Experiments with Degradation Data for Improving Reliability and for Achieving Robust Reliability

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ABSTRACT

Statistically designed experiments provide a proactive means for improving reliability. Moreover, they can be used to design products that are robust to noise factors, that is, those which are hard or impossible to control. Traditionally, failure time has been collected, but for high reliability products, it is likely that no failures will occur and therefore the experiment will not be informative. An alternative is to collect degradation data, however. Take for example, a fluorescent lamp whose light intensity decreases over time. The degradation path, given that it is smooth, provides information about the product's reliability and does not require the failure of the product. Thus, this paper considers experiments with degradation data for improving reliability as well as for achieving robust reliability using Taguchi's robust parameter design paradigm. Pseudo generalized least squares and pseudo maximum likelihood analyses based on a random coefficient model are proposed and illustrated by two experiments. One experiment on fluorescent lamps considers reliability improvement. The other experiment focuses on robust reliability improvement of light emitting diodes.

Key Words: Taguchi's parameter design, Control and noise factors, Product array, Loss function, Pseudo generalized least squares, Pseudo maximum likelihood.

1 Introduction

Both "reliability improvement" and "robust reliability achievement" are effective strategies for enhancing a product's competitive position in the marketplace. Statistically designed experiments have been adopted extensively for attacking these goals; e.g., *Reliability Improvement with Design of Experiments* (Condra, 1993) provides several examples from the electronics industry. See also Hamada (1995) who reviews analysis methods for such experiments. For an already highly reliable product, the fact that few or even no failures are likely to occur during a reasonable period of reliability testing makes it difficult with traditional time-to-failure data to identify the important factors that affect reliability out of many potentially important ones.

If a product has characteristics whose degradation over time can be related to the its reliability, then collecting "degradation data" can provide important information about product reliability. Nelson (1990, Chapter 11) surveys the scant literature on the subject and documents various applications. Lu and Meeker (1993) give an updated literature survey and provide methods which use the degradation data to estimate the failure time distribution. While degradation data have been used to assess a product's reliability, it has received little attention as a means for improving reliability. Tseng, Hamada and Chiao (1995) show how the degradation of a fluorescent lamp's luminosity over time provides a practical way to improve fluorescent lamp reliability. Degradation data can also be used to achieve robust reliability using Taguchi's strategy of making a product robust to *noise factors*, e.g., manufacturing variation or product use conditions over which there is little control or which may be difficult to control.

One purpose of this paper is to show how statistically designed experiments which collect degradation data can be used to improve reliability and to achieve robust reliability. Tseng et al. (1995) propose a simple procedure using predicted lifetimes based on the degradation data which can be analyzed using standard methods for fractional factorial experiments. Because the procedure assumes that the predicted lifetimes have the same distribution as the lifetimes, it is not recommended when there is significant measurement error. Instead, this paper focuses on modeling the degradation rates directly and proposes an analysis methodology which accounts for measurement errors.

An outline of the paper is as follows. In Section 2, a random coefficient model for degradation rates is introduced. Section 3 presents separate classes of experimental plans for improving reliability and for achieving robust reliability, with examples of actual experiments involving fluorescent lamps and Light Emitting Diodes (LEDs). In Section 4, some approaches for extracting the information from the degradation data are discussed. Here, a new methodology which analyzes estimated degradation rates using pseudo generalized least squares and pseudo maximum likelihood estimation is proposed. The proposed methodology is then illustrated in Sections 5 and 6 using the fluorescent lamp and LED experimental data. The paper concludes with a discussion in Section 7.

2 A Random Coefficient Model for Degradation Rates

A key quality characteristic of a lighting product (e.g., fluorescent lamps and LEDs) is its luminosity or luminous flux as measured in lumens, say $Y(t)$, which is measured at time t. Because luminosity degrades over time, industry has traditionally defined failure in terms of the amount of degradation. Let the observed relative degradation $V(t) = v(Y(t), Y(t_0))$, where $Y(t_0)$ is the initial (or base) luminosity measured at time t_0. Then, industry's definition for lifetime is defined as the time t^* when $V(t^*)$ falls below a prespecified level D.

A useful and simple model for observed relative degradation is:

$$V(t) = \tau(t, \lambda) + e_t, \tag{1}$$

where $\tau(t, \lambda)$ is the actual relative degradation and is observed (i.e., $V(t)$) with error e_t. To describe a population of degradation paths, the degradation rate λ is random with distribution function $G_\lambda(\cdot)$. The error e_t is assumed to be independent normal with mean zero and variance σ_e^2 (denoted by $N(0, \sigma_e^2)$). The so-called linear random coefficient model is a special case of (1) for which $\tau(t,\lambda) = \lambda t$. Following Lu and Meeker (1993), the lifetime distribution can be derived for a particular $G_\lambda(\cdot)$. For example, they show for the linear random coefficient model that if λ is lognormally distributed then the lifetime is also lognormally distributed.

Example 1. A degradation model for fluorescent lamps

In Tseng et al. (1995), which analyzes degradation data collected to improve the reliability of fluorescent lamps, the observed degradation paths are modeled using (1) with:

$$V(t) = \log(Y(t)/Y(t_0)) \text{ and } \tau(t,\lambda) = -\lambda t, \tag{2}$$

where $t_0 = 100$ hours and λ is distributed as lognormal (μ, σ^2) (denoted by $LN(\mu, \sigma^2)$). That is,

$$G_\lambda(\omega) = \Phi(\frac{\log \omega - \mu}{\sigma}),$$

where Φ is the standard normal cumulative distribution function. The Chinese National Standard defines that a failure occurs at time t^* if $Y(t^*) \leq 0.6Y(100)$, i.e., the critical level $D = \log(0.6)$. Under this model, it can be shown that the reliability function $R(t)$ is

$$R(t) = 1 - \Phi(\frac{\log(t/-D) + \mu}{\sigma}). \tag{3}$$

Example 2. A degradation model for LEDs

To improve the reliability of LEDs (type GaAlAs), an experiment was performed in which the LEDs were monitored periodically for ten inspection times. Here, observed relative degradation is defined in terms of the third inspection luminosity because the degradation paths stabilized by then and can be modeled using (1) with:

$$V(t) = Y(t) - Y(t_3) \text{ and } \tau(t,\lambda) = \lambda(t_3 - t), \tag{4}$$

where t_3 denotes the third inspection time. Assuming that λ is random and distributed as $N(\mu, \sigma^2)$, Lu and Meeker (1993) show that the reliability function based on critical level D is approximately:

$$R(t) \approx 1 - \Phi(\frac{t - D/\mu}{\sigma t/\mu}). \qquad (5)$$

3 Classes of Degradation Experiments

Separate classes of experiments which collect degradation data for improving reliability and for achieving robust reliability, respectively, will be considered next.

3.1 Experiments for Improving Reliability

Statistically designed experiments provide systematic and efficient plans for experimentation to identify the important factors that affect the reliability of a product. Besides identifying the important factors, values (or levels) for these factors can be recommended that yield reliability gains. Such information can be obtained using an appropriately chosen experimental plan. Typically in industry, a large number of factors may need to be studied in a relatively small number of runs. Thus, fractional factorial designs are often used.

Tseng et al. (1995) use a 2^{3-1} fractional factorial design in the experiment to improve the reliability of fluorescent lamps mentioned in Example 1; i.e., three factors were studied at two different levels (denoted by $-$ and $+$) in four different combinations of factors levels. From the description of a seven step manufacturing process, there are many factors that could potentially affect the reliability of fluorescent lamps. Based on the process engineer's experience, the following factors were chosen for the experiment: (i) the amount of electric current ($Factor\ A$) in the exhaustive step which coats the lamp filament, (ii) the concentration of the mercury dispenser ($Factor\ B$) in the mercury dispenser coating step and (iii)

Table 1. $2^{(3-1)}$ Fractional Factorial Design for the Fluorescent Lamp Experiment

	Factors			
	A	B	C	
Run	Current	Mercury	Argon	
1	−	−	−	
2	−	+	+	Degradation Paths
3	+	−	+	See Figure 1.
4	+	+	−	

the concentration of argon (*Factor* C) in the argon filling step; (ii) and (iii) produce the gas mixture of mercury and argon which fills the fluorescent tube.

The $2^{(3-1)}$ fractional factorial design is given in Table 1. From a production run of each of the four lamp types (as defined by the rows of Table 1), five lamps were randomly selected for testing. A spectrophotometer was used to measure luminous flux (in lumens) for each of the 20 lamps at 100, 500, 1000, 2000, 3000, 4000, 5000 and 6000 hours. The same engineer took the measurements and used a single spectrophotometer throughout the experiment. Note that the Chinese National Standard for fluorescent lamps require a 100 hour burn-in (about four days) and that the reading at 500 hours is useful for detecting defective lamps. Because the luminous flux degradation stabilized by 1000 hours, the readings after 1000 hours were used in the modeling. Readings for lamps from Runs 2 and 4 continued to be taken up to 12000 hours at 1000 hour intervals because of limited testing equipment availability. See Figure 1 which presents the sample luminous flux degradation paths.

Using the path model (1) for the observed relative degradation given in (2), the degradation data can be analyzed under the following assumptions:

- The measurement error e_t is independent $N(0, \sigma_e^2)$, where σ_e^2 is constant and does not depend on the particular run, unit or inspection.

- The degradation rate λ is distributed as $LN(\mu, \sigma^2)$, where:
  - σ^2 is constant.
  - μ depends on the particular run (denoted by μ_i for the ith run) and has the following linear structure:

$$\mu_i = \alpha_0 + \sum_{j=1}^{3} x_{ij}\alpha_j,$$

    where x_{ij} is given in Table 1 (columns 2–4) and α_j ($j = 1, 2, 3$) are the main effects for factors A, B and C, respectively.

In terms of the degradation path model, the experimental objectives are to identify the important (i.e., nonzero) α_j and recommend factor levels that minimize the mean log degradation rate μ.

3.2 Experiments for Achieving Robust Reliability

The goal of Taguchi's robust design is to find levels of control (or design) factors that yield a robust product, i.e., that make the product insensitive to the variation of so-called noise factors, which are hard or impossible to control. Taguchi's tactics for carrying out robust design are to specify a criterion for assessing noise factors effects, estimate the criterion using experimentation and then identify product designs that optimize the criterion. The criterion for assessing the effect of the noise factors at a particular combination of control factor levels $z_{control}$ (termed the loss and denoted by $L(z_{control})$) can be defined for a general loss function $l(\cdot)$ following Welch, Yu, Kang and Sacks (1990) as:

$$L(z_{control}) = \int l(z_{control}, z_{noise}) f(z_{noise}) dz_{noise}, \qquad (6)$$

where $l(z_{control}, z_{noise})$ is the loss function evaluated at a particular combination of control and noise levels $(z_{control}, z_{noise})$ and $f(\cdot)$ is the joint probability density function of the noise factors. The objective of robust design is then to find a product design $z_{control}$ with minimum loss. An appropriate $l(\cdot)$ will be discussed in Section 6.

As an example, consider an experiment for achieving the robust reliability of LEDs mentioned in Example 2 in Section 2. The experimental plan used was a product array consisting of an 8-run control array and a 2-run noise array to study three control factors and one noise factor as displayed in Table 2. An LED is composed of a die, a solid-state component, which is attached by silver epoxy to the lead frame which carries the electric current. The control factors chosen for the experiment include die vendor ($Factor\ A$), type of epoxy material ($Factor\ B$) and lead frame design ($Factor\ C$). In the manufacturing process, the location on the lead frame where the die is attached varies. One goal of this experiment is to study how to make LED reliability robust to this manufacturing variation of the die location; hence die location ($Factor\ N$) is a noise factor. Note that all factors are studied at two levels (denoted by $+$ and $-$) giving eight different LED designs (as specified by the control factor array), each using two different die locations (as specified by the noise factor array); for the experiment, the die location needs to be specially controlled so that information about die location can be obtained. 30 LEDs were made for each of the 16 configurations indexed by (i, j) for run i of the control factor array and noise factor level j ($i = 1, 2, \ldots, 8$ and $j = 1, 2$). A UDT-81 optometer was used to measure the LED luminous flux (in lumens). The sample degradation paths are given in Figure 2 which plot the readings defined in (4) for inspection times 3 to 10.

Using the path model (1) for the observed relative degradation from (4), the degradation data can be analyzed under the following assumptions:

- The measurement error e_t is independent $N(0, \sigma^2_{e(i)})$, where $\sigma^2_{e(i)}$ depends on run i, i.e., the control factors but not the noise factor.
- The degradation rate λ is distributed as $N(\mu, \sigma^2)$, where:
  - σ depends on only the control factors (denoted by σ_i^2 for the ith run); all $Control$ main effects and possibly some $Control \times Control$ interactions can be estimated.

Table 2. Product Array Design for the LED Experiment

	Control Array Factors			Noise Array N	
	A	*B*	*C*	−	+
Run	Dice	Epoxy	Lead Frame		
1	−	−	−		
2	+	−	−		
3	−	+	−		
4	+	+	−	Degradation Paths	
5	−	−	+	See Figure 2.	
6	+	−	+		
7	−	+	+		
8	+	+	+		

A log-linear structure is assumed as follows:

$$\log(\sigma_i^2) = \beta_0 + \sum_{j=1}^{6} x_{ij}\beta_j, \tag{7}$$

where x_{ij} for $j = 1, 2, 3$ are columns 2–4 in Table 2 and correspond to the A, B and C main effects β_1–β_3; the x_{ij} for $j = 4, 5, 6$ which correspond to the two-factor interactions $A \times B$, $A \times C$ and $B \times C$, are obtained using $x_{i4} = x_{i1} * x_{i2}$, $x_{i5} = x_{i1} * x_{i3}$ and $x_{i6} = x_{i2} * x_{i3}$.

– μ depends on both the control and noise factors (denoted by μ_{ik} for the ith run and the kth column of noise array); the product array design allows all *Control* main effects (possibly some *Control* $\times$ *Control* interactions), all *Control* $\times$ *Noise* interactions and all *Noise* main effects to be estimated, where *Control* and *Noise* denote control and noise factors, respectively. A linear structure is assumed as follows:

$$\mu_{ik} = \alpha_0 + \sum_{j=1}^{10} x_{ij}\alpha_j, \tag{8}$$

where x_{ij} for $j = 1, \ldots, 6$ are the same given above for σ^2 and correspond to the *Control* and *Control* $\times$ *Control* interactions which here are the A, B and C main effects and the $A \times B$, $A \times C$ and $B \times C$ two-factor interactions. Letting $x_{i7}=$ -1 or $+1$ depend on whether the noise factor level is $-$ $(k = 1)$ or $+$ $(k = 2)$, corresponds to the *Noise* main effect N and $x_{i8} = x_{i1} * x_{i7}$, $x_{i9} = x_{i2} * x_{i7}$ and $x_{i10} = x_{i3} * x_{i7}$ correspond to the *Control* $\times$ *Noise* interactions, $A \times N$, $B \times N$ and $C \times N$.

The *Control* $\times$ *Noise* interactions play an important role because the fact that the loss $L(\cdot)$ in (6). Changes for different control factor combinations means that these interactions must exit. Thus, robust design exploits the existence of interactions between control and noise factors. The *Control* main effects and *Control* $\times$ *Control* interactions give the mean response value about which the response varies as the noise factors vary according to their distribution; the amount of variation depends on the magnitudes of the $Noise$ main effects and $Control \times Noise$ interactions.

Next, some methodology is considered for analyzing degradation data from reliability and robust reliability experiments.

4 Analysis Methodology

For highly reliable products, the only available data may be degradation paths which nevertheless contain important reliability information, namely, the degradation rates (λs). Based on the path model (1) (and specifically for the examples given in (2) and (4)), the distribution of the random effect λ depend on the experimental factors. Consequently, this section focuses on estimators for the factorial effects (i.e., main effects and interactions) as well as their statistical properties.

Let $\boldsymbol{\theta}$ denote the vector of factorial effects in the model; $\boldsymbol{\theta}$ consists of the α_i for the fluorescent lamp experiment and consists of the α_i and β_i for the LED experiment. Using the degradation data, estimators for $\boldsymbol{\theta}$ and their corresponding sampling distribution are needed to identify the important (i.e., non-zero) effects.

Before proposing a new methodology, consider some other approaches based on previous work.

4.1 A One-Stage Approach

One immediate approach might be to analyze the degradation data $V(t)$ directly by forming an appropriate likelihood function and calculating maximum likelihood estimates. This so-called one-stage approach may not be simple because the likelihood might not be expressable in a closed form. For example, for the fluorescent lamp experiment, the distribution of $V(t)$ is a convolution of a lognormal (from the random λ) and a normal (from the measurement error). Another drawback is that many nuisance parameters associated with the measurement error (e.g., the $\sigma^2_{e(ij)}$ in the LED experiment) need to be estimated. Consequently, some two-stage approaches will be considered next.

4.2 Some Two-Stage Approaches

In a two-stage approach, first the individual degradation rate λ for a single unit based on its degradation path is estimated using least squares, say $(\hat{\lambda}|\lambda)$. The estimated degradation rates or some function of them is then analyzed. For example, Tseng et al. (1995) obtained predicted lifetimes $\hat{t}(= -\log(0.6)/\hat{\lambda})$ for the fluorescent lamp experiment and analyzed the $\hat{t}$ as lognormal lifetimes. When the measurement error is small (i.e., small σ^2_e), the procedure should perform well. However, for substantial measurement error, the procedure is not recommended because the distribution of $\hat{t}$ will no longer be approximately lognormal. Instead, a methodology which accounts for the measurement error is needed.

Lu and Meeker (1993) do account for measurement error in the single population case. They assume the random effect λ or some appropriate reparameterization $\theta = H(\lambda)$ (e.g., let H be the log transformation in the fluorescent lamp experiment.) is normally distributed. The normality assumption allows the information in the sample paths to be summarized by the first two moments without substantial loss of information. They propose method of moment estimators which are used to obtain pointwise and interval estimates for the time-to-failure distribution.

Briefly consider an extension of Lu and Meeker (1993) to the experimental design framework which is the focus of this paper. The point estimates for each run can be calculated as in the single population case and used as the "response" in subsequently computing estimates for the factorial effects, i.e., the main effects and interactions; this extension is really a three-stage approach. Interval estimates to assess the significance of the effects are also possible by bootstrapping. Some potential drawbacks of this extension include: (i) $H(\hat{\lambda})$ may not be normally distributed (e.g., for the fluorescent lamp experiment, $\log \lambda$ is normally distributed but not $\log \hat{\lambda}$), (ii) asymptotic moments for $\log(\hat{\lambda})$ are used rather than exact moments and (iii) obtaining interval estimates by bootstrapping will be computationally involved which is true even in the single population case.

4.3 A New Two-Stage Approach

Consider a different two-stage approach which models the estimated degradation rates directly and avoids the drawbacks mentioned above. As above, the individual degradation rate λ for a single unit based on its degradation path is estimated using least squares, say $(\hat{\lambda}|\lambda)$. Then the density function of $\hat{\lambda}$ for the population of degradation paths can be expressed as:

$$f_{\hat{\lambda}}(\omega, \boldsymbol{\theta}, \boldsymbol{\eta}) = \int_0^{\infty} f_{\hat{\lambda}|\lambda}(\omega \mid \delta) f_{\lambda}(\delta)\, d\delta, \tag{9}$$

where $\boldsymbol{\eta}$ are the variance parameters associated with the measurement error (i.e., σ_e^2 for the fluorescent lamps and $\sigma_{e(ij)}^2$ for the LEDs). In principle, then maximum likelihood estimators (MLEs) for $\boldsymbol{\theta}$ can be found by maximizing

$$\prod_{\forall \text{ runs } i} \; \prod_{\forall \text{ units } j} f_{\hat{\lambda}}(\hat{\lambda}_{ij}, \boldsymbol{\theta}, \boldsymbol{\eta}). \tag{10}$$

There are two difficulties with this approach, however:

(i) There may be many parameters associated with the measurement error ($\boldsymbol{\eta}$) to estimate.

(ii) The density function of $\hat{\lambda}$ may be very complicated. For example, it does not have a closed form when λ has a lognormal distribution.

For analyzing the estimated degradation rates $\hat{\lambda}$, the following general procedure overcomes these difficulties:

(I) To handle (i), estimates of $\boldsymbol{\eta}$ based on the least squares fits of the degradation paths are treated as true values in step II.

(II) – If (ii) does not hold, i.e., can be evaluated easily, the likelihood (10) can be maximized. Note that because of step I, this is not the exact likelihood so that the resulting estimators have been called pseudo MLEs or PMLEs (Gong and Samaniego, 1981, and Parke, 1986).

– If (ii) holds, but the mean and variance of $\hat{\lambda}$ are easy to calculate, then a generalized least squares (GLS) approach can be used. (See Seber and Wild, 1989, for details.) In view of step I, the resulting estimators are called pseudo GLS estimators or PGLSEs. Note that if $\hat{\lambda}$ is normally distributed, then the PGLSEs are PMLEs.

Next, this general procedure will be used to analyze the fluorescent lamp and LED experiments in Sections 5 and 6, respectively.

5 Analysis of the Fluorescent Lamp Experiment

Because λ is lognormally distributed, the density of $\hat{\lambda}$ cannot be easily computed. Therefore, the GLS approach will be taken here for analyzing the fluorescent lamp degradation data.

The mean and variance of $\hat{\lambda}$ can be computed as follows:

$$E(\hat{\lambda}_{ij}) = E[E(\hat{\lambda}_{ij} \mid \lambda_{ij})] = E(\lambda_{ij}) = exp(\frac{\mu_i + \sigma^2}{2})$$

and

$$\begin{aligned} Var(\hat{\lambda}_{ij}) &= E[Var(\hat{\lambda}_{ij} \mid \lambda_{ij})] + Var[E(\hat{\lambda}_{ij} \mid \lambda_{ij})] \\ &= \frac{\sigma_e^2}{\sum t^2} + (exp(\sigma^2) - 1)(exp(2\mu_i + \sigma^2)), \end{aligned}$$

for the jth lamp from the ith run and $\sum t^2$ is the sum of squared inspection times. Note that here, both σ_e^2 and σ^2 are treated as nuisance parameters. An estimate for σ_e^2 is easily obtained from the least squares fitting of the degradation paths. Also, the orthogonal structure of the experimental plan given in Table 1 provides an estimate for σ^2 using $\hat{\sigma}^2 = \frac{1}{2}\log(\nu_1\nu_2\nu_3\nu_4) - \hat{\alpha}_0$, where $\nu_i = \sum_{j=1}^{n} \hat{\lambda}_{ij}/n$ and n is the number of units sampled at each of the runs.

Thus, the PGLSEs for $\boldsymbol{\theta}$ are obtained by minimizing

$$Q(\boldsymbol{\theta}, \boldsymbol{\Sigma}(\boldsymbol{\theta})) = [\hat{\boldsymbol{\lambda}} - \boldsymbol{\mu}(\boldsymbol{\theta})]^{'}\boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta})[\hat{\boldsymbol{\lambda}} - \boldsymbol{\mu}(\boldsymbol{\theta})]. \tag{11}$$

For the lamp experiment, $\hat{\boldsymbol{\lambda}}^{'} = (\hat{\lambda}_{11}, \ldots, \hat{\lambda}_{15}, \ldots, \hat{\lambda}_{41}, \ldots, \hat{\lambda}_{45})$, $\boldsymbol{\theta}^{'} = (\alpha_0, \ldots, \alpha_3)$, $\boldsymbol{\mu}^{'}(\boldsymbol{\theta}) = (E(\hat{\lambda}_{11}), \ldots, E(\hat{\lambda}_{15}), \ldots, E(\hat{\lambda}_{41}), \ldots, E(\hat{\lambda}_{45}))$ and $\boldsymbol{\Sigma}(\boldsymbol{\theta}) = diag(Var(\hat{\lambda}_{11}), \ldots, Var(\hat{\lambda}_{15}), \ldots Var(\hat{\lambda}_{41}), \ldots, Var(\hat{\lambda}_{45}))$ assuming that all paths are independent.

One method for minimizing (11) is to use an iterative algorithm. Letting $\boldsymbol{\theta}^{(l)}$ denote the estimate of $\boldsymbol{\theta}$ at the l^{th} iteration, the next estimate $\boldsymbol{\theta}^{(l+1)}$ is obtained by minimizing $Q(\boldsymbol{\theta}, \boldsymbol{\Sigma}(\boldsymbol{\theta}^{(l)}))$. Starting values $\boldsymbol{\theta}^{(0)}$ can be obtained using the methods of moments which solves the following equations:

$$\nu_i = exp(\frac{\hat{\mu}_i + \hat{\sigma}^2}{2}),$$

$$S_i^2 = \frac{\hat{\sigma}_e^2}{\sum t^2} + (exp(\hat{\sigma}^2) - 1)(exp(2\hat{\mu}_i + \hat{\sigma}^2)),$$

where

$$\nu_i = \frac{\sum_{j=1}^{n} \hat{\lambda}_{ij}}{n},$$

and

$$S_i^2 = \frac{1}{n-1}\sum_{j=1}^{n}[\hat{\lambda}_{ij} - \nu_i]^2.$$

Here, there are four runs ($i = 1, \ldots, 4$) and five units per run ($n = 5$).

The next estimate can be obtained as follows:

$$\boldsymbol{\theta}^{(l+1)} = \boldsymbol{\theta}^{(l)} + \{[\frac{\partial \boldsymbol{\mu}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^{'}}]^{'}_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(l)}} \boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta}^{(l)})[\frac{\partial \boldsymbol{\mu}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^{'}}]_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(l)}}\}^{-1}[\frac{\partial \boldsymbol{\mu}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^{'}}]^{'}_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(l)}} \boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta}^{(l)})[\hat{\boldsymbol{\lambda}} - \boldsymbol{\mu}^{'}(\boldsymbol{\theta}^{(l)})].$$

Fedorov (1974) shows that if $\boldsymbol{\theta}^{(l)}$ converges to the solution $\hat{\boldsymbol{\theta}}_G$, then

$$\sqrt{4n}(\hat{\boldsymbol{\theta}}_G - \boldsymbol{\theta}) \sim N(\boldsymbol{0}, \boldsymbol{\Omega}) \text{ asymptotically},$$

where

$$\boldsymbol{\Omega} = \frac{1}{4n}\{[\frac{\partial \boldsymbol{\mu}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^{'}}]^{'}_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_G} \boldsymbol{\Sigma}^{-1}(\hat{\boldsymbol{\theta}}_G)[\frac{\partial \boldsymbol{\mu}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^{'}}]_{\boldsymbol{\theta}=\hat{\boldsymbol{\theta}}_G}\}^{-1}$$

and 4 represents four runs.

Table 3 gives the PGLSEs and 95% confidence intervals for the three main effects A, B and C which indicates that all are important. Residual plots not shown here are consistent with the model assumed here. Based on the negative main effects, reliability can then be improved by setting all factors at their high levels $(+,+,+)$, i.e., at which the mean log degradation rate is minimized. Note that the estimated mean lifetime at the original process settings (all at their low levels) is 15000 hours, so that at these recommended levels, the estimated mean lifetime is 26000 hours, a 73% improvement! The recommended levels $(+,+,+)$ do not correspond to any of the four runs in the experiment which shows the power of using fractional factorial experimental plans.

Contrast the above results with those of Tseng et al. (1995) which analyzed predicted lifetimes using standard methods and missed the A main effect under the above model; subsequently, factor settings $(-,+,+)$ were recommended, whose estimated mean lifetime is 24600 hours.

Table 3. PGLSEs and 95% Confidence Intervals
for the Fluorescent Lamp Experiment

Parameter	PGLSE	95% Confidence Interval
α_0 *Intercept*	2.572	$(2.569, 2.576)$
α_1 A	-0.056	$(-0.059, -0.052)$
α_2 B	-0.329	$(-0.333, -0.326)$
α_3 C	-0.166	$(-0.170, -0.162)$

6 Analysis of the LED Experiment

In analyzing the data from the LED experiment, the goal of achieving robust reliability improvement must be addressed, i.e., identifying control factor combinations whose reliability is insensitive to the noise factor. Recall that the degradation rate λ is random and its distribution depends on the control and noise factors. Because small degradation rates are desired and ideally zero, the following loss function $l(\cdot)$ is used:

$$E[(\lambda - 0)^2] = E[(\lambda)]^2 + Var(\lambda), \tag{12}$$

which is a function of the control and noise factor levels $(z_{control}, z_{noise})$. The loss given in (6) can then be optimized once estimates for the model parameters are obtained from the degradation paths.

Recall that the path model (1) for the LED experiment data takes the form (4) with error variance $\sigma^2_{e(ij)}$ and

$$\lambda_{ij} \sim N(\mu_{ij}, \sigma_i^2),$$

where the dependence on control factor run i and noise factor level j are incorporated in the notation.

For each degradation path (30 for each of the 16 (i,j) combinations), a least squares fit gives $(\hat{\lambda}_{ij}|\lambda_{ij})$ whose distribution is $N(\lambda_{ij}, \gamma_{ij}^2 = \sigma^2_{e(ij)}/\sum(t_3 - t)^2)$. Then, $\hat{\lambda}_{ij}$ can be shown to also be distributed as $N(\mu_{ij}, \gamma_{ij}^2 + \sigma_i^2)$.

Substituting pooled estimates for γ_{ij}^2 from the least squares fit of the appropriate degradation paths, the PMLEs of $\boldsymbol{\theta}' = (\alpha_0, ..., \alpha_{10}, \beta_0, ..., \beta_6)$, the factorial effects for (μ_{ij}, σ_i^2) (refer to (7) and (8)), can be obtained by maximizing the pseudo log likelihood:

$$\mathcal{L} \propto \sum_i \sum_j \sum_k [-\frac{1}{2}\log(\sigma_i^2 + \hat{\gamma}_{ij}^2) - \frac{(\hat{\lambda}_{ijk} - \mu_{ij})^2}{2(\sigma_i^2 + \hat{\gamma}_{ij}^2)}],$$

where $\hat{\lambda}_{ijk}$ is the estimate based on the kth unit's degradation path. Thus, the PMLEs can be obtained by solving

$$\frac{\partial \mathcal{L}}{\partial \alpha_\ell} = 0 \text{ and } \frac{\partial \mathcal{L}}{\partial \beta_\ell} = 0.$$

The variance-covariance matrix can then be easily obtained by taking second partial derivatives and has the form:

$$\boldsymbol{\Sigma} + \boldsymbol{\Sigma}\boldsymbol{\vartheta}_{12} diag(\hat{Var}(\gamma_{ij}^2))\boldsymbol{\vartheta}_{12}'\boldsymbol{\Sigma},$$

where

$$\boldsymbol{\Sigma} = \begin{bmatrix} [-E(\frac{\partial^2 \mathcal{L}}{\partial \alpha_\ell \partial \alpha_{\ell'}})] & [-E(\frac{\partial^2 \mathcal{L}}{\partial \alpha_\ell \partial \beta_{\ell'}})] \\ & [-E(\frac{\partial^2 \mathcal{L}}{\partial \beta_\ell \partial \beta_{\ell'}})] \end{bmatrix}^{-1}$$

and

$$\boldsymbol{\vartheta}_{12} = \begin{bmatrix} [-E(\frac{\partial^2 \mathcal{L}}{\partial \alpha_\ell \partial \gamma_{ij}^2})|_{\hat{\gamma}_{ij}^2}] \\ [-E(\frac{\partial^2 \mathcal{L}}{\partial \beta_\ell \partial \gamma_{ij}^2})|_{\hat{\gamma}_{ij}^2}] \end{bmatrix};$$

the notation $[\cdot]$ indicates a matrix with the typical element given in the brackets.

Table 4 presents the PMLEs as well as their respective standard errors and 95% confidence intervals. Residual plots not shown here are consistent with the model assumed here. Based on these PMLEs, the loss (6) can be calculated for a specified distribution of the noise factor. Here, for illustration, it is assumed that the distribution of the noise factor levels is $N(0, (\frac{1}{3})^2)$ so that the bulk of the distribution is within the experimental region $[-1, 1]$. Table 5 presents the estimated loss for each run.

Based on the results from Table 5, a good choice of factor levels would be $(-,-,-)$ or $(-,-,+)$ for factors A, B and C, respectively; that is, Runs 1 and 5 have the smallest estimated losses. To see the impact of these factors in terms of reliability, the reliability function (5) averaged over the noise factor distribution can be computed. As an approximation, an average reliability ((5))using three positive and negative standard deviations region, that is $[-1, 1]$ for $N(0, (\frac{1}{3})^2)$, was computed with $D = 250$. The average reliability functions are displayed in Figure 3 for time units $t = 10(1)100$. Compared with the current process (Run 8), the recommended levels (Run 1) is much better.

7 Conclusions

In this paper, some ideas and methods for improving reliability and for achieving robust reliability for highly reliable products have been discussed. Namely, the usefulness of statistically designed experiments which collect degradation data has been shown. While the experimental plans are the same ones used for improving any quality characteristic, it is the

Table 4. PMLEs, Standard Errors and 95% Confidence Intervals for the LED Experiment

Parameter		PMLE	Standard Error	95% Confidence Interval
α_0	*Intercept*	8.203	0.021	$(7.920, 8.487)$
α_1	A	3.180	0.014	$(2.952, 3.408)$
α_2	B	5.278	0.021	$(4.993, 5.562)$
α_3	C	−3.435	0.017	$(-3.692, -3.178)$
α_4	$A \times B$	2.155	0.014	$(1.922, 2.387)$
α_5	$A \times C$	−0.435	0.004	$(-0.560, -0.311)$
α_6	$B \times C$	−3.277	0.016	$(-3.527, -3.027)$
α_7	$N(Noise)$	−0.075	0.010	$(-0.266, 0.117)$
α_8	$A \times N$	−0.010	0.004	$(-0.130, 0.110)$
α_9	$B \times N$	0.107	0.009	$(-0.080, 0.294)$
α_{10}	$C \times N$	−0.065	0.003	$(-0.179, 0.049)$
β_0	*Intercept*	1.821	0.003	$(1.720, 1.922)$
β_1	A	0.562	0.003	$(0.461, 0.663)$
β_2	B	1.402	0.003	$(1.301, 1.503)$
β_3	C	−0.320	0.003	$(-0.421, -0.219)$
β_4	$A \times B$	0.137	0.022	$(-0.156, 0.430)$
β_5	$A \times C$	0.055	0.017	$(-0.203, 0.313)$
β_6	$B \times C$	−0.552	0.022	$(-0.846, -0.258)$

Table 5. Estimated Losses for the LED Experiment

Run	Factors			Estimated Loss	Rank
	A	B	C		
	Dice	Epoxy	Lead Frame		
1	−	−	−	3.320	1
2	+	−	−	22.765	5
3	−	+	−	242.252	7
4	+	+	−	779.386	8
5	−	−	+	5.839	2
6	+	−	+	14.646	4
7	−	+	+	9.481	3
8	+	+	+	154.559	6

analysis of the degradation data from these experiments that provide new considerations. In this context, pseudo GLSEs and MLEs were used in analyzing the estimated degradation rate parameters from the individual degradation paths. These methods avoid problems that arise from evaluating complicated probability density functions or for handling a large number of nuisance parameters, i.e., mainly those associated with the measurement errors.

It is important that engineers applying these methods be aware of the methods' limitations. For example, the methods depend on the asymptotic properties of the pseudo GLSEs and MLEs. Further study is needed to determine how often and long should the units be inspected. Other types of degradation models, such as (1) nonlinear random effects models and (2) models with more than one random effect, need to be explored. For (1), the choice of degradation model is important. For (2), defining a suitable quality characteristic which combines the random effects is one approach. It will be interesting to see how the methods proposed here in this paper can be adapted in these other situations.

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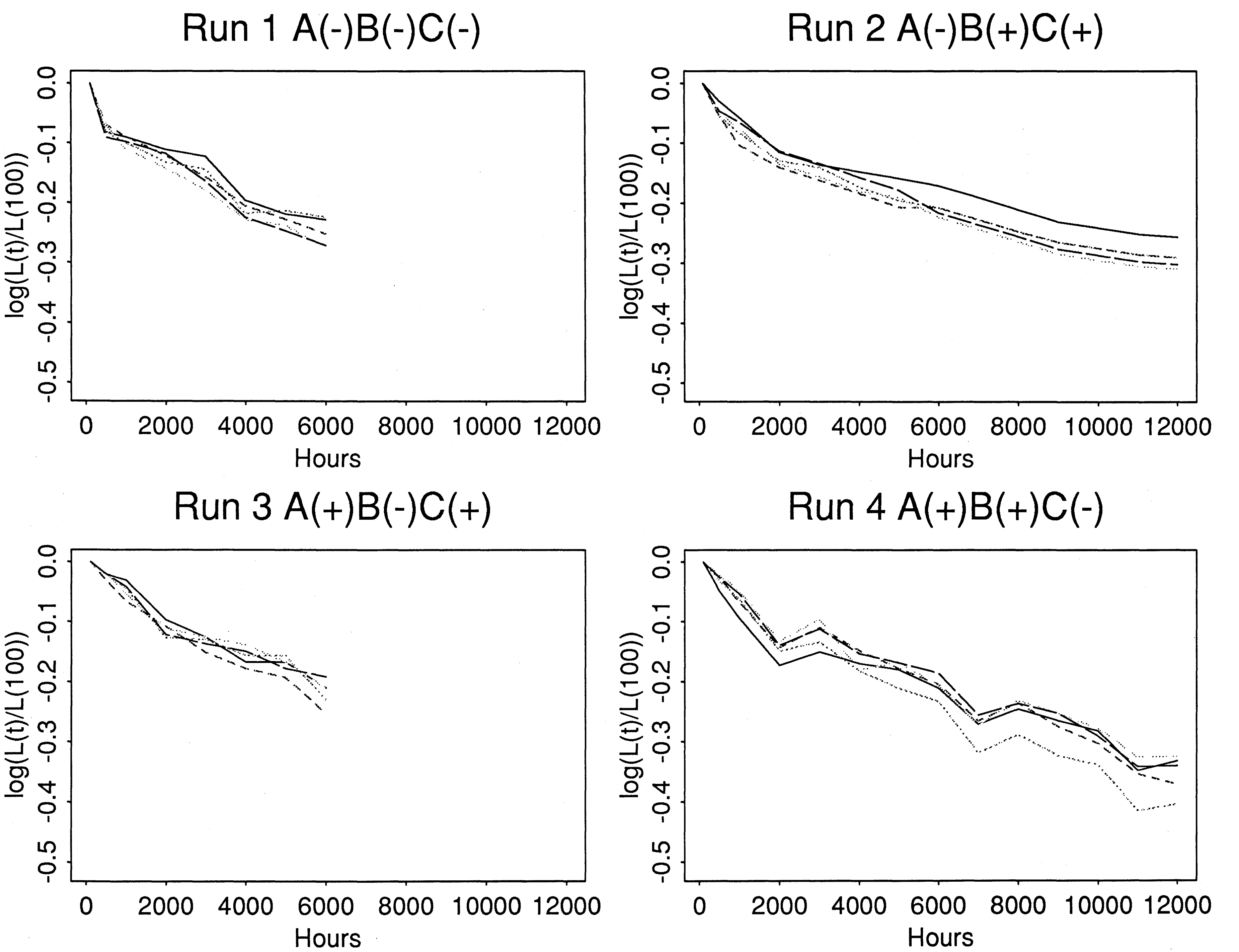

Figure 1. Sample Degradation Paths From Fluorescent Lamp Experiment

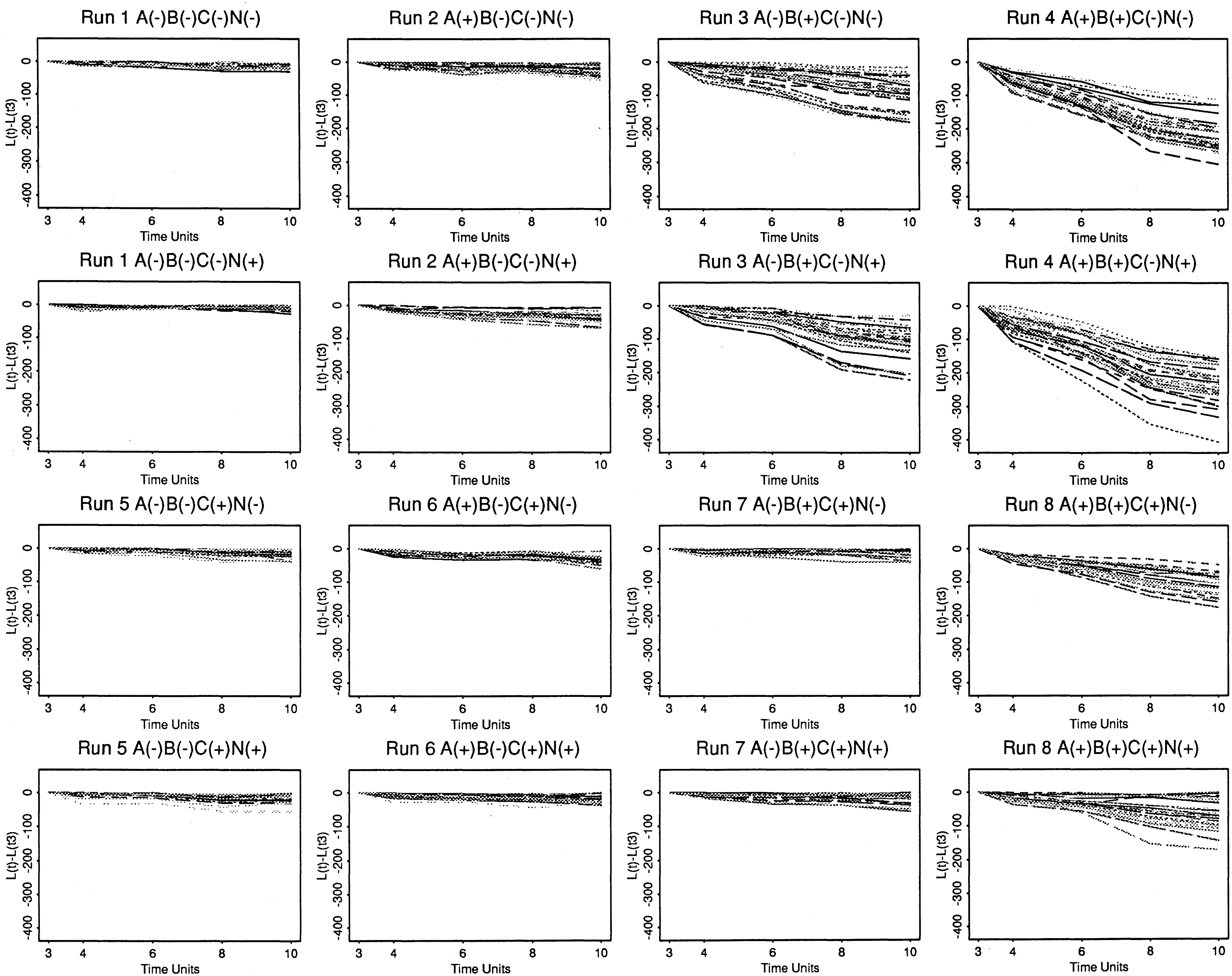

Figure 2. Sample Degradation Paths From LED Experiment

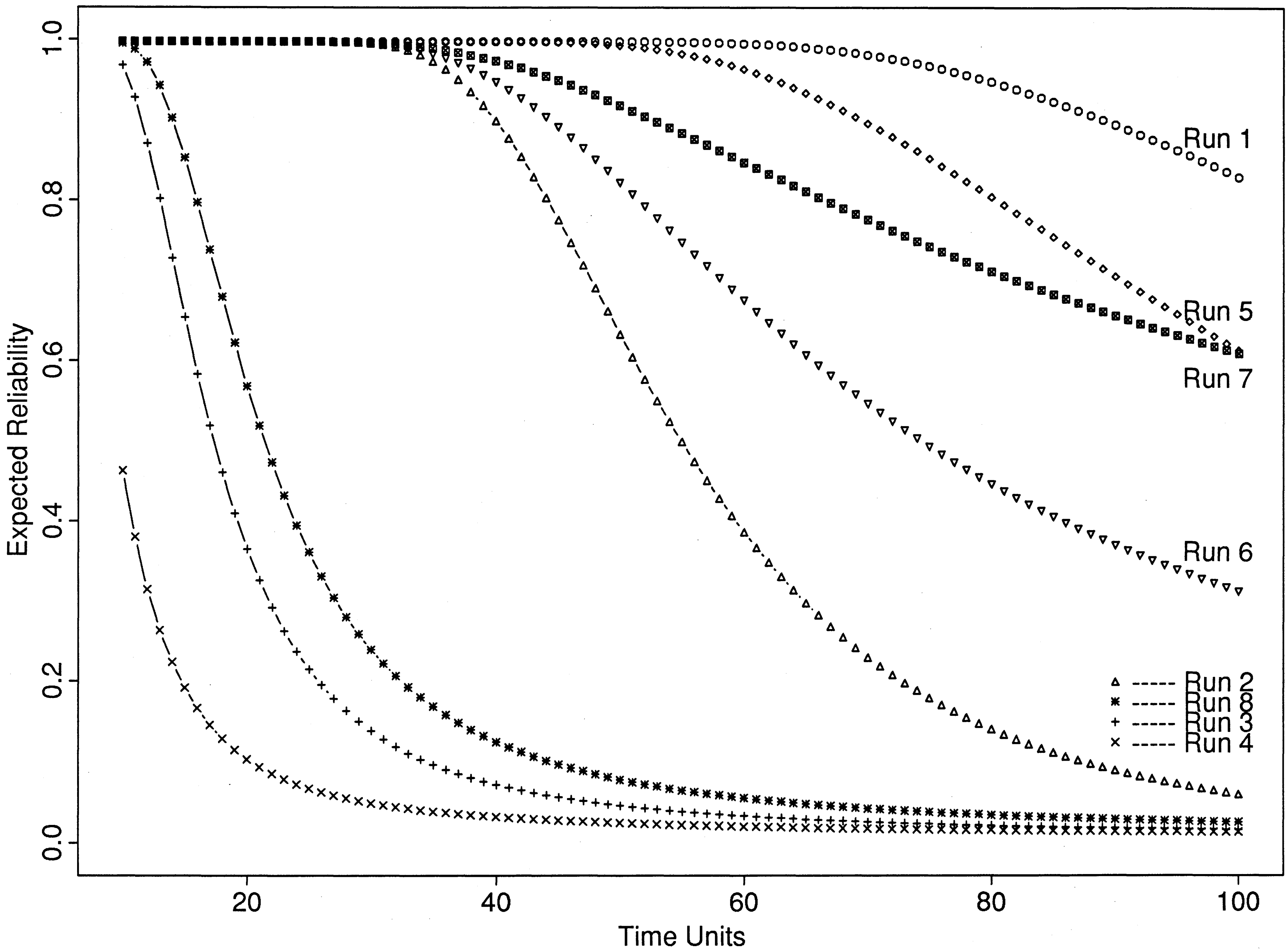

Figure 3. Average Reliability Curves For LED Experiment