PARAMETER DESIGN FOR SIGNAL-RESPONSE SYSTEMS

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IIQP Research Report
RR-95-08

June 1995

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ABSTRACT

Most of the literature on robust parameter design is concerned with simple response or "static characteristics" systems. A more recent trend in industrial practice is to consider more complex systems which are called signal-response systems in this paper or alternatively "systems with dynamic characteristics" in Taguchi's terminology. This potentially important tool in quality engineering lacks a solid basis on which to build a rigorous body of theory and methodology. The purpose of this paper is to provide a suitable basis. We argue that the problem has two distinct types, measurement systems and multiple target systems, and that three issues are of fundamental importance. First a proper performance measure needs to be chosen for system optimization and this choice depends on the type of system. Second there are two modelling and analysis strategies for such data: performance measure modelling and response function modelling. Finally the proper design of such experiments should take into account the modelling and analysis strategy. The proposed methodology is illustrated with a real experiment on injection moulding.

Key Words: measurement systems, multiple target systems, robust parameter design, performance measures, response function modelling.

1 Introduction

Taguchi (1987) introduced the robust parameter design (RPD) methodology as a method of improving the quality of a process or product by making it less sensitive to noise variation. There has been much discussion in the literature about his methodology and alternative procedures have been proposed. See Nair et al. (1992) and Box (1988) for general discussions. Most of the statistical literature concerning robust parameter design involve situations where the characteristic of interest can be described as a single quantity, Y, which has a specified optimal value. For example, Shoemaker, Tsui, and Wu (1991) presented a case study involving the plating of silicon layers on wafers. In this case, the response was the thickness of the silicon layer. A target value of 14.5 micrometers had been identified and the goal of the experiment was to determine conditions which would result in the distribution of Y being as concentrated as possible around this value. Another example, presented by Pignatiello and Ramberg (1985), involves a heat treating process of leaf springs used on trucks. The objective was to develop a process which would result in springs whose free height was as close as possible to the target value of 8 inches. Taguchi used the somewhat confusing term "static characteristic" to refer to this type of application. We call this type of application a *simple response system.*

Taguchi also identified a second type of application for robust parameter design methodology which he called "dynamic characteristics". Simply stated, this refers to situations where the response is required to assume different values as a result of changes in a signal factor, M. For example a method for determining the amount of calcium in a water sample is required to produce different responses for different amounts of calcium. The performance of the system can only be evaluated by considering the *relationship* between the response and the actual amount of calcium present in samples (signal). As the term dynamic is somewhat misleading we will call it a *signal-response system* because the interest focuses on the relationship between the output response and the signal factor. Since many engineering systems can be adequately described as signal-response systems, this methodology has become increasingly important in engineering applications. See, for example, many case studies

in the American Supplier Institute Symposia on Taguchi Methods(Dearborn, Michigan). In spite of the practical impact, signal-response systems have received little attention in the research literature.

This article presents a broad view and systematic development of the RPD methodology as applied to signal-response systems. First we classify the signal response systems into two distinct types: measurement systems and multiple target systems (Section 2). This classification, which was not considered by Taguchi, plays an important role in our methodological development. For example, the choice of performance measure (for parameter design optimization) varies with the type of system. Taguchi's ubiquitous dynamic signal-to-noise (SN) ratio (see (2)) turns out to be appropriate for the measurement system, but not for the multiple target system (see Section 3). With the system identified and the performance measure chosen, two remaining issues are modeling and analysis of data (Section 4), and strategies for designing experiments (Section 5). Two modeling strategies are considered: performance measure modeling and response function modeling. The former includes Taguchi's SN ratio analysis as a special case, while the latter is a more flexible approach and can be tailored for each type of system. The analysis strategies are illustrated in Section 6 on a real experiment on injection molding. Some concluding remarks are made in Section 7.

2 Classification of Signal-Response Systems

It is useful to classify signal-response systems into two broad types based on the function of the system. We focus on two common types of signal response systems in this article: *measurement systems* and *multiple target systems.*

A measurement system is the process used to obtain an estimate of some quantity of interest for a given unit or sample. This may include sampling, sample preparation, and calibration, as well as the actual measurement process. The true amount of the quantity present can be considered as an input signal, M, which the system converts into a measured value or response, Y. The precision with which M can be estimated based on Y is determined by the characteristics of the relationship between M and Y.

As an example, Miller and Wu (1991) analyzed data from an experiment originally described by Taguchi (1987), whose purpose was to identify levels of controllable factors which produced the most precise measurements of drive shaft imbalance in automotive manufacturing.

A multiple target system is a system whose function requires that the value of a response quantity can be adjusted by changing the level of a signal factor. For example, an injection molding machine may be used to produce a number of different parts by attaching different molds to the main apparatus. The function of the system is to inject material into the mold. Obviously, molds of different sizes would require the system to inject different amounts of material. Therefore, some method of controlling the amount of material delivered to the mold is required. DeMates (1990) described an experiment involving an injection molding process. As the system was used to produce a number of different parts, it was required to inject different amounts of material for different applications. Therefore, a reliable method of controlling the amount of material injected was needed.

Factor	$X = 1$	$X = -1$
A: Injection Speed	2.0	0.0
B: Clamp Time	44 s	49 s
C: High Injection Time	6.3 s	6.8 s
D: Low Injection Time	17 s	20 s
E: Clamp Pressure	1900 psi	1700 psi
F: Water Cooling	70 F	80 F
G: Low Injection Pressure	650 psi	550 psi

Table 1: Control Factors for Injection Molding Case Study

Part weight was adopted as the response and high injection pressure was chosen as the signal factor due to its known ability to change the amount of material injected. Eight levels of high injection pressure were used in the experiment (see column entries in the Appendix). Seven control factors, each at two levels, were included in the experiment (see Table 1). The term *control factors* designates factors which can be readily adjusted to different levels by the process operator and once set remain

Label	Factor	Levels
$X_N = 1$	Melt Index	22
	Percent Re-grind	0%
	Operator	Experienced
	Resin Moisture	Low
$X_N = -1$	Melt Index	18
	Percent Re-grind	5%
	Operator	New
	Resin Moisture	High

Table 2: Compound Noise Factor for Injection Molding Case Study

constant during the operation of the system. These factors were chosen since they were thought to have the potential of affecting variability. One compound noise factor which represents the settings of four confounded noise factors was used (see Table 2). A *noise factor* is a factor which may vary during the operation of the system (and thus contribute to variability in the response) but can be held constant for the purposes of an experiment. Noise factors are often included in RPD experiments to insure that important sources of variability are investigated. In some experiments compound noise factors are used to reduce the required number of runs (see Phadke, 1989). A discussion of compound noise factors is given in Section 5.

Another example of a multiple target system can be found in Yano (1991, page 293). In this case, the quality characteristic of interest is the surface roughness of parts after the surface has been machined using a lathe. Since different applications require a different degree of surface roughness, some method of controlling the surface roughness of machined parts is required. Experience indicated that the feed rate of the tool bit could effectively be used to alter the roughness of the machined surface. Therefore, it was chosen as the signal factor. It was thought that factors such as lathe, cutting speed, depth of tool cut, type of tool cut, corner radius, cutting edge angle, front escape angle, and side scoop angle may affect the relationship between feed rate and surface roughness. An experiment was conducted to select settings for these control factors which would allow surface roughness to be reliably controlled by

the feed rate of the tool bit.

3 Performance Measures

The logical first step in examining a signal-response system is to develop a performance measure (PM) which evaluates the suitability of a given signal-response relationship for the intended application. By optimizing the chosen measure, control factor settings which achieve the desired engineering objectives can be identified.

One approach to identifying a suitable PM is to specify an ideal or target signal-response relationship and penalize for deviations from this target function. (This relationship is called an *ideal function* in Taguchi's terminology.) Taguchi recommends a PM for signal-response systems (see Phadke 1989, page 114) that can be motivated in the following manner. Suppose the target (or ideal) function is of the form

$$E(Y) = \beta_t M,$$

where β_t is the target slope and the actual signal-response relationship can be represented by

$$Y = f(M) + \epsilon\,, \quad \text{where} \quad \mathrm{E}(\epsilon) = 0, \;\; \mathrm{V}(\epsilon) = \sigma^2\,. \tag{1}$$

A PM can then be generated by averaging the mean square error (MSE) over a specified range for the signal factor, say (m_a, m_b),

$$\begin{aligned} \mathrm{PM} &= \int_{m_a}^{m_b} \mathrm{MSE}\;\, dm \\ &= \int_{m_a}^{m_b} \mathrm{E}\left[f(m) - \beta_t m + \epsilon\right]^2 \;\, dm \\ &= \int_{m_a}^{m_b} \left[(f(m) - \beta_t m)^2 + \sigma^2\right] \;\, dm \end{aligned}$$

Taguchi's dynamic SN ratio is based on the average MSE with one modification, i.e. he assumed the existence of a special type of control factor called an *adjustment factor* which affects the system in the same manner as a change of scale (see Leon, Shoemaker, and Kacker 1987 for a discussion of adjustment factors). Suppose for a fixed set of control factor levels we have the true signal-response relationship given

in (1). Then by changing the adjustment factor we can obtain any signal-response relationship of the form $Y = c\,[f(M) + \epsilon]$, where c is any positive constant. To see how the adjustment factor is used in practice, suppose a set of observations are made for a fixed set of control factor levels, and let y_{ij} represent the jth observed response at the ith signal level (M_i). Now consider the least squares fit to the model

$$\mathrm{E}(y_{ij}) = \beta M_i.$$

Let $\hat{\beta}$ represent the least squares estimate of β and $s^2 = (n-1)^{-1} \sum_i \sum_j (y_{ij} - \hat{\beta} M_i)^2$, where n is the total number of observations. So $\hat{\beta}$ represents the estimated slope for the best fitting linear model and s^2 represents the estimated MSE for this model (averaged over the signal levels). If we assume the target function is $\beta_t M$ then we would wish to use the adjustment factor to scale the signal response relationship by a factor of $\beta_t/\hat{\beta}$. Given this adjustment, the projected MSE would be $(\beta_t/\hat{\beta})^2 s^2$. Minimizing this MSE is equivalent to maximizing

$$\log(\hat{\beta}^2/s^2), \tag{2}$$

which he called the *dynamic SN ratio.* Taguchi's dynamic SN ratio is based on the objectives that the ideal signal-response relationship should be:

1. linear,

2. robust to uncontrolled factors.

Figure 1 illustrates the difference between what Taguchi would consider a good signal-response system (a) and poor signal response systems (b and c). Each line represents an observed signal response curve for a given set of noise factor conditions. The first system is linear and relatively insensitive to noise factors whereas the second system is sensitive to noise and the third system is non-linear.

Two major criticisms can be made of Taguchi's dynamic SN ratio. First the assumption that an "ideal" signal response relationship can always be identified seems too restrictive. For example, we might ask whether is it necessary for a measurement system to be based on a linear signal-response relationship? Consider a situation

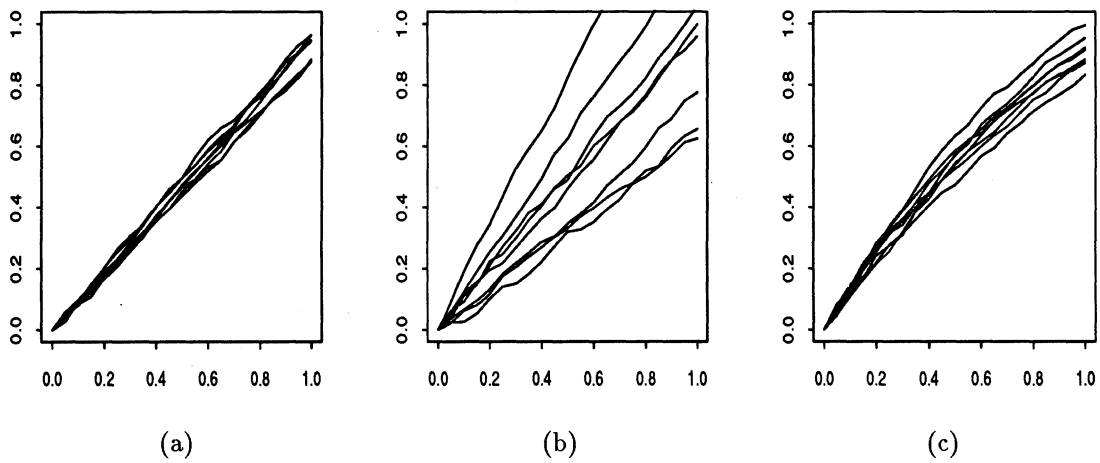


Figure 1: Examples of Signal-Response Relationships

where the relationship is not linear in the original metrics of the response and signal factor but can be made linear by a suitable transformation of the signal factor. In this case, a PM based on a linear ideal function may well yield different values depending on whether the original or transformed metric is used for the signal factor. Second, the assumption that an adjustment factor which acts like a scaling factor exists does not hold in many practical situations. An adjustment factor, if available, is usually identified based on physical knowledge.

Rather than developing a performance measure by identifying an ideal signal response relationship and then penalizing departures from this ideal, we prefer to base performance measures directly on the ability of a system to perform its designated function. In the following subsections these performance measures will be developed separately for two types of systems: measurement systems and multiple target systems.

3.1 Measurement Systems

As the purpose of a measurement system is to obtain an estimate of some quantity of interest, it is reasonable that the system should be evaluated with respect to the

precision of estimates obtained.

Mandel (1964, page 366) developed a "criterion for technical merit", as a method of comparing the relative merits of measurement systems. Mandel only considered systems where the variation of the response was constant across signal levels. Let $\mathrm{E}(Y) = g(M)$ and $\mathrm{V}(Y) = \sigma^2$, where g is an invertible function with inverse g^{-1}. In this case, assuming $g(M)$ is known, the variance of $\hat{M} = g^{-1}(y_{obs})$ can be approximated by $\sigma^2/\left[g'(M)\right]^2$. Mandel defined $\sigma/|g'(M)|$ as his criterion. Although Mandel only considered the comparison of different systems, his criterion could also be used to optimize a particular system.

Taguchi (1987, page 629) specifically considered the optimization of measurement systems which are based on linear calibration curves. Suppose $\mathrm{E}(Y) = \beta_0 + \beta_1 M$ and $\mathrm{V}(Y) = \sigma^2$. Taguchi, in essence, adopted $\omega = \beta_1^2/\sigma^2$ as a performance measure (SN ratio) which he justified on the basis that it is the reciprocal of the estimation variance for $\hat{M} = (y_{obs} - \beta_0)/\beta_1$, provided β_0, β_1, and σ^2 are assumed known. Clearly, Taguchi's SN ratio is a special case of Mandel's criterion.

The measures developed by Taguchi and Mandel assume that the error variance is constant over the levels of the signal factor. There are applications which require this assumption to be relaxed. For example, Bocek and Novak (1970) discussed the use of gas chromatography for quantitative analysis and indicated that under certain circumstances this involves non-constant variance. Suppose the system can be meaningfully represented by a model consisting of a location function and a variance function as follows:

$$\begin{aligned} \mathrm{E}(Y) &= g(M), \\ \mathrm{V}(Y) &= h(M), \end{aligned} \tag{3}$$

where g is a monotonic function of M. Then for an observed value of $Y = y_{obs}$, the classical estimator of M is

$$\hat{M}(y_{obs}) = g^{-1}\left(y_{obs}\right) ,$$

where g^{-1} is the inverse function of g. The variance of $\hat{M}$ is approximately

$$\mathrm{V}(\hat{M}|y_{obs}) \approx \mathrm{V}(Y|M = m_t)/\left[g'(m_t)\right]^2$$

$$= \quad h(m_t)/\left[g'(m_t)\right]^2,$$

where $g'(M) = dg(M)/dM$ and m_t is the true value of M for the unit being tested.

As the variance of the estimate depends on the true signal level, a performance measure can be obtained by integrating $\mathrm{V}(\hat{M})$ over the required range of signal values, (m_a, m_b), for the application being considered. A general form for a performance measure would be

$$\mathrm{PM} = \int_{m_a}^{m_b} \frac{h(m)}{\left[g'(m)\right]^2}\, dm. \tag{4}$$

The above development of performance measures for measurement systems was based on the assumption that the location function, $g(M)$, can be estimated with sufficient precision that it is reasonable to treat it as being known. For many practical situations this is not true and therefore the validity of the developed measures may be questioned. An alternative and more rigorous justification for the SN ratio in (2) is based on the length of Fieller intervals for the true value of M. Consider a linear calibration system described by the model

$$\begin{aligned} Y &= \alpha + \beta M + \sigma\epsilon, \\ \epsilon &\sim N(0,1). \end{aligned} \tag{5}$$

Let y_j represent the measured values of Y and m_j represent the known values of M for the standards ($j = 1, \ldots, p$). The classical estimates for α, β, and σ^2 are $\hat{\beta} = S_{ym}/S_{mm}$, $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{m}$, and $s^2 = (p-2)^{-1}(S_{yy} - \hat{\beta}S_{ym})$, where $S_{yy} = \sum_1^p (y_j - \bar{y})^2$, $S_{mm} = \sum_1^p (m_j - \bar{m})^2$, and $S_{ym} = \sum_1^p (y_j - \bar{y})(m_j - \bar{m})$. For a specific value of $M = m_o$, the $100(1-\gamma)\%$ prediction interval for Y is given by

$$\hat{\alpha} + \hat{\beta}m_o \pm ts\sqrt{1 + \frac{1}{p} + \frac{(m_o - \bar{m})^2}{S_{mm}}}, \tag{6}$$

where $t = t_{\gamma/2,p-2}$. Suppose that y_o is the measured value of W for a sample which has an unknown value of M. A $100(1-\gamma)\%$ confidence interval, called a Fieller interval, for m_o can be obtained by using the set of values of M for which y_o is in the $100(1-\gamma)\%$ prediction interval of m. Therefore, the interval will contain all values of m that satisfy:

$$(y_o - \hat{\alpha} - \hat{\beta}m)^2 \le t^2 s^2 \left(1 + \frac{1}{p} + \frac{(m - \bar{m})^2}{S_{mm}}\right). \tag{7}$$

These values can form: (i) a finite interval, (ii) a semi-infinite interval, (iii) two semi-infinite intervals, or (iv) the entire real line. Cases (ii), (iii) and (iv) would imply no clear evidence of a relationship between Y and M (see Miller and Wu, 1991, for details). So only case (i) is of practical interest. It is shown in Miller and Wu (1991) that for a finite interval, (m_L, m_U), its length is

$$m_U - m_L = 2t\left[\left(1+\frac{1}{p}\right)\left(\hat{\omega}-\frac{t^2}{S_{mm}}\right)+\hat{\omega}\frac{(m_o-\bar{m})^2}{S_{mm}}\right]^{1/2}\left(\hat{\omega}-\frac{t^2}{S_{mm}}\right)^{-1}, \qquad (8)$$

which depends on $\hat{\omega} = \hat{\beta}^2/s^2$, S_{mm}, and $m_o - \bar{m}$. This is equivalent to a result shown by Hoadley(1970) that the width of the Fieller interval depends on the magnitude of the F statistic, $f = S_{mm}\hat{\omega}$, for testing H:$\beta = 0$.

The length of the Fieller interval decreases as $\hat{\omega}$ increases for $\hat{\omega} > t^2/S_{mm}$. This can easily be seen by rewriting (8) as

$$m_U - m_L = 2t\left[1+\frac{1}{p}+\hat{\omega}\left(\hat{\omega}-\frac{t^2}{S_{mm}}\right)^{-1}\frac{(m_o-\bar{m})^2}{S_{mm}}\right]^{1/2}\left(\hat{\omega}-\frac{t^2}{S_{mm}}\right)^{-1/2}. \qquad (9)$$

The result is evident since both $\hat{\omega}(\hat{\omega}-t^2/S_{mm})^{-1}$ and $(\hat{\omega}-t^2/S_{mm})^{-1/2}$ are decreasing functions for $\hat{\omega} > t^2/S_{mm}$.

Since the observed length of the Fieller interval is a random variable, the goal of the experiment can be thought of as making the distribution of this variable as favorable as possible. Noting that $S_{mm}\omega$ has a non-central F distribution with 1 and $\nu = p-2$ degrees of freedom, $\mathrm{E}(\hat{\omega}) = \nu(\nu-2)^{-1}(S_{mm}^{-1}+\omega)$, which means that the expected length of the Fieller interval will decrease as ω decreases. This justifies the maximization of the dynamic SN ratio $\log\hat{\omega}$ as described in (2).

3.2 Multiple Target Systems

For multiple target systems the signal factor is used to adjust the function of the system to accommodate different target values for the response. The shape of the signal-response function is not of direct concern for these applications as long as all the desired target values can be realized. Let $\mathcal{M} = (m_a, m_b)$ represent the useful

range of signal which in practice can be applied to the system, and $\mathcal{T}$ represent the required target values. $\mathcal{T}$ may either be a set of discrete values or an interval. In this section only results for discrete $\mathcal{T}$ will be presented as the extensions to continuous $\mathcal{T}$ are straightforward.

As was the case for measurement systems, assume that the signal-response relationship can be modelled by (3). An obvious way to obtain a performance measure in this situation is to take an average (possibly weighted) of performance for the individual elements of $\mathcal{T}$. Assume that $\mathrm{V}(Y)$ given $\mathrm{E}(Y) = y_t$ is a suitable performance measure, as would be the case if a quadratic loss function is applicable. Then a suitable performance measure can be defined by

$$\mathrm{PM} = \sum_{y_t \in \mathcal{T}} \mathrm{V}(Y|M = m_t)\, w(y_t) \tag{10}$$

for discrete $\mathcal{T}$, where $m_t = g^{-1}(y_t)$ and $w(y_t)$ is a weighting function based on the relative importance of the various targets. In some cases it may not be possible to set M so that $\mathrm{E}(Y) = y_t$ for all y_t in $\mathcal{T}$. In these cases, the PM can be modified by replacing $\mathrm{V}(Y)$ by the minimum obtainable mean square error.

The PM in (10) is suitable for multiple target systems which are used for a single purpose. Occasionally, one may encounter a multi-purpose system. For example, an injection molding machine may be used to inject a number of different molding materials. As the physical properties of these materials may be quite diverse, the signal-response relationship may vary substantially with respect to the materials. One approach would be to identify compromise settings of the control factors such that the system performs reasonably well over the range of materials. In this case, a weighted average of the performance for individual materials could be used as an overall performance measure. Alternatively, the control factor settings could be optimized for each material individually. This approach may not be practical for control factors which are very difficult to set. In practice, compromise settings can be obtained for those control factors which are difficult to set and, customized settings be obtained for the rest.

In view of Taguchi's recommendation of the dynamic SN ratio (2) and its prevalent use in some industrial sectors, it is important to point out why it is not appropriate

for multiple target systems. Roughly speaking, maximizing $\log \hat{\omega}$ has the effect of minimizing s^2 and maximizing $\hat{\beta}^2$. The former is always desirable, while the latter can lead to undesirable results. For fixed $M = (m_a, m_b)$, a larger $|\hat{\beta}|$ value can give a wider range of the Y values, which may be outside the specification limits of the target. Furthermore, if there is error in the setting of M (as will be case in the example of Section 6), this error will be propagated through a larger slope $|\hat{\beta}|$, again resulting in a bigger variation in Y.

4 Modeling and Analysis

The purpose of a RPD experiment is to identify the manner in which control factors affect the performance of the system. Therefore, the goal is to model the chosen PM as a function of the control factors. However, there are two distinct approaches to developing such a model which we refer to as performance measure modeling (PMM) and response function modeling (RFM).

PMM requires a two-stage modeling procedure. The first stage is to obtain an estimate of the PM for each combination of control factors used in the experiment. For a fixed combination of control factor levels, the response is measured for various combinations of signal and noise factor levels and these observations are used to estimate the PM. The second stage involves using these estimates to model the PM as a function of the control factors. The preferred settings of the control factors are determined directly from this fitted model.

PMM can be illustrated using Taguchi's dynamic SN ratio approach. Consider a full factorial experiment involving three 2-level control factors (C_1, C_2, C_3), two 2-level noise factors (N_1, N_2) and a 4-level signal factor (M). For each of the eight distinct control factor combinations there are sixteen observations corresponding to the noise and signal factor combinations. These sixteen observations are used to fit a linear model, $\mathrm{E}(Y) = \beta M$, and the parameter estimates are then used to estimate the PM, in this case $\hat{\mathrm{PM}} = \hat{\beta}^2/s^2$. Now the eight $\hat{\mathrm{PM}}$s are treated as the set of observations for a 2^3 experiment where C_1, C_2, and C_3 are the experimental factors

and standard analysis techniques can be applied to them. A typical analysis would be to use a normal plot to identify active effects and thus identify a suitable model for PM as a function of the control factors.

RFM uses the experimental data to model the signal-response relationship as a function of the control and noise factors. The specified performance measure is then evaluated with respect to the fitted models in order to select preferred levels of the control factors. This approach in essence treats the signal-response relationship as the response and models this relationship as a function of the control and noise factors. This is an extension of the response modeling approach recommended by Welch, Yu, Kang, and Sacks (1990) and Shoemaker, Tsui, and Wu (1991) for simple response applications.

To illustrate the RFM approach consider the previous example. The experiment contains 32 combinations of control and noise factor levels. For each of these combinations there are 4 observations corresponding to the levels of the signal factor. These 4 observations are used to fit a parametric model for the signal-response relationship. Suppose in our case a model of the form

$$Y = \beta M + \epsilon, \quad \epsilon \sim \mathrm{N}(0, \sigma^2)$$

is suitable. Then β and σ^2 are estimated for each of the 32 control/noise factor combinations, and models for β and σ^2 as functions of the control and noise factors are produced. The chosen PM is then evaluated for different combinations of the control factors using these models. A systematic development of RFM is given in Miller and Wu (1991).

We do not, in general, recommend the PMM procedure since it can often obscure useful information present in the data. Box (1988) illustrates for simple response systems how very different data sets can give the same estimated values of performance measures. Therefore, modeling the PM directly can result in the loss of valuable information regarding the problem. Clearly, the same argument applies to signal-response applications. The PPM approach only provides information on how control factors affect the overall performance of the system. Any information in the data on how specific control factors affect the shape of the signal response system or interact with

specific noise factors is lost. It is this type of information which can be most valuable in suggesting directions for future research. The response based approaches do not suffer from this deficiency. The initial modeling of the response will often provide useful insight into the system and may suggest directions for future research. The PM is then applied to this model to identify preferred settings of control factors. In many cases, it will not be necessary to formally do this step since it will be straightforward to deduce the preferred settings directly from the response model. The advantages of the RFM approach will be clearly demonstrated in Section 6 using the injection molding experiment.

5 Designing Experiments

In traditional experiments the expectation of the response is of primary importance and the variability is mainly considered with respect to inference on the expectation (Box, Hunter, and Hunter 1978). For RPD experiments the variability of the response is of as much interest as the expectation.

An initial step in any experiment is to select the experimental factors. For signal-response RPD experiments this means identifying the signal, noise, and control factors. In almost all cases, the choice of the signal factor is clear and is a factor which has been traditionally used to control the response. The selection of appropriate noise factors, which represent major sources of variability in the system, is extremely important. The traditional approach of using replicate observations to assess variability is ineffective. In order for estimation of variability based on replications to be valid, it must be assumed that the observations reflect the variability in the entire population of interest. This is seldom reasonable in practice since real systems tend to change over time. For example, key parts wear over time, operators change from shift to shift, batches of raw materials vary, machine settings may drift, and climatic conditions vary over days as well as over weeks. Even supposing the sample is representative of the population, the number of replicates required to get a sufficiently precise estimate of variability for most applications will be too large to be economi-

cally feasible (see Gunter 1988). The solution to this problem is to use noise factors which represent major causes of variability in the experiment. Available knowledge of the system, observational studies, and screening experiments can be used to identify the important noise factors. Finally, the selection of control factors should be done keeping in mind that factors which either affect the system directly or influence the way noise factors affect the system are of interest.

Once the experimental factors have been selected, the design matrix can be considered. For simplicity of discussion, consider a system whose signal-response relationship is linear. There are two ways in which a factor (control or noise) can affect this relationship: (1) it may cause a uniform shift in the relationship or (2) it may cause the slope of the relationship to change. Clearly, if an additive model is used, influences of type (1) are represented by a main effect for the factor and influences of type (2) are represented by an interaction between the factor and the signal factor. For control factors these effects indicate the ability to adjust the average response or to adjust the sensitivity of the response to changes in the signal factor. Noise factor main effects indicate that the factor is inducing variability by shifting the average response and signal$\times$noise interactions indicate variability is being induced by causing the sensitivity of the response to the signal factor to change. To make the system robust (insensitive) to a noise factor we need to identify control factors which interact with the noise factor in the first case and which interact with the noise$\times$signal interaction in the second case. As a result, the ability to detect interactions is of critical importance for an RPD experiment. This is especially true for interactions which involve the signal factor or those which involve both control and noise factors.

Full factorial designs are certainly suitable for RPD experiments as they allow the detection of all interactions involving the experimental factors. A problem which occurs in many situations is that they require too many runs to be economically feasible. A viable alternative is a fractional factorial design which is selected to ensure that all interactions deemed potentially important can be detected. Typically these will include control$\times$signal, control$\times$noise, noise$\times$signal, and control$\times$noise$\times$signal interactions. The modeling procedure (PMM or RFM) also becomes a consideration in selecting a fractional factorial design.

Suppose RFM is to be used to analyze the data. This procedure requires that a fitted model of the signal-response relationship be obtained for each combination of control and noise factors used in the experiment. The overall experiment can be thought of as being composed of two parts. The first part consists of a "primary experiment" where the experimental factors are the control and noise factors and the response is the estimated signal-response relationship. The design array for the primary experiment represents combinations of control and noise factors. For each of these combinations, a secondary experiment is run which simply involves taking observations of the response variable for specified signal levels and using these to fit a model for the signal-response relationship. The design array for the secondary experiment consists simply of the signal levels. The number of signal levels usually should not exceed five and can be as low as three. The same set of signal levels should be used for each combination in the primary array, since the set of signal levels can influence the fitted model for the signal-response relationship. For example, a linear model may be suitable for a set of levels which are restricted to a particular range of the signal factor, while a more complicated model will be required over a different range of levels.

The overall design array consists of replicating the secondary design array for each run in the primary experiment. Shoemaker et al. (1991) referred to this type of design as a product array design. The number of runs required for the experiment will be the number of rows in the primary array times the number of runs in the secondary array. To reduce the size of the experiment, the size of one of these arrays must be reduced. The secondary array can only be reduced by decreasing the number of signal levels and this will be limited by the complexity of the signal-response relationship. The primary array can be reduced by adopting a fractional factorial design. Consider the example used in the modeling section which involved factors C_1, C_2, C_3, N_1, N_2, and M. A 2^{5-1} design with defining relation $I = C_1C_2C_3N_1N_2$ could be used which would reduce the size of the experiment from $32 \times 4 = 128$ runs to $16 \times 4 = 64$ runs. This design would allow all main effects and 2 factor interactions involving the control and noise factors to be estimated provided that 3 factor and higher order interactions involving only control and noise factors are negligible. It also would allow all 2 and

3 factor interactions which involve the signal factor to be estimated provided that 4 factor and higher order interactions involving the signal factor are negligible. In general, a good design for the primary array can be chosen by using standard criteria such as maximum resolution or minimum aberration (see Chen, Sun and Wu, 1993).

Now suppose PMM is to be used to analyze the data. This requires that an estimate of the PM be obtained for each combination of control factors used in the experiment. Again the overall experiment is composed of two parts, but in this case only the control factors are included in the primary experiment. The noise and signal factors are varied in the secondary experiments which are used to estimate the PM for each control factor combination of the primary experiment. The same design array should be used for each of the secondary experiments since the results may be quite misleading otherwise. This can be readily demonstrated by considering the design in Table 3 (the combinations of factors used are denoted by •) which has the defining relation $I = C_1C_2N_1N_2$.

				1	1	1	1	1	1	1	1	-1	-1	-1	-1	-1	-1	-1	-1	N_1
				1	1	1	1	-1	-1	-1	-1	1	1	1	1	-1	-1	-1	-1	N_2
	C_1	C_2	C_3	-3	-1	1	3	-3	-1	1	3	-3	-1	1	3	-3	-1	1	3	M
1	1	1	1	•	•	•	•									•	•	•	•	
2	1	1	-1	•	•	•	•									•	•	•	•	
3	1	-1	1					•	•	•	•	•	•	•	•					
4	1	-1	-1					•	•	•	•	•	•	•	•					
5	-1	1	1					•	•	•	•	•	•	•	•					
6	-1	1	-1					•	•	•	•	•	•	•	•					
7	-1	-1	1	•	•	•	•									•	•	•	•	
8	-1	-1	-1	•	•	•	•									•	•	•	•	

Table 3: Design for $I = C_1C_2N_1N_2$

Assume we are investigating a linear measurement system and therefore adopt PM $= \beta_1^2/\sigma^2$. For each control factor combination we fit a model of the form $Y = \beta_0 + \beta_1 M + \epsilon$ where $\epsilon \sim N(0, \sigma^2)$. Then for the ith run, $\hat{\text{PM}}_i = \hat{\beta}_{1i}^2/s_i^2$ where $\hat{\beta}_{1i}$ and s_i^2 are the least squares estimates of slope and variance. Notice what happens if the

true relationship is of the form

$$Y = 50 + 5M + 1.0N_1 + 1.0N_2 + \epsilon, \tag{11}$$

with $V(\epsilon) = \sigma_\epsilon^2$ where σ_ϵ^2 represents the residual variation after the components due to N_1 and N_2 have been removed. In this model, the control factors have no effect on the system and therefore the estimated PM should be about the same for all 8 rows of the primary array. However, for rows 1, 2, 7, and 8, all the observations have the effects of the noise factors reinforcing each other (either $N_1 = 1$ & $N_2 = 1$ or $N_1 = -1$ & $N_2 = -1$) whereas for rows 3, 4, 5, and 6, the effects cancel each other (either $N_1 = 1$ & $N_2 = -1$ or $N_1 = -1$ & $N_2 = 1$). It is easy to show that for rows 1, 2, 7, 8 $E(\hat{\beta}) = 5$ and $E(s^2) = \sigma_\epsilon^2 + 32/6$ whereas for rows 3, 4, 5, 6 $E(\hat{\beta}) = 5$ and $E(s^2) = \sigma_\epsilon^2$. Clearly, the estimated PM for combinations 3, 4, 5, and 6 will be larger than those for 1, 2, 7, and 8. As a result, the analysis will indicate a C_1C_2 interaction since the C_1C_2 contrast has a level of 1 for rows 1, 2, 7, and 8, and a level of -1 for rows 3, 4, 5, and 6. This is strictly an artifact of different combinations of noise factors being used for the different combinations of control factors. Therefore, it is necessary to require that the same secondary design array be used for each combination of control factors if a PMM procedure is to be used.

Continuing with our example to select a suitable half fraction design for a PMM procedure, it is necessary to adopt a half fraction for either the primary array or the secondary array. For the primary array, the maximum resolution (III) design would be obtained by using $I = C_1C_2C_3$ as the defining relation. This choice necessitates the assumption that there are no control factor interactions with respect to the PM. If this assumption (which seems rather severe in this situation) is not tenable, we need to consider reducing the size of the secondary array. The secondary array consists of noise factors N_1 and N_2 each with two levels and the signal factor M with 4 levels. It is useful to consider M as being composed of pseudo-factors M_a, M_b, and $M_{ab}(= M_a \times M_b)$ each having two levels. The levels of M are represented by the four combinations of levels for M_a and M_b. The maximum resolution designs are obtained by using $I = N_1N_2M_a$, as the defining relation (see Wu and Zhang 1993 for results on $4^1 2^p$ and $4^2 2^p$ designs with maximum resolution or minimum aberration). However for

PMM it is not necessary to estimate the individual noise factor effects and therefore resolution is not the most suitable criterion for the secondary array. What is necessary is that the secondary experiment provide a reliable indication of the performance of the system. For this to occur, we need to ensure: (1) signal effects are not confounded with active interactions, and (2) if active effects involving noise factors are confounded with each other, these effects reinforce rather than cancel. Consider the maximum resolution design defined by $I = N_1N_2M_a$ which confounds $N_1 = N_2M_a$, $N_2 = N_1M_a$, and $N_1N_2 = M_a$. For this design to be suitable, it is necessary to assume N_1N_2 is not active, N_1 reinforces N_2M_a, and N_2 reinforces N_1M_a. A more appealing alternative would be to use the design defined by $I = \pm N_1N_2$ which is equivalent to what Taguchi refers to as a compound noise factor. The defining relation ($I = N_1N_2$ or $I = -N_1N_2$) is chosen so that the effects of N_1 and N_2 reinforce rather than cancel. That is, if both N_1 and N_2 have positive (or negative) effects then $I = N_1N_2$ should be used but if one has a positive effect and the other has a negative effect then $I = -N_1N_2$ should be used. Note that this only requires knowledge about the signs of the noise factors.

The above discussion focuses on reducing the size of the experiment. However, in some situations cost is not directly proportional to run size since some factors (e.g. furnace temperature) are more difficult or expensive to set than others. In these cases, cost is determined by the number of level changes of the "difficult-to-change" factors. Designs which involve restricted randomization such as split-plot or strip-plot designs may be useful in reducing cost. Difficult-to-change factors are assigned to main-units which reduces the number of times these factors are reset. Further discussion can be found in Box and Jones (1992).

For most signal-response applications the signal factor is relatively easy to change compared to control and noise factors. For measurement systems, signal levels often represent different standards and, in most cases, once a measurement system has been set up the increase in cost to make additional measurements is minimal compared with the cost of reconfiguring the system. For multiple target systems, the signal factor has been specifically chosen because it provides a convenient way of controlling the response. Therefore, from an economic standpoint it usually makes sense to assign the

signal factor to sub-units of a split-plot design. The situation is not as clear-cut with respect to noise factors. Noise factors are often, but not always, easier to change than control factors. For example, the noise factors in the injection molding experiment are not easy to change, which explains why a compound noise factor (see Table 2) was chosen to reduce the length and cost of the experiment. Therefore the individual application will determine whether noise factors should be assigned to sub-units or whole units.

In considering fractional factorial design matrices which are suitable for split plot designs, the key characteristic is the number of distinct combinations of the difficult-to-change factors in the design matrix. This number can be reduced by having as many generators as possible in the defining relation made entirely from the difficult-to-change factors. Consider combining this requirement with the restrictions imposed by the modeling procedures. For RFM all control and noise factors are included in the primary array and therefore in selecting the primary array as many generators as possible should be made from difficult-to-change factors. For PMM the control and noise factors are split between the primary and secondary arrays and so the difficult-to-change factors may also be split. This can severely restrict the ability to select generators made entirely from difficult-to-change factors.

6 Analysis

In this section, data from the injection molding experiment is used to demonstrate the PMM and RFM approaches. The Appendix contains the data from the original experiment (DeMates, 1990).

First, we must identify a suitable PM for the system. As an example, suppose it is required that the system be able to produce target values from 650 to 700 for the response. Since this range of target values is obtainable for all combinations of control and noise factors used in the experiment, a reasonable PM would be the average variation of the response over this range.

The original experiment used a product array design. The primary array was used

	Control Factors						
Row	A	B	C	D	E	F	G
1	1	1	1	1	1	1	1
2	1	1	1	-1	-1	-1	-1
3	1	-1	-1	1	1	-1	-1
4	1	-1	-1	-1	-1	1	1
5	-1	1	-1	1	-1	1	-1
6	-1	1	-1	-1	1	-1	1
7	-1	-1	1	1	-1	-1	1
8	-1	-1	1	-1	1	1	-1

Table 4: Inner Array for Injection Molding Case Study

to vary the levels of the control factors and the secondary array was used to vary the signal and noise factors. The primary array was a 2^{7-4} fractional factorial design (see Table 4). The secondary array consisted of all 16 combinations of the eight signal factor levels and the two levels of the compound noise factor. This layout is more suited to a PMM approach than a RFM approach. The experiment was run over two days using a split-plot randomization procedure. On the first day, the compound noise factor was set to its low level. The control factors were then varied according to the rows of the primary array. For each control factor combination, the signal factor was varied over its eight levels and four observations were taken at each level. On the second day, the procedure was repeated using the high level of the compound noise factor.

Figure 2 contains scatter plots of the data (Y, M) and the fitted quadratic models for several runs, where the response Y is the part weight and the signal factor M is the high injection pressure (see Section 2). Figure 3 contains the residual plots (after the quadratic model fit) against M. Although the quadratic model appears reasonably satisfactory, there appears to be a systematic pattern in the residual plots. In order to investigate this further, the total sum of squares for each control/noise factor combination was divided into contrasts using orthogonal polynomials of M. Then a forward selection procedure was used to sequentially test the addition of higher order

terms (at the 0.05 significance level) to the models.

	Signal Factor Levels							
	650	700	750	800	850	900	950	1000
$P1(M)$	-7	-5	-3	-1	1	3	5	7
$P2(M)$	7	1	-3	-5	-5	-3	1	7

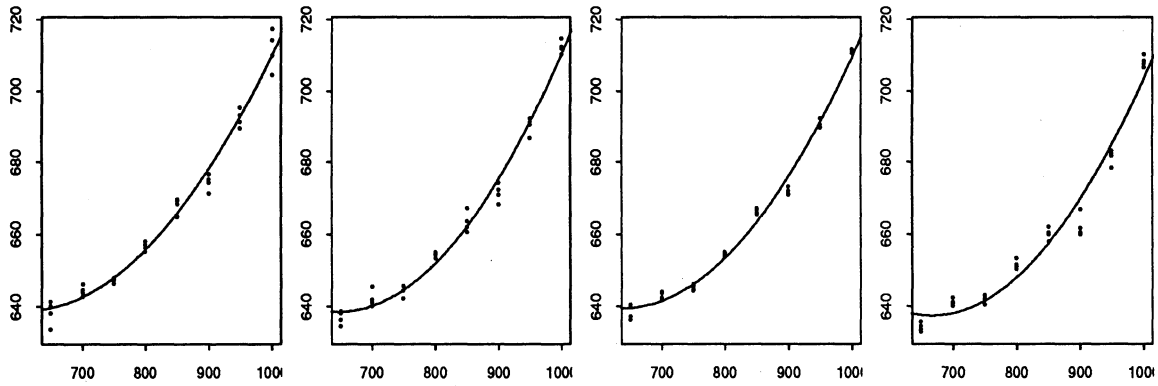
Table 5: Levels for Orthogonal Polynomials

In twelve of the sixteen cases, the selection procedure indicated that a quadratic polynomial was adequate. In the other four cases, the cubic term was also added. An interesting aspect of this analysis was that in over half the runs there was an unusually large sum of squares attributed to the 6th degree term (it was typically more than a factor of 10 larger than the 4th or 5th degree terms). As it is unlikely that the 6th degree term is important this suggests that at least part of the systematic pattern in the residuals (after quadratic fit) seen in Figure 3 may not be due to model inadequacy. One possible explanation is that systematic difference exists between the settings of the signal factor and the actual high injection pressure delivered by the system. For example, in most of the residual plots, the residuals corresponding to $M = 900$ are negative. This could be explained if the system consistently delivered a high injection pressure of less than 900 at a nominal setting of 900, as this would cause the observed part weights to be lower than expected. Whatever the real explanation for this phenomenon, it is clear it will not be corrected by using a more complicated model.

Figure 2 indicates that the quadratic model adequately captures the essential features of the signal-response relationship. Although for certain runs, the above analysis indicated that the cubic term was significant, from a practical point of view it does not appear necessary. Therefore, a quadratic location model based on orthogonal polynomials was used,

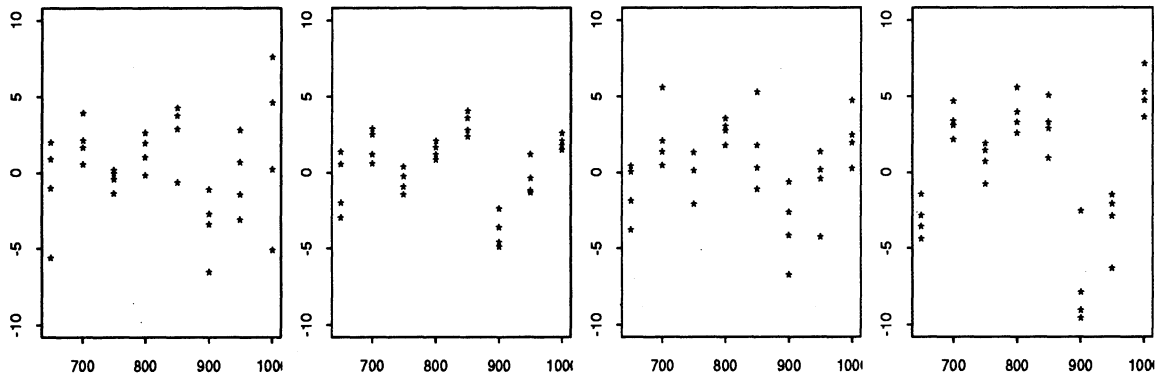
$$\mathrm{E}(Y) = \beta_0 + \beta_1 P_1(M) + \beta_2 P_2(M) \ , \tag{12}$$

where the values $P_1(M)$ and $P_2(M)$ for the various signal levels are given in Table 5.



$X_N = 1$ Row 1 $X_N = 1$ Row 2 $X_N = -1$ Row 1 $X_N = -1$ Row 2

Figure 2: Fitted Quadratic Models for Injection Molding Data



$X_N = 1$ Row 1 $X_N = 1$ Row 2 $X_N = -1$ Row 1 $X_N = -1$ Row 2

Figure 3: Residual Plots for Injection Molding Data

This type of model was adopted as it makes the interpretation of results easier with β_1 representing the linear component of the signal-response relationship and β_2 the quadratic departure from linearity.

6.1 PMM Analysis

The first step is to estimate the PM for each combination of control factors. From the residual plots it seems reasonable to assume variation is constant across levels of the signal factor. For each row in the control factor array, the standard least squares estimate of variance, s^2, for the fitted quadratic model (12) was used as the estimated PM. The next step is to treat s^2 as the response for the control factor design array. Actually, $\log(s^2)$ will be used in order to stabilize the variation of these estimates. Figure 4 gives the half normal plot of factor effects for $\log(s^2)$. This plot does not clearly indicate that any control factors have an effect on the PM. However, it does appear that A, B, and D warrant further consideration.

Row	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	s^2
1	665.8	5.00	1.25	8.39
2	662.2	4.91	1.46	19.70
3	666.8	4.93	1.25	9.06
4	666.4	4.66	1.40	10.40
5	665.3	4.56	1.38	4.34
6	674.3	4.33	1.34	9.75
7	666.4	4.92	1.31	1.54
8	664.3	4.96	1.27	3.27

Table 6: Estimated Parameters for PMM

6.2 RFM Analysis

For RFM, we first fit location and dispersion models for each combination of control and noise factors used in the experiment. For the location model, we use the quadratic

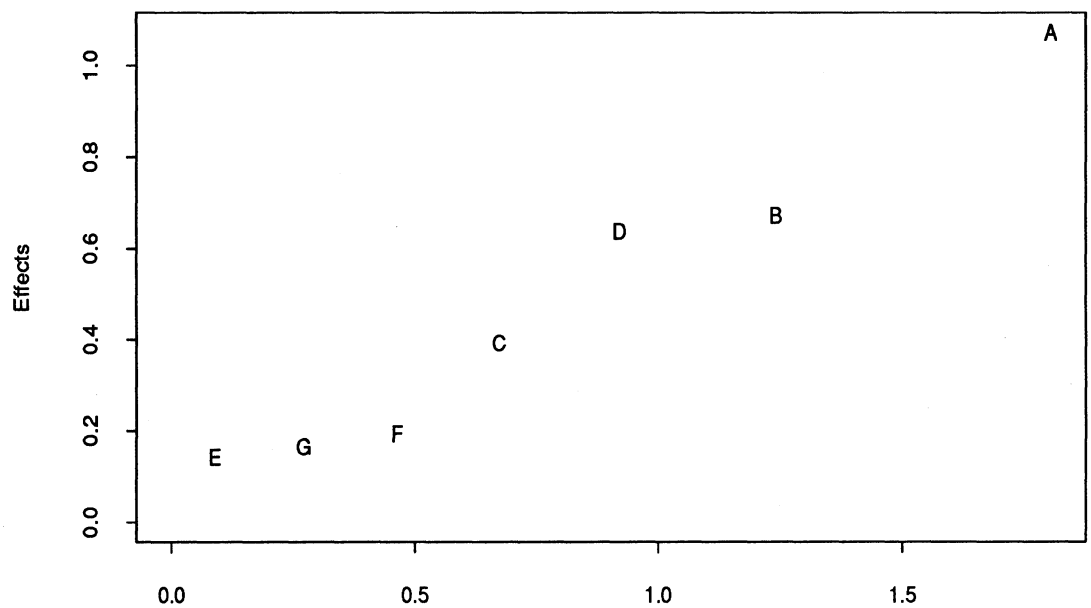


Figure 4: Half Normal Plot for $\log(s^2)$

model in (12). Since replicate observations were made at each signal level, it is possible to separate variation into a lack-of-fit component and a replicate component. The replicate component, $\hat{\sigma}_p^2$, will reflect part-to-part variation for observations taken over a short time interval. The lack-of-fit component, $\hat{\sigma}_l^2$, will represent longer term variation and as was noted previously, contains a systematic component which may be due to systematic errors in the recorded values for the signal factor. Therefore, we will analyze these two components of variation separately. The estimated values for each of the location parameters and the two components of variation are given in Table 7.

Next, the effects of the control and noise factors on these parameters are evaluated. Figure 5 contains half-normal plots for the location model parameters and Figure 6 contains half-normal plots for the two components of variation. For β_0 there are six effects which appear to be significant: G, C, E, A, N, and F. Only C stands out as being clearly significant for β_1. There are no clearly significant effects for β_2, but E, N, D, B, AN, FN, and GN appear marginal. None of the estimated effects are significant for lack-of-fit variation, while for the part-to-part variation A is clearly significant and N, EN, B, and C are all large enough to warrant further attention.

	$X_N = 1$						$X_N = -1$				
Run	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\sigma}_l^2$	$\hat{\sigma}_p^2$	Run	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\sigma}_l^2$	$\hat{\sigma}_p^2$
1	666.5	5.02	1.16	5.61	7.78	1	665.0	4.98	1.33	6.87	1.20
2	664.2	5.12	1.44	7.10	4.45	2	660.0	4.69	1.48	26.81	3.20
3	668.2	4.98	1.22	4.28	4.99	3	665.2	4.86	1.26	6.34	2.70
4	668.4	4.76	1.25	4.81	3.53	4	664.2	4.55	1.54	3.64	2.64
5	666.3	4.66	1.35	4.93	0.67	5	664.2	4.46	1.39	2.54	0.56
6	674.4	4.32	1.32	14.78	1.00	6	674.1	4.33	1.36	13.27	0.30
7	666.6	4.92	1.31	2.30	0.21	7	666.1	4.91	1.30	1.76	0.18
8	664.9	4.90	1.25	3.21	0.75	8	663.6	5.02	1.29	3.96	0.12

Table 7: Estimated Parameters for Injection Molding Data

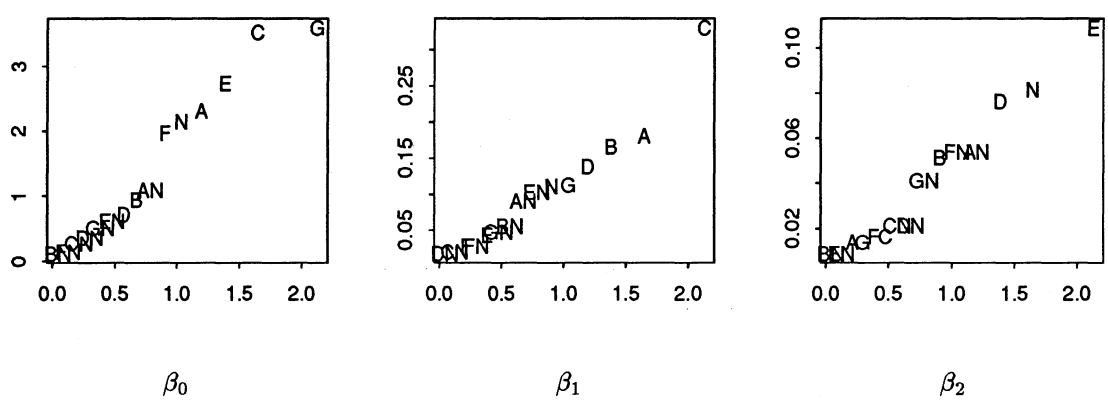


Figure 5: Half Normal Plots for Location Model Parameters

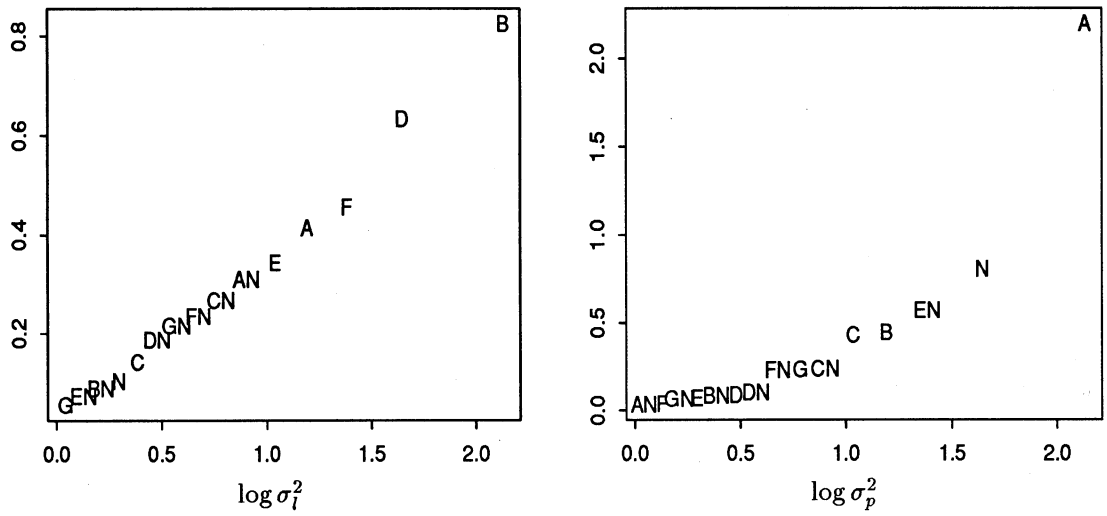


Figure 6: Half Normal Plot for Variance Parameters

The fitted models for the parameters are

$$
\begin{aligned}
\hat{\beta}_0 &= 666.4 + 1.2X_A - 1.8X_C \\
&\quad +1.4X_E - 1.0X_F + 1.8X_G + 1.1X_N, \\
\hat{\beta}_1 &= 4.79 + 0.16X_C, \\
\hat{\beta}_2 &= 1.33 + 0.03X_B - 0.04X_D - 0.05X_E \\
&\quad -0.04X_N + 0.01X_AX_N - 0.03X_FX_N - 0.02X_GX_N, \\
\hat{\nu} &= 0.12 + 1.10X_A + 0.22X_B - 0.21X_C, \\
&\quad +0.40X_N + 0.28X_EX_N + 0.04X_E,
\end{aligned}
$$

where $\nu = \log\ \sigma_p^2$.

To begin, consider the fitted model for β_1. Factor C can be used to adjust the sensitivity of the response to the signal. In this case C would be set to level $X_C = 1$ if a wider range of attainable targets was necessary. Otherwise the level of C could be determined by other considerations.

Next consider the model for $\log \sigma_p^2$. Part-to-part variation is not the only type of variation which is relevant to the process. However it is clearly desirable to reduce this type of variation as much as possible. In this case, A should be set to the $X_A = -1$

	$X_E = -1$	$X_E = 1$
$X_N = -1$	-1.57	-2.05
$X_N = 1$	-1.33	-0.69

Table 8: Estimated Values for $\log \sigma_p^2$

level. It also appears worthwhile to set B to the $X_B = -1$ level and C to the $X_C = 1$ level. The compound noise factor, N, and the EN interaction also affect σ_p^2. Since the EN interaction was significant, E was also included in the model. Table 8 contains the estimated values of $\log \sigma_p^2$ for the combinations of levels of E and N assuming $X_A = -1$, $X_B = -1$, and $X_C = 1$. The table indicates that for E set to $X_E = -1$ the part-to-part variation will be more consistent with respect to changes in the noise factors than for $X_E = 1$.

Now consider the model for β_0. Due to previous considerations the levels of A, C, and E have already been determined. This leaves factors F and G which could be used to make adjustments to β_0 (if necessary). Notice, that N does affect β_0 but no interaction has been identified which could offset this.

Finally consider the model for β_2. In this case the levels for B, D, and E have been previously determined. Consider the effect of N. The estimated observed coefficient (given fixed levels of control factors) for N would be

$$(-0.04 + 0.01X_A - 0.03X_F - 0.02X_G)X_N.$$

In order to make the system insensitive to changes in N, we would like to make the absolute value of this coefficient as small as possible. As the setting $X_A = -1$ has already been determined this suggests setting $X_F = -1$ and $X_G = -1$.

Notice that the PMM approach did not clearly indicate that any control factors could be adjusted to improve system performance. Even if certain factors had been identified, it would not have provided insight into how these factors affect the system. RFM on the other hand, not only indicated certain control factors could be used to improve the system, it also provided insight into how these factors affected

the system. In particular a quadratic model was identified as suitably describing the signal-response relationship and control factors which could be used to alter the parameters of this relationship were identified. Further, the residual plots produced from the fitted models for the signal-response relationship indicated the possibility of a systematic error in the signal levels. Finally, the flexibility of the RFM procedure allowed variation to be divided into two components, σ_p^2 and σ_l^2, which led to the conclusion that factor A could be used to reduce part-to-part variability.

The advantages of the RFM approach can be better appreciated by contrasting the main findings summarized above with what Taguchi's approach would lead to. First, Taguchi's dynamic SN ratio assumes a linear relation while the data clearly exhibit a quadratic relation. Second, use of data analysis techniques, like residual plots suggests the possibility of errors in the signal factor setting. Third, analyzing components of variation allows us to identify factor A as effective in reducing part-to-part variability, which can have major engineering implications. Since Taguchi's approach does not encourage the use of modern data analysis, following his analysis method will unlikely lead to the last two findings.

7 Conclusion

This paper explored the use of designed experiments to improve the performance of signal-response systems. These are systems whose function depends on the causal relationship between a signal factor and a response variable. Measurement systems and multiple target systems are examples of such systems and were considered in detail. To summarize we recommend the following steps for investigating signal-response systems.

1. Identify a suitable performance measure which reflects the ability of the system to perform its designated function.

2. Adopt a response function modeling approach. In essence this means that the signal-response relationship is to be modelled as a function of both control and

noise factors. Then the identified performance measure is applied to the fitted model to determine preferred settings for the control factors.

3. The experiment should be designed using a two stage strategy. First, a design array is adopted for the control and noise factors. Then, for each row in this array, the signal factor is varied over a number of levels.

4. The analysis is quite straightforward. For each row in the control-noise array, parametric location and dispersion models are fitted for the response. The fitted parameters for these models are then modelled as functions of the control and noise factors. Standard procedures, such as half normal plots and regression analysis, are used to identify significant effects and produce a fitted model. The PM is then applied to this model and preferred settings of the control factors identified.

Acknowledgments

This research was supported by the Natural Sciences and Research Council of Canada, the University of Auckland Research Committee (for A.M.), and the Center for Display Technology and Manufacturing, University of Michigan (for C. F. J. W.).

Appendix: Data from the Injection Molding Experiment

Run	Signal Factor Levels 650	700	750	800	850	900	950	1000	X_N
1	640.1	644.4	647.6	655.2	664.8	674.4	693.2	709.8	1
	641.2	646.2	646.3	657.3	669.7	671.3	689.4	714.2	
	633.6	642.8	647.2	656.4	668.3	676.7	691.1	717.2	
	638.2	643.9	647.8	658.0	669.2	675.1	695.3	704.5	
2	638.6	645.3	645.5	655.1	662.1	670.8	692.3	711.8	1
	636.3	640.2	642.1	654.3	663.6	668.2	691.1	712.3	
	634.4	641.8	642.1	653.3	660.7	672.3	690.5	714.6	
	638.2	641.1	644.3	654.6	667.1	674.3	686.7	710.1	
3	642.6	648.3	650.0	657.3	666.3	675.2	695.2	714.6	1
	640.2	642.9	648.2	659.4	667.3	674.4	691.4	713.7	
	641.6	646.1	647.9	658.1	670.1	676.6	689.9	714.2	
	639.9	645.2	649.9	660.0	671.5	678.2	699.2	709.9	
4	643.8	649.8	650.6	658.3	666.2	673.2	696.6	713.8	1
	641.6	646.3	649.7	657.9	666.8	675.8	691.2	711.7	
	642.2	645.2	648.2	659.1	670.2	675.8	690.2	711.8	
	643.6	647.2	650.1	660.0	671.8	678.2	690.6	712.2	
5	642.6	645.6	647.9	654.6	666.8	672.3	687.9	709.8	1
	641.8	645.8	648.2	655.2	665.7	674.6	688.8	710.2	
	642.0	645.7	648.0	654.7	665.8	673.9	689.3	711.3	
	642.3	646.0	647.8	654.9	669.2	675.4	688.6	710.7	
6	650.6	655.7	660.2	667.8	671.1	678.9	694.7	718.4	1
	650.2	656.2	659.7	666.5	672.0	679.3	693.3	720.2	
	651.3	655.5	659.4	666.7	671.7	679.1	696.8	716.6	
	650.1	656.0	658.9	666.6	671.4	678.6	692.1	717.0	
7	639.9	644.1	647.6	656.3	664.8	675.3	693.1	709.9	1
	640.2	644.6	648.0	656.0	665.2	674.7	692.8	711.4	
	640.3	645.0	648.2	656.4	665.1	674.9	691.9	712.2	
	640.1	644.7	647.8	656.7	665.5	675.2	692.4	711.6	
8	637.7	642.9	647.3	651.1	665.0	673.2	689.6	710.9	1
	638.1	643.4	647.3	655.4	664.7	672.8	689.9	709.3	
	638.2	643.0	646.8	655.4	664.5	673.4	690.7	708.6	
	638.4	642.9	647.0	655.2	664.8	672.8	690.2	709.1	
1	639.7	642.3	645.5	653.9	666.6	672.1	692.2	711.6	-1
	640.5	641.7	644.8	655.1	665.8	670.8	690.6	710.8	
	636.2	643.6	646.1	654.7	667.1	673.3	689.7	711.1	
	637.2	644.0	644.3	654.2	665.4	671.1	689.8	710.5	
2	634.4	639.9	642.6	650.2	659.9	666.8	678.4	708.3	-1
	632.9	640.8	640.4	651.6	660.3	660.3	682.6	710.1	
	633.7	641.1	643.1	650.9	657.9	659.8	681.8	707.7	
	635.8	642.4	641.9	653.2	662.1	661.5	683.2	706.6	
3	640.2	646.1	647.2	655.5	666.2	671.0	688.6	708.9	-1
	638.1	644.4	646.0	654.0	667.3	673.6	687.5	710.0	
	637.3	644.4	647.5	653.8	669.1	672.4	691.0	711.3	
	639.1	641.2	644.3	652.8	664.7	672.2	693.1	708.4	
4	641.1	644.5	647.2	652.0	665.3	669.2	688.7	709.8	-1
	642.1	647.3	644.8	654.6	661.0	671.1	690.4	710.1	
	642.0	642.8	646.0	653.8	659.7	670.1	686.3	707.7	
	641.8	643.9	646.3	651.7	662.4	671.1	685.8	706.4	
5	640.8	644.7	647.6	652.3	661.1	673.0	685.7	706.4	-1
	641.1	645.3	646.8	654.5	662.8	673.2	686.7	707.7	
	641.2	644.6	647.3	653.9	659.2	672.5	686.2	706.9	
	641.6	645.0	647.5	653.6	659.9	673.7	686.1	706.3	
6	650.4	655.4	659.7	665.8	671.0	677.7	695.6	716.5	-1
	650.8	655.0	660.2	665.9	670.8	677.5	696.8	717.0	
	651.2	654.6	660.3	665.9	671.2	678.2	694.3	718.3	
	650.7	654.9	659.3	666.4	670.5	677.8	696.1	717.6	
7	639.6	643.8	648.2	655.7	665.2	674.8	691.7	710.1	-1
	639.4	644.2	647.3	656.0	664.8	675.3	691.4	711.4	
	639.9	644.1	647.2	655.5	664.3	675.0	691.8	710.3	
	640.0	644.4	647.8	656.2	663.9	675.1	692.3	711.1	
8	636.5	641.8	645.2	653.8	662.8	671.8	689.4	709.7	-1
	636.2	640.6	646.1	653.9	662.3	671.6	689.1	709.6	
	635.7	640.5	645.5	653.9	662.1	671.6	689.6	709.7	
	636.1	640.3	645.0	653.6	662.4	671.6	689.3	709.3	

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