245	1245	345	1345	2345	12345
		4 Runs		8 Runs							16 Runs												32 Runs								
1	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-
2	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+
3	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+
4	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-
5	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+
6	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-
7	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-
8	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
9	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+
10	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-
11	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-
12	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+
13	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-
14	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
15	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
18	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
19	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
20	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
21	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
22	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
23	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
24	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
25	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
26	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
27	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
28	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
29	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
30	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
31	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
32	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 2: Fractional Factorial Designs for Up to Seven Factors in Eight Runs

Effects (Contrasts)	Exactly Four Factors (Resolution IV)	Five to Seven Factors (Resolution III)			
		5 Factors		6 Factors*	7 Factors*
--------	----------	---------		←--------	←--------
1		24	35		67
2		14		36	57
12	34	***4=12***		56	37
3			15	26	47
13	24		***5=13***	45	27
23	14		45	***6=23***	17
123	***4=123***	34	25	16	***7=123***

* And including all columns to the left *except* that for *exactly* four factors.

Table 3: Fractional Factorial Designs for Up to Twelve Factors in Sixteen Runs

Effects (Contrasts)	*Exactly* Five Factors (Resolution V)	Six to Eight Factors (Resolution IV)				Nine or More Factors (Resolution III)			
		6 Factors		7 Factors*	8 Factors*	9 Factors*	10 Factors*	11 Factors*	12 Factors*
1						69	$2\ \overline{10}$	$3\ \overline{11}$	$8\ \overline{12}$
2						79	$1\ \overline{10}$	$5\ \overline{11}$	$4\ \overline{12}$
12		35		67	48		***$\overline{10}$=12***		
3						89	$5\ \overline{10}$	$1\ \overline{11}$	$6\ \overline{12}$
13		25		47	68			***$\overline{11}$=13***	$9\ \overline{12}$
23		15	46		78			$\overline{10}\ \overline{11}$	
123	45	***5=123***				49	$3\ \overline{10}$	$2\ \overline{11}$	$7\ \overline{12}$
4						59	$8\ \overline{10}$	$7\ \overline{11}$	$2\ \overline{12}$
14			56	37	28				$\overline{10}\ \overline{12}$
24			36	57	18			$9\ \overline{11}$	***$\overline{12}$=24***
124	35				***8=124***	39	$4\ \overline{10}$	$6\ \overline{11}$	$1\ \overline{12}$
34			26	17	58		$9\ \overline{10}$		
134	25			***7=134***		29	$6\ \overline{10}$	$4\ \overline{11}$	$5\ \overline{12}$
234	15		***6=234***			19	$7\ \overline{10}$	$8\ \overline{11}$	$3\ \overline{12}$
1234	***5=1234***	45	16	27	38	***9=1234***			$\overline{11}\ \overline{12}$

* And including all columns to the left *except* that for *exactly* five.

TABLE 4: Fractional Factorial Designs for Up to Ten Factors in Thirty Two-Runs

Effects (Contrasts)	*Exactly* Six Factors (Resolution V)	Seven to Ten Factors (Resolution IV)				
		7 Factors		8 Factors*	9 Factors*	10 Factors*
1						
2						
12			67			
3						
13				68		
23				78		
123					59	4 $\overline{10}$
4						
14					69	
24					79	
124				58		3 $\overline{10}$
34					89	
134			57			2 $\overline{10}$
234		56				1 $\overline{10}$
1234	56					***$\overline{10}$=1234***
5						
15						6 $\overline{10}$
25						7 $\overline{10}$
125				48	39	
35						8 $\overline{10}$
135			47		29	
235		46			19	
1235	46				***9=1235***	
45						9 $\overline{10}$
145			37	28		
245		36		18		
1245	36			***8=1245***		
345		26	17			
1345	26		***7=1345***			
2345	16	***6=2345***				
12345	***6=12345***	16	27	38	49	5 $\overline{10}$

* And including all columns to the left *except* that for *exactly* six.

TABLE 5: Fractional Factorial Designs for Eleven or Twelve Factors in Thirty-two Runs

Effects (Contrasts)	Eleven or More Factors (Resolution IV)						
	11 Factors						12 Factors*
1							
2							
12	36			79		$\overline{10}\,\overline{11}$	$5\,\overline{12}$
3							
13	26			49	$8\,\overline{10}$		
23	16	47				$8\,\overline{11}$	
123	***6=123***						
4							
14		67		39	$5\,\overline{10}$		$\overline{11}\,\overline{12}$
24		37		69		$5\,\overline{11}$	$\overline{10}\,\overline{12}$
124							
34		27	58	19			
134				***9=134***			
234		***7=234***					
1234	46	17		29			$8\,\overline{12}$
5							
15				89	$4\,\overline{10}$		$2\,\overline{12}$
25			78			$4\,\overline{11}$	$1\,\overline{12}$
125							$\overline{12}=125$
35			48		$9\,\overline{10}$	$7\,\overline{11}$	$6\,\overline{12}$
135							
235							
1235	56				$7\,\overline{10}$	$9\,\overline{11}$	$3\,\overline{12}$
45			38		$1\,\overline{10}$	$2\,\overline{11}$	
145					$\overline{10}=145$		
245						$\overline{11}=245$	
1245			68		$2\,\overline{10}$	$1\,\overline{11}$	$4\,\overline{12}$
345			***8=345***				
1345			18	59	$3\,\overline{10}$	$6\,\overline{11}$	$7\,\overline{12}$
2345		57	28		$6\,\overline{10}$	$3\,\overline{11}$	$9\,\overline{12}$
12345							

* And all columns to the left.

March 29, 1995

Mr. Frank Caplin
Editor in Chief
Quality Engineering
22531 S.E. 42nd Court
Issaquah, WA
98027-7241

Dear Mr. Caplin:

I have enclosed two copies of my paper ***A Catalogue of Confounding Schemes for Eight-, Sixteen-, and Thirty Two-Run Fractional Factorial Designs*** for consideration for publication in **Quality Engineering**.

This short paper contains concise, compact tables that several of my industrial clients have found to be very helpful when designing fractional factorial experiments. In fact, they have recommended that I make the tables more readily available by publishing them. I felt your journal would make the tables available to the right audience.

Although other confounding tables exist (for example, Appendix XII in D.C. Montgomery's book "Design and Analysis of Experiments"), those in the enclosed paper are much more efficiently set up with, for instance, for sixteen-run designs, one table covering the equivalent of eight separate tables *as far as the confounding structure of most interest is concerned.* Software is also available, but only a few engineers have access to such software. Most have only Lotus or Excel (if that).

I have included some initial comments reflecting my concern regarding the decline in interest in designed experiments. You may not feel these are necessary as far as your readers are concerned.

As you can see, I feel the tables will be useful and hope you will consider them for publication.

Yours Truly,

J. C. Young

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A drawback of these designs is that the differences in means (contrasts) that measure the effects of interest may be influenced by two or more effects at the same time. Such effects are said to be confounded with each other (Box, Hunter and Hunter, 1978, Chapter 12). The tables at the end of this note are intended to save time in the planning of fractional factorial experiments by providing a very compact catalogue of the confounding patterns (alias structure) for the main effects and two-factor interactions for the most common designs. Higher order interactions are not included since they are less common and their inclusion would make the tables far too cumbersome.

The added complexity of confounding should not discourage people from using fractional factorial experiments. Their systematic or balanced approach to the study of several factors simultaneously makes them one of the most efficient and effective methods yet devised for the study of complex industrial processes when many factors are of interest and time for experimentation limited.

Unfortunately, as had already happened with control charts, factorial designs have often been promoted as being the solution to all of industries' problems. This has lead to designed experiments being attempted where they should not have been used. The result has been disappointment in the results of factorial experimentation and a corresponding decline in interest in their application. In fact, strict adherence to standard operating procedure and simple data analysis using stratification by shift, raw material lot, line, time of day and so on will solve many of the problems of excess variation in industry.

Even in situations where designs should have been effective, they have often failed. The reason is usually a lack of careful initial planning: People miss the significance of the word "design". Such basic requirements as the careful choice of factors and levels are often missing. Fundamental requirements of good design such as blocking, randomization and replication are rarely incorporated because they are rarely covered in adequate detail in industrial short courses and are very poorly understood by many industrial practitioners. An outstanding source of information on these and other fundamentals of the planning of experiments is the book by Cox (1958). Case studies illustrating their impact on industrial experimentation can be found in Young, Abraham and Whitney (1991) and Young (1994). The importance of these techniques cannot be over-emphasized. It is unfortunate to see this powerful tool being abandoned because of a lack of understanding of these fundamentals.

THE TABLES AND THEIR USE

The designs in the tables are essentially an extension of the designs in table 12.15 in Box, Hunter and Hunter (1978). In the tables, factor names and the corresponding contrasts for estimating their effects are indicated by numerals; thus 1, 2,...9, $\overline{10}$, $\overline{11}$, $\overline{12}$, and so on represent the specific factors to be studied. Interactions are then represented by number combinations; thus 235 represents the interaction (and corresponding contrast) for the three-factor interaction between factors 2, 3, and 5. Note that $\overline{12}$ is used to distinguish factor twelve and its contrast from the two-factor interaction 12. Thus 3 $\overline{12}$ represents the two-factor interaction between factors 3 and $\overline{12}$.

For convenience, Table 1 provides contrasts for all effects that can be measured in fractions of the 2^n series of designs in up to 32 runs. If preferred, the "+" and "-" signs can be replaced with the numerals "1" and "2" to provide designs equivalent to the L_4, L_8, L_{16} and L_{32} series of designs discussed in, for instance, Chapter 6 of Taguchi (1987). Note that, when the signs are used, interaction contrast signs are obtained as products of main effect signs. Thus the signs of the 235 interaction constrast are given by the product of the signs of the 2, 3, and 5 main effect contrasts. Many people find this fact useful in motivating intuitively the nature of the various interactions and the corresponding alias structure.

The designs in the tables are chosen to ensure that main effects and two-factor interactions are confounded with other effects of as high an order as possible. They thus maximize the *resolution* (Box, Hunter and Hunter, 1978, Section 12.4) of the design. The designs with resolution V in the tables ensure that main effects are confounded with four-factor interactions or higher and two-factor interactions with three-factor interactions or higher. At worst, resolution IV designs confound main effects with three-factor interactions and two-factor interactions with each other. In resolution III designs at least some main effects are confounded with two-factor interactions (but not each other). Normally, designs of maximum possible resolution are to be preferred.

For example, we see in Table 3 that a resolution V design for studying five factors in sixteen runs is possible by setting factor 5 using the signs of the four-factor interaction 1234. However, to achieve the highest possible resolution (IV) for six factors in sixteen runs we set factors 5 and 6 using the 123 and 234 interactions. Had we set 5=1234 and 6=123 then the 56 interaction would have been confounded with the 4 main effect and the resolution would have been III.

Although, in Table 3, levels of 5 and 6 have been set using the signs of 123 and 234, any pair of three-factor interactions could have been used. The designs shown are, for the most part, based on Table 12.15 of Box, Hunter and Hunter. There are, for the sake of consistency, a few slight departures. For instance, with six or more factors in a sixteen-run experiment, the fifth factor is always set using with the same three-factor interaction (234).

The actual settings of the additional factors in any design can be set using either the same signs as the interaction contrast used or exactly the opposite. For example, in order to study eight factors in thirty two runs, the extra factors can be set using

$$6 = \pm 2345$$

$$7 = \pm 1345$$

$$8 = \pm 1245$$

Further details on the choice of signs and the ramifications of the choice are found in Box, Hunter and Hunter.

AN EXAMPLE

Following the introduction of a new design of the control arm in the suspension of a motor vehicle, an undesirable level of cracks and foldbacks were found in a rubber bushing that was part of the assembly. After much discussion, management agreed to free up the line time necessary to perform a sixteen-run experiment. Brainstorming sessions resulted in the choice of the following seven factors and levels:

	Factor	Levels	
		-	+
A:	Position of Stop	3/8"	5/8"
B:	Bushing Radius	Current	Modified
C:	Insertion Speed	Low	High
D:	Alignment	No Shim	Shim
E:	Cone Design	Current	New
F:	Air Pressure	30 psi	45 psi
G:	Bushing Lubrication	Standard	High

In order to ensure that the conclusions would be reasonably reliable (accurate) it was decided to carry out the experiment in four blocks each consisting of a different batch of bushings. To further ensure that the experiment was truly representative of normal operating conditions the four blocks were spread out over two weeks. The four runs within each block were to be run under as nearly identical process conditions as possible to ensure high precision. The inclusion of the four blocks requires the use of two more contrasts which we will label H and I. Thus the design effectively involves nine factors. As well, the HI contrast will be affected by block differences, so a total of ten contrasts are used up by the basic design.

Since a sixteen-run design results in information on fifteen separate (orthogonal) contrasts, and since the above requirements have used up ten of them, only 5 are left to estimate interactions that are unconfounded with each other. After much discussion it was decided that the AE, AC, CE, CF and CG interactions were the most likely candidates, BE and BD interactions were also thought to be remotely possible. Although it was rather optimistic to hope that we could determine the most likely five of the twenty one possible interactions, it was agreed that the balanced factorial approach was the most reliable way to narrow down the possibilities.

The design was set up using Table 3 and assigning the blocking factors H and I to contrasts 1 and 2 and thus losing the 12 (HI) contrast to blocks as well. It was then noted that if factor C was set using the 7=134 contrast and A, E, F and G set using contrasts 3, 4, 5=123 and 8=124 then the required interactions AE, AC, CE, CF and CG would be estimated by the five unconfounded (with each other) contrasts 34, 14, 13, 24 and 23 respectively. The design thus has resolution IV in the main effects and interactions of interest. Factors B and D were then set using contrasts 6=234 and 9=1234 with the result that BD and BE were confounded with H (one of the blocking factors) and CG. The final design is shown in Table 6. The runs within each block should be carried out in randomized order.

TABLE 6: The Final Design for the Sixteen-Run Experiment on Seven Factors in Four Blocks

	Factor Names								
	H	I	A	E	F	B	C	G	D
	1	2	3	4	5=123	6=234	7=134	8=124	9=1234
Block 1	-	-	-	-	-	-	-	-	+
	-	-	+	-	+	+	+	-	-
	-	-	-	+	-	+	+	+	-
	-	-	+	+	+	-	-	+	+
Block 2	+	-	-	-	+	-	+	+	-
	+	-	+	-	-	+	-	+	+
	+	-	-	+	+	+	-	-	+
	+	-	+	+	-	-	+	-	-
Block 3	-	+	-	-	+	+	-	+	-
	-	+	+	-	-	-	+	+	+
	-	+	-	+	+	-	+	-	+
	-	+	+	+	-	+	-	-	-
Block 4	+	+	-	-	-	+	+	-	+
	+	+	+	-	+	-	-	-	-
	+	+	-	+	-	-	-	+	-
	+	+	+	+	+	+	+	+	+

It should be noted that it took time and juggling to come up with the final design.

The experiment was run and factor F was found to be much more influential than any of the other factors. The result was a substantial reduction in the problem.

Although it was not needed in this example, the complete listing of the two factor interaction aliases of all the required main effects and interactions is given in Table 7. These were found using Table 3. If the predicted yield using settings recommended on the basis of significant two-factor interactions that the design was set up to measure is not obtained in follow-up work, further small experiments can be designed using the alias relationships in Table 7 and the methods described in Appendix 12B of Box, Hunter and Hunter (1978).

TABLE 7: Alias Relationships Between the Two-Factor Interactions for the Example

Contrast Name		Effects Measured
	1	H (Block Differences), BD
	2	I (Block Differences), CD
	12	HI (Block Differences), AF, EG, CB
A	3	A, DG
	13	CE, BG
	23	CG, BE
F	5=123	F, DE
E	4	E, DF
	14	AC, BF
	24	CF, AB
G	6=124	G, AD
	34	AE, FG
C	7=134	C
B	8=234	B
D	9=1234	D AG, EF

It can be seen in the table that, with the exception of the aliases of factor D, the only aliases of main effects involve interactions with factor D. This was done on purpose since interactions with factor D were expected to be negligible except for the remote possibility of an interaction with factor B. The BD interaction is aliased with blocks (contrast H).

ACKNOWLEDGEMENT

I would like to thank Alison Burnham for her help in preparing the tables.

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TABLE 1: Signs Indicating Factor Settings for Four-, Eight-, Sixteen-, and Thirty Two-Run (Fractional) Factorial Designs

	EFFECT (CONTRAST) NAMES																													
Run	1	2	12	3	13	23	123	4	14	24	124	34	134	234	1234	5	15	25	125	35	135	235	1235	45	145	245	1245	345	1345	2345	12345
	4 Runs			8 Runs							16 Runs												32 Runs								
1	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-
2	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+
3	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+
4	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-
5	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+
6	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-
7	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-
8	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
9	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+
10	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-
11	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-
12	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+
13	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-
14	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
15	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
18	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
19	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
20	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
21	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
22	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
23	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
24	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
25	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
26	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
27	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
28	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
29	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
30	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
31	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
32	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 2: Fractional Factorial Designs for Up to Seven Factors in Eight Runs

Effects (Contrasts)	Exactly Four Factors (Resolution IV)	Five to Seven Factors (Resolution III)			
		5 Factors		6 Factors*	7 Factors*
1		24	35		67
2		14		36	57
12	34	***4=12***		56	37
3			15	26	47
13	24		***5=13***	45	27
23	14		45	***6=23***	17
123	***4=123***	34	25	16	***7=123***

* And including all columns to the left *except* that for *exactly* four factors.

Table 3: Fractional Factorial Designs for Up to Twelve Factors in Sixteen Runs

Effects (Contrasts)	*Exactly* Five Factors (Resolution V)	Six to Eight Factors (Resolution IV)			Nine or More Factors (Resolution III)			
		6 Factors	7 Factors*	8 Factors*	9 Factors*	10 Factors*	11 Factors*	12 Factors*
1					69	$2\ \overline{10}$	$3\ \overline{11}$	$8\ \overline{12}$
2					79	$1\ \overline{10}$	$5\ \overline{11}$	$4\ \overline{12}$
12		35	67	48		$\overline{10}=12$		
3					89	$5\ \overline{10}$	$1\ \overline{11}$	$6\ \overline{12}$
13		25	47	68			$\overline{11}=13$	$9\ \overline{12}$
23		15 46		78			$\overline{10}\ \overline{11}$	
123	45	***5=123***			49	$3\ \overline{10}$	$2\ \overline{11}$	$7\ \overline{12}$
4					59	$8\ \overline{10}$	$7\ \overline{11}$	$2\ \overline{12}$
14		56	37	28				$\overline{10}\ \overline{12}$
24		36	57	18			$9\ \overline{11}$	$\overline{12}=24$
124	35			***8=124***	39	$4\ \overline{10}$	$6\ \overline{11}$	$1\ \overline{12}$
34		26	17	58		$9\ \overline{10}$		
134	25		***7=134***		29	$6\ \overline{10}$	$4\ \overline{11}$	$5\ \overline{12}$
234	15	***6=234***			19	$7\ \overline{10}$	$8\ \overline{11}$	$3\ \overline{12}$
1234	***5=1234***	45 16	27	38	***9=1234***			$\overline{11}\ \overline{12}$

* And including all columns to the left *except* that for *exactly* five.

TABLE 4: Fractional Factorial Designs for Up to Ten Factors in Thirty Two-Runs

Effects (Contrasts)	*Exactly* Six Factors (Resolution V)	Seven to Ten Factors (Resolution IV)				
		7 Factors		8 Factors*	9 Factors*	10 Factors*
1						
2						
12			67			
3						
13				68		
23				78		
123					59	4 $\overline{10}$
4						
14					69	
24					79	
124				58		3 $\overline{10}$
34					89	
134			57			2 $\overline{10}$
234		56				1 $\overline{10}$
1234	56					*$\overline{10}$=1234*
5						
15						6 $\overline{10}$
25						7 $\overline{10}$
125				48	39	
35						8 $\overline{10}$
135			47		29	
235		46			19	
1235	46				*9=1235*	
45						9 $\overline{10}$
145			37	28		
245		36		18		
1245	36			*8=1245*		
345		26	17			
1345	26		*7=1345*			
2345	16	*6=2345*				
12345	*6=12345*	16	27	38	49	5 $\overline{10}$

* And including all columns to the left *except* that for *exactly* six.

TABLE 5: Fractional Factorial Designs for Eleven or Twelve Factors in Thirty-two Runs

Effects (Contrasts)	Eleven or More Factors (Resolution IV)						
	11 Factors						12 Factors*
1							
2							
12	36			79		$\overline{10}\,\overline{11}$	5 $\overline{12}$
3							
13	26			49	8 $\overline{10}$		
23	16	47				8 $\overline{11}$	
123	***6=123***						
4							
14		67		39	5 $\overline{10}$		$\overline{11}\,\overline{12}$
24		37		69		5 $\overline{11}$	$\overline{10}\,\overline{12}$
124							
34		27	58	19			
134				***9=134***			
234		***7=234***					
1234	46	17		29			8 $\overline{12}$
5							
15				89	4 $\overline{10}$		2 $\overline{12}$
25			78			4 $\overline{11}$	1 $\overline{12}$
125							$\overline{12}=125$
35			48		9 $\overline{10}$	7 $\overline{11}$	6 $\overline{12}$
135							
235							
1235	56				7 $\overline{10}$	9 $\overline{11}$	3 $\overline{12}$
45			38		1 $\overline{10}$	2 $\overline{11}$	
145					$\overline{10}=145$		
245						$\overline{11}=245$	
1245			68		2 $\overline{10}$	1 $\overline{11}$	4 $\overline{12}$
345			***8=345***				
1345			18	59	3 $\overline{10}$	6 $\overline{11}$	7 $\overline{12}$
2345		57	28		6 $\overline{10}$	3 $\overline{11}$	9 $\overline{12}$
12345							

* And all columns to the left.

November 28, 1994

Dr. Douglas C. Montgomery
Editor-Elect
Journal of Quality Technology
Dept. of Industrial & Management
Systems Engineering
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Tempe, AZ 85287-5906

Dear Dr. Montgomery:

I have enclosed a short note which inlcudes confounding tables for fracitonal factorial designs for up to twelve factors in 32 runs. The tables provide a very concise suppliment to table 12.15 in Box, Hunter and Hunter. I use it with my students here on campus and in industrial short courses that I teach. Students find it very helpful and have recommended that I make it more broadly available. I felt your journal would make the tables available to the right audience. I hope you will consider it for publication.

Yours Truly,

J. Clifton Young

A CATALOGUE OF CONFOUNDING SCHEMES FOR EIGHT-, SIXTEEN-, AND THIRTY TWO-RUN FRACTIONAL FACTORIAL DESIGNS

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Abstract

A set of tables is provided showing maximum resolution designs and the alias structure for all main effects and two-factor interactions for fractional factorial designs with eight-, sixteen-, and thirty two-runs. An example of their use is included.

Key words: Fractional factorial designs, Confounding schemes, Alias structure, Planning of experiments, Experimental design, Interactions.

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INTRODUCTION

Engineers involved in industrial process improvement are often confronted with many potentially influential factors and, because of pressure to meet production schedules, a severely limited amount of line time available for experimentation. In order to screen out those few factors with the greatest influence, fractions of 2^n factorial experiments are frequently used. Eight-, sixteen-, and occasionally thirty two-run fractions are most common.

A drawback of these designs is that the differences in means (contrasts) that measure the effects of interest may be influenced by two or more effects at the same time. Such effects are said to be confounded with each other (Box, Hunter and Hunter, 1978, Chapter 12). The tables at the end of this note are intended to save time in the planning of fractional factorial experiments by providing a catalogue of the confounding patterns (alias structure) for the main effects and two-factor interactions for the most common designs. Higher order interactions are not included since they are less common and their inclusion would make the tables far too cumbersome. Methods for designing small follow-up studies that will resolve any uncertainty regarding two or more confounded effects are described in Appendix 12B of Box, Hunter, and Hunter. The added complexity of confounding should not discourage people from using fractional factorial experiments. Their systematic or balanced approach to the study of several factors simultaneously makes them one of the most efficient and effective methods yet devised for the study of complex industrial processes when many factors are of interest and time for experimentation limited.

Unfortunately, as had already happened with control charts, factorial designs have often been promoted as being the solution to all of industries' problems. This has lead to designed experiments being attempted where they should not have been used. The result has been disappointment in the results of factorial experimentation and a corresponding decline in interest in their application. In fact, strict adherence to standard operating procedure and simple data analysis using stratification by shift, raw material lot, line, time of day and so on will solve many of the problems of excess variation in industry.

Even in situations where designs should have been effective, they have often failed. The reason is usually a lack of careful initial planning: People miss the significance of the word "design". Such basic requirements as the careful choice of factors and levels are often missing. Fundamental requirements of good design such as blocking, randomization and replication are rarely incorporated because they are rarely covered in adequate detail in industrial short courses and are very poorly understood by many industrial practitioners. An outstanding source of information on these and other fundamentals of the planning of experiments is the book by Cox (1958). Case studies illustrating their impact on industrial experimentation can be found in Young, Abraham and Whitney (1991) and Young (1994). The importance of these techniques cannot be over-emphasized. It is unfortunate to see this powerful tool being abandoned because of a lack of understanding of these fundamentals.

THE TABLES AND THEIR USE

The designs in the tables are essentially an extension of the designs in table 12.15 in Box, Hunter and Hunter (1978). In the tables, factor names and the corresponding contrasts for estimating their effects are indicated by numerals; thus 1, 2,...9, $\overline{10}$, $\overline{11}$, $\overline{12}$, and so on represent the specific factors to be studied. Interactions are then represented by number combinations; thus 235 represents the interaction (and corresponding contrast) for the three-factor interaction between factors 2, 3, and 5. Note that $\overline{12}$ is used to distinguish factor twelve and its contrast from the two-factor interaction 12. Thus 3 $\overline{12}$ represents the two-factor interaction between factors 3 and $\overline{12}$.

For convenience, Table 1 provides contrasts for all effects that can be measured in fractions of the 2^n series of designs in up to 32 runs. If preferred, the "+" and "-" signs can be replaced with the numerals "1" and "2" to provide designs equivalent to the L_4, L_8, L_{16} and L_{32} series of designs discussed in, for instance, Chapter 6 of Taguchi (1987). Note that, when the signs are used, interaction contrast signs are obtained as products of main effect signs. Thus the signs of the 235 interaction constrast are given by the product of the signs of the 2, 3, and 5 main effect contrasts. Many people find this fact useful in motivating intuitively the nature of the various interactions and the corresponding alias structure.

The designs in the tables are chosen to ensure that main effects and two-factor interactions are confounded with other effects of as high an order as possible. They thus maximize the *resolution* (Box, Hunter and Hunter, 1978, Section 12.4) of the design. The designs with resolution V in the tables ensure that main effects are confounded with four-factor interactions or higher and two-factor interactions with three-factor interactions or higher. At worst, resolution IV designs confound main effects with three-factor interactions and two-factor interactions with each other. In resolution III designs at least some main effects are confounded with two-factor interactions (but not each other). Normally, designs of maximum possible resolution are to be preferred.

For example, we see in Table 3 that a resolution V design for studying five factors in sixteen runs is possible by setting factor 5 using the signs of the four-factor interaction 1234. However, to achieve the highest possible resolution (IV) for six factors in sixteen runs we set factors 5 and 6 using the 123 and 234 interactions. Had we set 5=1234 and 6=123 then the 56 interaction would have been confounded with the 4 main effect and the resolution would have been III.

Although, in Table 3, levels of 5 and 6 have been set using the signs of 123 and 234, any pair of three-factor interactions could have been used. The designs shown are, for the most part, based on Table 12.15 of Box, Hunter and Hunter. There are, for the sake of consistency, a few slight departures. For instance, with six or more factors in a sixteen-run experiment, the fifth factor is always set using with the same three-factor interaction (234).

The actual settings of the additional factors in any design can be set using either the same signs as the interaction contrast used or exactly the opposite. For example, in order to study eight factors in thirty two runs, the extra factors can be set using

$$6 = \pm\ 2345$$

$$7 = \pm\ 1345$$

$$8 = \pm\ 1245$$

Further details on the choice of signs and the ramifications of the choice are found in Box, Hunter and Hunter.

AN EXAMPLE

Following the introduction of a new design of the control arm in the suspension of a motor vehicle, an undesirable level of cracks and foldbacks were found in a rubber bushing that was part of the assembly. After much discussion, management agreed to free up the line time necessary to perform a sixteen-run experiment. Brainstorming sessions resulted in the choice of the following seven factors and levels:

	Factor	Levels	
		-	+
A:	Position of Stop	3/8"	5/8"
B:	Bushing Radius	Current	Modified
C:	Insertion Speed	Low	High
D:	Alignment	No Shim	Shim
E:	Cone Design	Current	New
F:	Air Pressure	30 psi	45 psi
G:	Bushing Lubrication	Standard	High

In order to ensure that the conclusions would be reasonably reliable (accurate) it was decided to carry out the experiment in four blocks each consisting of a different batch of bushings. To further ensure that the experiment was truly representative of normal operating conditions the four blocks were spread out over two weeks. The four runs within each block were to be run under as nearly identical process conditions as possible to ensure high precision. The inclusion of the four blocks requires the use of two more contrasts which we will label H and I. Thus the design effectively involves nine factors. As well, the HI contrast will be affected by block differences, so a total of ten contrasts are used up by the basic design.

Since a sixteen-run design results in information on fifteen separate (orthogonal) contrasts, and since the above requirements have used up ten of them, only 5 are left to estimate interactions that are unconfounded with each other. After much discussion it was decided that the AE, AC, CE, CF and CG interactions were the most likely candidates, BE and BD interactions were also thought to be remotely possible. Although it was rather optimistic to hope that we could determine the most likely five of the twenty one possible interactions, it was agreed that the balanced factorial approach was the most reliable way to narrow down the possibilities.

The design was set up using Table 3 and assigning the blocking factors H and I to contrasts 1 and 2 and thus losing the 12 (HI) contrast to blocks as well. It was then noted that if factor C was set using the 7=134 contrast and A, E, F and G set using contrasts 3, 4, 5=123 and 8=124 then the required interactions AE, AC, CE, CF and CG would be estimated by the five unconfounded (with each other) contrasts 34, 14, 13, 24 and 23 respectively. The design thus has resolution IV in the main effects and interactions of interest. Factors B and D were then set using contrasts 6=234 and 9=1234 with the result that BD and BE were confounded with H (one of the blocking factors) and CG. The final design is shown in Table 6. The runs within each block should be carried out in randomized order.

TABLE 6: The Final Design for the Sixteen-Run Experiment on Seven Factors in Four Blocks

	Factor Names								
	H	I	A	E	F	B	C	G	D
	1	2	3	4	5=123	6=234	7=134	8=124	9=1234
Block 1	-	-	-	-	-	-	-	-	+
	-	-	+	-	+	+	+	-	-
	-	-	-	+	-	+	+	+	-
	-	-	+	+	+	-	-	+	+
Block 2	+	-	-	-	+	-	+	+	-
	+	-	+	-	-	+	-	+	+
	+	-	-	+	+	+	-	-	+
	+	-	+	+	-	-	+	-	-
Block 3	-	+	-	-	+	+	-	+	-
	-	+	+	-	-	-	+	+	+
	-	+	-	+	+	-	+	-	+
	-	+	+	+	-	+	-	-	-
Block 4	+	+	-	-	-	+	+	-	+
	+	+	+	-	+	-	-	-	-
	+	+	-	+	-	-	-	+	-
	+	+	+	+	+	+	+	+	+

It should be noted that it took time and juggling to come up with the final design.

The experiment was run and factor F was found to be much more influential than any of the other factors. The result was a substantial reduction in the problem.

Although, as it turned out, it was not needed in this example, the complete listing of the two factor interaction aliases of all the required main effects and interactions is given in Table 7. These were found using Table 3. If the predicted yield using settings recommended on the basis of significant two-factor interactions that the design was set up to measure is not obtained in follow-up work, further small experiments can be designed using the alias relationships in Table 7 and the methods described in Appendix 12B of Box, Hunter and Hunter (1978).

TABLE 7: Alias Relationships Between the Two-Factor Interactions for the Example

	Contrast Name	Effects Measured
	1	H (Block Differences), BD
	2	I (Block Differences), CD
	12	HI (Block Differences), AF, EG, CB
A	3	A, DG
	13	CE, BG
	23	CG, BE
F	5=123	F, DE
E	4	E, DF
	14	AC, BF
	24	CF, AB
G	6=124	G, AD
	34	AE, FG
C	7=134	C
B	8=234	B
D	9=1234	D AG, EF

It can be seen in the table that, with the exception of the aliases of factor D, the only aliases of main effects involve interactions with factor D. This was done on purpose since interactions with factor D were expected to be negligible except for the remote possibility of an interaction with factor B. The BD interaction is aliased with blocks (contrast H).

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TABLE 1: Signs Indicating Factor Settings for Four-, Eight-, Sixteen-, and Thirty Two-Run (Fractional) Factorial Designs

	EFFECT (CONTRAST) NAMES																														
Run	1	2	12	3	13	23	123	4	14	24	124	34	134	234	1234	5	15	25	125	35	135	235	1235	45	145	245	1245	345	1345	2345	12345
	4 Runs			8 Runs							16 Runs												32 Runs								
1	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-
2	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+
3	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+
4	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-
5	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+
6	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-
7	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-
8	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+
9	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+
10	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-
11	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-
12	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+
13	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-
14	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+
15	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+
16	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
17	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+
18	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-
19	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-
20	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+
21	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-
22	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+
23	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+
24	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-
25	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-
26	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+
27	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+
28	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-
29	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+
30	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-
31	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-
32	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+

Table 2: Fractional Factorial Designs for Up to Seven Factors in Eight Runs

Effects (Contrasts)	Exactly Four Factors (Resolution IV)	Five to Seven Factors (Resolution III)			
		5 Factors		6 Factors*	7 Factors*
--------	----------	---------		←--------	←--------
1		24	35		67
2		14		36	57
12	34	***4=12***		56	37
3			15	26	47
13	24		***5=13***	45	27
23	14		45	***6=23***	17
123	***4=123***	34	25	16	***7=123***

* And including all columns to the left *except* that for *exactly* four factors.

Table 3: Fractional Factorial Designs for Up to Twelve Factors in Sixteen Runs

Effects (Contrasts)	*Exactly* Five Factors (Resolution V)	Six to Eight Factors (Resolution IV)				Nine or More Factors (Resolution III)			
		6 Factors		7 Factors* ←	8 Factors* ←	9 Factors* ←	10 Factors* ←	11 Factors* ←	12 Factors* ←
1						69	2 $\overline{10}$	3 $\overline{11}$	8 $\overline{12}$
2						79	1 $\overline{10}$	5 $\overline{11}$	4 $\overline{12}$
12		35		67	48		***$\overline{10}$=12***		
3						89	5 $\overline{10}$	1 $\overline{11}$	6 $\overline{12}$
13		25		47	68			***$\overline{11}$=13***	9 $\overline{12}$
23		15	46		78			$\overline{10}$ $\overline{11}$	
123	45	***5=123***				49	3 $\overline{10}$	2 $\overline{11}$	7 $\overline{12}$
4						59	8 $\overline{10}$	7 $\overline{11}$	2 $\overline{12}$
14			56	37	28				$\overline{10}$ $\overline{12}$
24			36	57	18			9 $\overline{11}$	***$\overline{12}$=24***
124	35				***8=124***	39	4 $\overline{10}$	6 $\overline{11}$	1 $\overline{12}$
34			26	17	58		9 $\overline{10}$		
134	25			***7=134***		29	6 $\overline{10}$	4 $\overline{11}$	5 $\overline{12}$
234	15		***6=234***			19	7 $\overline{10}$	8 $\overline{11}$	3 $\overline{12}$
1234	***5=1234***	45	16	27	38	***9=1234***			$\overline{11}$ $\overline{12}$

* And including all columns to the left *except* that for *exactly* five.

TABLE 4: Fractional Factorial Designs for Up to Ten Factors in Thirty Two-Runs

Effects (Contrasts)	*Exactly* Six Factors (Resolution V)	Seven to Ten Factors (Resolution IV)				
		7 Factors		8 Factors*	9 Factors*	10 Factors*
				←	←	←
1						
2						
12			67			
3						
13				68		
23				78		
123					59	$4\ \overline{10}$
4						
14					69	
24					79	
124				58		$3\ \overline{10}$
34					89	
134			57			$2\ \overline{10}$
234		56				$1\ \overline{10}$
1234	56					$\overline{10}=1234$
5						
15						$6\ \overline{10}$
25						$7\ \overline{10}$
125				48	39	
35						$8\ \overline{10}$
135			47		29	
235		46			19	
1235	46				*9=1235*	
45						$9\ \overline{10}$
145			37	28		
245		36		18		
1245	36			*8=1245*		
345		26	17			
1345	26		*7=1345*			
2345	16	*6=2345*				
12345	*6=12345*	16	27	38	49	$5\ \overline{10}$

* And including all columns to the left *except* that for *exactly* six.

TABLE 5: Fractional Factorial Designs for Eleven or Twelve Factors in Thirty-two Runs

Effects (Contrasts)	Eleven or More Factors (Resolution IV)						
	11 Factors						12 Factors*
1							
2							
12	36			79		$\overline{10}\ \overline{11}$	5 $\overline{12}$
3							
13	26			49	8 $\overline{10}$		
23	16	47				8 $\overline{11}$	
123	***6=123***						
4							
14		67		39	5 $\overline{10}$		$\overline{11}\ \overline{12}$
24		37		69		5 $\overline{11}$	$\overline{10}\ \overline{12}$
124							
34		27	58	19			
134				***9=134***			
234		***7=234***					
1234	46	17		29			8 $\overline{12}$
5							
15				89	4 $\overline{10}$		2 $\overline{12}$
25			78			4 $\overline{11}$	1 $\overline{12}$
125							***$\overline{12}$=125***
35			48		9 $\overline{10}$	7 $\overline{11}$	6 $\overline{12}$
135							
235							
1235	56				7 $\overline{10}$	9 $\overline{11}$	3 $\overline{12}$
45			38		1 $\overline{10}$	2 $\overline{11}$	
145					***$\overline{10}$=145***		
245						***$\overline{11}$=245***	
1245			68		2 $\overline{10}$	1 $\overline{11}$	4 $\overline{12}$
345			***8=345***				
1345			18	59	3 $\overline{10}$	6 $\overline{11}$	7 $\overline{12}$
2345		57	28		6 $\overline{10}$	3 $\overline{11}$	9 $\overline{12}$
12345							

* And all columns to the left.