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Another Look at Supersaturated Designs

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Abstract

A design of an experiment in which a number of factors are at least as large as the number of runs is referred to as a supersaturated (SS) design. Recently these designs have received increased attention. Construction of such design and analysis of data from these design have been discussed be several authors. All the discussions assume the sparsity of effects: although many factors are initially considered only a small number of factors is really important. With the sparsity of effect assumption, it is claimed that one can identify the real effects in design with 12 or 14 runs when there are as many as many as 60 factors under consideration. Our objective in this paper is to examine these claims so that practitioners get an opportunity to see what is going on.

Key Words: Rubber data, Hadamard matrices, Placket and Burnam Designs

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1. INTRODUCTION

In industrial experimentation it is natural to look at many factors simultaneously. It is impractical from cost consideration to design experiments so that effects of all these factors are simultaneously estimated. Considering that only a few of these factors are active in the sense of having large effects it is suggested that these active factors could be identified with high probability using smaller number of observations than the number of factors considered. Such designs are referred to as supersaturated designs (see Booth and Cox (1962)) and the objective in this context is to identify all or most of the active factors. The intention of this paper is to investigate these designs and their ability to identify active factors.

It is assumed that there is no interaction between the factors and that the effect is linear on the level of each factor. Under this assumption we have the basic model for observations y_i:

$$y_i = \beta_0 + \sum_{j=1}^{m} \beta_j X_{ij} + \epsilon_i \ , \ i = 1, 2, \cdots, n \qquad n < m \tag{1.1}$$

where m is the number of factors and n is the number of observations (runs). Here X_{ij} is the level of factor $j(X_j)$ in run i and is restricted to two values $+1$ or -1. The parameters $\beta_j (j = 1, 2, \ldots, m)$ stand for the effect of factor j and it is assumed that only a few of these parameters are different from 0.

As the number of parameters is more than the number of observations usual least square methods are not applicable here. One suggestion is to use a forward stepwise regression approach (see Lin (1993)) and we adopt this strategy in our discussions. The success of this method depends on the choice of the design matrix. Lin (1993) suggested using half fractions of Hadamard matrices to construct supersaturated designs. The procedure can be summarized as follows. In a standard Hadamard matrix whose first column entries are all $+1$, take any other column which may be called a branching column. Consider the matrix formed by the rows which have the same sign in the branching columns. Assign factors to the columns other than the first and branching columns. The resulting design is recommended.

Lin (1993) used one such design on a real data set reported by Williams (1968) and the results obtained supported the use of the design.

The construction procedure described before implies that the design obtained is not unique. For example, suppose we are interested in investigating 23 factors and we consider a Hadamard matrix of order 28 (28 rows, 28 columns with +1 in the first column). The construction method suggested would lead to a design with 14 rows and 23 columns (excluding column 1). There are 8 such designs possible. Which one should we use?

In Section 2 we re-analyse the Rubber Data (Williams (1968) and Lin (1993)). Section 3 gives a simulation study to investigate the merits of the half replicate supersaturated design. Section 4 discusses some of the design issues and gives another possible construction method. Section 5 gives some concluding remarks. The main purpose of the paper is to make practitioners aware of some of the difficulties involved in the use of supersaturated designs.

2. RUBBER DATA

In order to illustrate the performance of the supersaturated design using the half replicate method, Lin (1993) considered the data originally reported by Williams (1968) who used a Plackett and Burman (PB) design with 24 factors and 28 runs in a rubber making process. PB design is a particular case of using a Hadamard matrix and Lin's half replicate method described in Section 1 can be used to construct a supersaturated design from the PB design for the Rubber data and then obtain a set of realistic data from this experiment. The design and data of Williams are given in Table A.1 in the Appendix. As was pointed out by Box and Draper (1987) there was a typographical error in one of the entries of the original data (8th element from top in column 20) and this is corrected in our table. Also columns 13 and 16 are identical and hence we eliminated column 16.

There are 8 possible SS designs that could be constructed using the half replicate method. (The PB design with 24 columns including column of +1's is part of a Hadamard matrix

Table 2.1 Rubber Data Factors included after Five Steps in a Stepwise Procedure

Design	Steps	1	2	3	4	5	R^2
1	SL = .15	17	15	4	22	10	.90
	SL=.075	17	15	4	22	10	.90
2	SL=.15	15	24	18	13	8	.97
	SL=.075	15	24	18	13	8	.97
3	SL=.15	15	20	3	4	22	.92
	SL=.075	15	20	-	-	-	.73
4	SL=.15	2	13	8	3	20	.84
	SL=.075	2	13	8	-	-	.69
5*	SL=.15	15	12	20	4	10	.97
	SL=.075	15	12	20	4	10	.97
6	SL=.15	4	22	23	18	24	.88
	SL=.075	4	22	23	-	-	.74
7	SL=.15	14	12	11	23	-	.89
	SL=.075	14	12	11	23	-	.89
8	SL=.15	15	22	8	17	1	.94
	SL=.075	15	22	8	17	1	.94

Notes: * Design 5 is the same as that used in Lin (1993)

"-" indicates that the procedure stopped before of order 28 and the remaining 4 columns can be uniquely determined as shown by Vijayan (1976); these 4 columns are used as branching columns to produce the 8 designs.) The rows corresponding to these 8 designs are given in Table A.2 in the Appendix.

Lin (1993) used design number 5 for his analysis, where a stepwise regression procedure was utilized to identify the active factors. We adopted the same procedure for each of these 8 designs using the stepwise procedure in SAS (Statistical Analysis System). For inclusion and exclusion we used the default option ("significance" level .15) first. Then we used the level .075 to get results for design 5 to match those of Lin (1993). All these are presented in Table 2.1. The numbers in the table indicate the factor number. For instance, with significance level $(SL) = .075$ in design 1 in the first step, factor 17 was selected as active and factors 15, 4, 22 and 10 were selected in subsequent steps. As can be seen from the table, different conclusions result from different designs. As expected Design 5 led to the same choices as in Williams; however, this is the only design which led to this choice. This example illustrates that it is very difficult to make general conclusions regarding the choice of active factors in a SS design.

3. SIMULATION STUDY

We conducted a simulation study to verify some of the features of the supersaturated designs given in Lin (1993, 1995). For this we consider two designs (14 run, 12 run) and the model given in (1.1).

Case 1: 14 run design with 23 factors

Step 1: Data generation:

We generate $n = 14$ $N(0,1)$ random variates, $\epsilon_i, i = 1, \ldots, 14$ and then using the model (1.1) generate $y_1, \cdots, y_{14}$ for specific sets of values of the β's.

Pure noise: We consider all the β's to be zero.

Then we consider the following situations.

(1) $$\beta_j = \begin{cases} .5, 1, 20 & i = 1 \\ 0 & i \neq 1 \end{cases}$$ (3 separate cases)

(2) $\beta_2 = \beta_7 = 1$, other β's zero

$\beta_2 = \beta_7 = 20$, other β's zero

(3) $\beta_1 = \beta_2 = \beta_3 = 1$, other β's zero

$\beta_2 = \beta_7 = \beta_{13} = .5$, other β's zero

$\beta_2 = \beta_7 = \beta_{13} = 1$, other β's zero

$\beta_2 = 5, \beta_7 = 10, \beta_{13} = 20$, other β's zero

$\beta_2 = 14, \beta_7 = 20, \beta_{13} = 20$, other β's zero

$\beta_2 = \beta_7 = \beta_{13} = 20$, other β's zero

Step 2: Analysis for factor selection.
We use the forward selection procedure with a specific significance level (SL) for including a factor, stop after step 5 and record the selected factors.

Step 3: Repeat Steps 1 and 2, $N = 200$ times and record the number of times each factor was selected.

In the Pure noise case with SL = .05, 79% of the times, at least one factor was selected as active in 5 steps. This was not unexpected. With 23 contrasts tested to zero individually and assuming them to be independent (this is not quite true in our case) there is a chance $1 - (1 - \alpha)^{23}$ of selecting at least one significant contrast where α is the SL for inclusion. When $\alpha = .05$ this probability = .70 which is in the vicinity of the simulated value. We cannot avoid this situation even if we have a large number of observations. On the other hand if the present experiment is considered as a device to reduce the number of factors

Table 3.1 Simulation Results Selection Pattern of Factors (Lin's 14 run Design)

Factors with Real Effects	Corresp-onding β values	Percent of times real effect factors selected in k steps $k = \#$ factors in col. 1		Percent of times real effect factors selected in 5 steps		Percent of time X_1 selected in the first step	
		SL=0.10	SL=0.05	SL=0.10	SL = 0.05	SL =0.10	SL=0.05
X_1	0.5	36.5	27.5	56.5	38		
	1	87	85.5	93	90.5		
	20	100	100	100	100		
X_2, X_7	1,1	55	56	67	62.5	24.5	22
	20, 20	78	74	78	74	22	26
X_1, X_2, X_3	1, 1, 1	40.5	38	55	45.5		
X_2, X_7, X_{13}	0.5, 0.5, 0.5	0	0	1	1	43.5	46.5
	1, 1, 1,	.5	.5	4	2	91.5	91
	5, 10, 20	100	100	100	100	0	0
	14, 20, 20	0	0	0	0	100	100
	20, 20, 20	0	0	0	0	100	100

which need to be carefully examined in a later experiment, then it is important to know whether the procedure leads to the active factors with high probability. For this purpose we consider cases 1-3 where one, two, or three β's are different from zero. The results are presented in Table 3.1. We note the following from the table:

(i) When only one β is different from zero or it is much larger than the rest, the corresponding factor gets selected in the first 5 steps. For instance, with a SL of 10%, X_1 gets selected 100% of the cases if $\beta_1 = 20$, 87% of the cases when $\beta_1 = 1$ and only 36.5% when $\beta_1 = .5$. When SL=.05 and $\beta_1 = .5$, X_1 gets selected only in 36% of cases. (Note that the standard error of $\beta_1 \simeq .27$).

(ii) When two β's are important and very large ($\beta_2 = \beta_7 = 20$), the corresponding factors get selected about 78% of the time. This drops to 67% when $\beta_2 = \beta_7 = 1$. It should also be noted that in all these cases X_1, which is not an active factor gets selected in step 1 in about 22% of the time.

(iii) When more than two β's are important, conclusions are different depending on which β's are non zero. For instance, with SL $= .1$ if $\beta_2 = \beta_7 = \beta_{13} = 1$ and $\beta_i = 0, i \neq 2$, 7, or 13, the combination (X_2, X_7, X_{13}) is selected only 4% of the time while X_1 is selected 91.5% of the time in step 1; however if $\beta_1 = \beta_2 = \beta_3 = 1$ and the rest are zero then (X_1, X_2, X_3) gets picked up 55% of the time. It should also be noted that when $\beta_2 = 14, \beta_7 = \beta_{13} = 20$ or $\beta_2 = \beta_7 = \beta_{13} = 20$ and the rest are zero the combination (X_2, X_7, X_{13}) is never selected while X_1 is selected 100% of the time. On the other hand, if $\beta_2 = 5, \beta_7 = 10, \beta_{13} = 20$ the combination gets selected 100% of the cases.

Factors selected as active depend on what columns in the design matrix are assigned to the real active factors. This is a result of the correlation structure of the columns of the design matrix.

Case 2: 12 run design

Lin (1995) presented an algorithm to generate certain SS designs and the ones for 12 runs are shown in the paper. General construction methods are available to produce these designs with number of factors as high as 66. Basically the design is constructed from the columns of a standard Hadamard matrix. Using the notation $\mathbf{1}' = (1,1,1), \mathbf{u}_1' = (1,-1,-1), \mathbf{u}_2' = (-1,1,-1)$, and $\mathbf{u}_3' = (-1,-1,1)$ we can write down one such Hadamard matrix of order 12 as given below:

1	2	3	4	5	6	7	8	9	10	11	12
$\mathbf{1}$	$\mathbf{1}$	$-\mathbf{u}_3$	$\mathbf{1}$	$-\mathbf{u}_3$	$-\mathbf{u}_3$	$-\mathbf{u}_2$	$-\mathbf{u}_2$	$-\mathbf{u}_2$	$\mathbf{u}_1$	$\mathbf{u}_1$	$\mathbf{u}_1$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{u}_3$	$-\mathbf{1}$	$\mathbf{u}_1$	$\mathbf{u}_2$	$\mathbf{u}_2$	$\mathbf{u}_1$	$\mathbf{u}_3$	$-\mathbf{u}_3$	$-\mathbf{u}_2$	$-\mathbf{u}_1$
$\mathbf{1}$	$-\mathbf{1}$	$\mathbf{1}$	$\mathbf{u}_1$	$\mathbf{u}_3$	$\mathbf{u}_2$	$-\mathbf{u}_2$	$-\mathbf{u}_1$	$\mathbf{u}_2$	$-\mathbf{u}_3$	$\mathbf{u}_1$	$\mathbf{u}_3$
$\mathbf{1}$	$-\mathbf{1}$	$-\mathbf{1}$	$-\mathbf{u}_1$	$-\mathbf{u}_2$	$-\mathbf{u}_3$	$\mathbf{u}_1$	$\mathbf{u}_2$	$-\mathbf{u}_2$	$\mathbf{u}_3$	$-\mathbf{u}_3$	$-\mathbf{u}_1$

The different columns of the design matrix would be obtained by taking componentwise products of two columns of this basic matrix leading to a design matrix with 66 columns. We denote the componentwise product of the columns i and j by "$i \times j$". Lin (1995) demonstrated the usefulness of SS designs by carrying out a simulation study using the above design. In the simulation study factors corresponding to columns $1 \times 4, 1 \times 8$, and 8×10 were given large effects (β values 17, 24 and 15 respectively), those corresponding to 1×11, and 8×11 were given moderate effects (β values 3), and others were taken as zero. The results of the simulation indicated that the 5 active factors were selected in the first five steps of a forward selection procedure always in all the repetitions.

We wish to demonstrate that the results depend on which columns of the X matrix correspond to the large and moderate effect factors. In our simulation study we assigned the β values $17, 24, 15$ (large effects in Lin's simulation) to columns $8 \times 10, 6 \times 12$, and 9×11 and the β value 3 to columns 3×5 and 4×7, and zero to the remaining columns. 12 observations were generated from model (1.1) using the β-values indicated before and a forward selection procedure as in Lin (1995) was used to pick up the active factors. In 200 repetitions, none of the active factors corresponding to columns $6 \times 12, 9 \times 11, 3 \times 5$ and 4×7 were picked up in the first five steps while the factor corresponding to column 8×10 was selected only 29% of the time. The factor corresponding to column 1×2 (inactive) was always the choice in the first step.

4. DISCUSSION AND SUGGESTIONS FOR IMPROVEMENT

When the number of factors is very large it is inevitable that some of the inactive factors would be selected with high probability. This is not a serious problem as long as active factors are also selected with high probability. We notice from simulations in Section 3 that sometimes the procedures fail to select active factors and select inactive factors instead. This is a very serious issue and others (see Chipman et al (1995), Hamada and Wu (1992) and Wu (1993)) have noticed this as well. The nonorthogonality of the columns of the design matrix X is the root of the problem.

Suppose that the first r factors are active and the rest inactive. For the sake of discussion let us assume that the error ϵ in (1.1) is negligible. Then the observations can be written in the form

$$\boldsymbol{y} = \beta_0 + \sum_{i=1}^{r} \boldsymbol{X}_i \beta_i \tag{4.1}$$

where $\boldsymbol{y}$ is the observation vector, $\boldsymbol{X}_i$ the ith column of the X matrix and $\mathbf{1}$ is a column of one's. Note that $\boldsymbol{X}_i (i = 1, 2, \cdots, r)$ is orthogonal to $\mathbf{1}$. Then the estimate of β_k in the first step of the forward selection procedure is

$$\hat{\beta}_k = (1/\boldsymbol{X}'_k \boldsymbol{X}_k) \boldsymbol{X}'_k \boldsymbol{y} = (1/n) \sum_{i=1}^{r} S_{ik} \beta_i$$

where $S_{ik} = \boldsymbol{X}_i{}' \boldsymbol{X}_k$. The first selected factor is the one for which

$$n \mid \hat{\beta}_k \mid = \mid \sum_{i=1}^{r} S_{ik} \beta_i \mid \qquad \text{is largest.}$$

Let us assume for discussion that the β_i $(0 = 1, 2, \cdots, r)$ have the same magnitude (say β) and that δ_i represent the sign of β_i. Then the factor for which

$$t_k = \left| \sum_{i=1}^{r} \delta_i S_{ik} \right| \tag{4.2}$$

is largest would be selected as active in Step 1 of the forward selection procedure. If the design is such that (4.2) is largest for a k different from $1, 2, \ldots, r$, the selection procedure would lead to an inactive factor to be active. Since the k^{th} factor X_k is very highly correlated with $X_i (i = 1, 2, \cdots, r)$, the residuals from Step 1 of the forward selection procedure would be almost uncorrelated with $X_i (i = 1, 2, \cdots, r)$ and hence the chance of the selection of the true active factors $X_1, \cdots, X_r$ in the subsequent steps is very small.

Now consider the covariance ("$X'X$") matrix in Table A.3, for the 14 run design given in Lin (1993). Suppose that the active factors are X_2, X_7 and X_{13} with the same β coefficients. Then $t_2 = t_7 = t_{13} = 10$ while t_1 is 18. Hence X_1 would be selected before factors 2, 7, 13 unless the error (ϵ) is very large in which case such a study would not be very conclusive anyway. Once X_1 is chosen, the adjusted estimates ($\hat{\beta}_{k \cdot a}$) for the remaining β's are obtained as

$$(14 - S_{1k}^2 / S_{11}) \hat{\beta}_{k \cdot a} = \beta (t_k - (S_{1k}/S_{11}) t_1)$$

leading to

$$\hat{\beta}_{k \cdot a} = \left(\frac{10 - 6 \times 18/14}{14 - 36/14} \right) \beta \quad = (1/5) \beta \qquad\qquad k = 2, 7, 13$$

Note that the estimate of β_k has gone down from what was to be (i.e. $10/14\ \beta$) if not adjusted for X_1. For factors which are not highly correlated with X_1, the effect of adjustment for X_1 would be small. For example for $k = 8$ or 24

$$\hat{\beta}_{k \cdot a} = \beta \left(\frac{2 - (-2 \times 18/14)}{14 - 4/14} \right) = (1/3) \beta$$

while $\hat{\beta}_k = (1/7)\beta$. This would again lead to selection of inactive factors in preference to the active ones.

The situation will be the same even if β_2, β_7, and β_{13} are different provided that these dominate over the others. For example, if $\beta_2 = (2/3)\beta, \beta_7 = \beta$ and $\beta_{13} = (1/2)\beta$ then

$\hat{\beta}_2 = (23/14)\beta$ while $\hat{\beta}_1 = (27/14)\beta$ and hence X_1 would be chosen rather than X_2. After adjusting for X_1, $\hat{\beta}_{2\cdot a} = \beta$ while $\hat{\beta}_{8\cdot a} = (111/94)\beta$ leading to the selection of X_8 in preference to active factors. Thus the high covariances S_{ij} cause the problem of selecting the inactive factors. One suggestion made to lessen the impact of these covariances is to consider a design in which the average value of S_{ij}^2, ($E(S^2)$ say) is as small as possible (for example, see Booth and Cox (1962), Wu (1993)).

Let us consider a 12 run design in 13 factors. It can be shown that for such a design the minimum value of $E(S^2) = 48$ and is attained by the following design (Table A.4) given by Wu (1993).

For this design

$$S_{ij} = \begin{cases} 12 & \text{if} \quad i = j \\ 4 & j = 12, 13 \quad i \neq 1 \text{ or } j - 10 \\ 0 & \text{otherwise} \end{cases}$$

Suppose now that X_4, X_5, X_6, and X_7 have large and equal effects while the rest have zero effects and that the error (ϵ) is negligible; we obtain

$$\hat{\beta}_k = \begin{cases} \beta & k = 4, 5, 6, 7 \\ (4/3)\beta & k = 12, 13 \end{cases}$$

Hence one of the factors X_{12}, X_{13} would be selected first and since the correlation between these factors is zero, the other would be selected in the second step.

The design given in Table A.5 gets around this problem. However, this design (Design 2) has a slightly higher $E(S^2)$. Design 1 (Table A.4) is less attractive because X_{12} and X_{13} have many factors correlated with them and their combined effect spuriously gave strength to these factors. One way to avoid this is to construct designs for which the maximum contribution made to a factor by the others through correlation is small. Thus a useful method for construction of a design is to consider the minimax criterion: Choose the design for which

$$\lambda = \max_i \sum_j | S_{ij} |$$

is as small as possible.

This is the criterion used to obtain the design in Table A.5. It should be noted that if $\lambda < 2n$ then the active factors will have a higher chance of selection than the inactive factors.

5. CONDLUDING REMARKS

Data analysis in Section 2 and the simulations in Section 3 indicate that one should be very cautious with the use of SS designs. The simulations, in particular show that there is a high chance of missing the real active factors and selecting the inactive ones instead. The assignment of factors to columns of these designs is crucial because of the correlation structure among the columns of the design.

We have not yet seen a SS design used in a real situation to collect data. It is interesting to note that the examples used for demonstrating SS designs in all the previous papers were originally considered for other purposes; other designs were used for data collection, and important factors were already identified.

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Table A.1

Rubber Data, Williams (1968)

Run Number	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Response y
1	+	+	+	−	−	−	+	+	+	+	+	−	+	−	−	+	+	−	−	+	−	−	−	+	133
2	−	+	−	−	−	−	+	+	+	+	−	+	−	−	+	−	+	+	+	+	−	+	+	−	49
3	+	−	−	−	−	−	+	+	+	−	−	−	+	+	+	+	−	+	−	−	+	+	−	−	62
4	+	+	−	+	+	−	−	−	−	+	−	+	+	+	+	+	+	−	−	−	−	+	+	−	45
5	+	+	−	−	+	+	−	−	−	−	−	−	−	−	−	−	+	+	−	+	+	+	−	+	88
6	+	+	−	+	−	+	−	−	−	+	+	−	+	−	+	+	−	+	+	+	−	−	−	−	52
7	−	−	+	+	+	+	+	+	−	+	−	+	+	−	−	+	−	+	−	−	−	−	−	−	300
8	−	−	+	+	+	+	−	+	+	−	−	−	+	−	+	+	+	−	−	+	−	+	+	+	56
9	−	−	+	+	+	+	+	−	+	+	+	−	−	+	+	−	+	+	+	+	+	+	−	−	47
10	−	−	−	−	+	−	−	+	−	+	−	+	+	+	−	+	+	+	+	+	+	−	−	+	88
11	+	−	+	−	−	+	−	−	+	−	−	−	+	+	−	+	+	+	+	−	−	−	+	−	116
12	−	+	+	+	−	−	+	−	−	+	+	−	+	+	−	+	+	+	−	−	+	+	+	+	83
13	−	+	+	−	−	+	−	+	−	+	−	−	−	−	−	−	−	−	+	−	+	+	+	−	193
14	−	−	−	+	−	−	−	−	+	+	−	−	−	−	+	−	−	−	−	−	+	−	−	+	230
15	+	−	+	−	+	−	+	−	−	+	−	−	−	+	+	−	−	−	+	+	−	−	+	+	51
16	−	+	+	−	+	−	−	−	+	−	+	+	+	−	+	+	+	−	+	−	+	−	−	−	82
17	−	−	−	−	−	+	+	−	−	−	+	+	−	−	+	−	+	+	−	−	−	−	+	+	32
18	+	−	+	+	−	−	−	+	−	−	+	+	+	−	+	+	−	+	+	+	+	+	+	+	58
19	+	−	−	+	+	−	+	+	−	−	+	−	−	−	−	−	+	−	+	−	−	+	−	−	201
20	+	+	+	−	+	+	−	+	+	+	+	+	−	+	+	−	−	+	−	−	−	+	−	+	56
21	−	+	−	+	−	+	+	−	+	−	−	+	+	+	−	+	−	−	+	+	−	+	−	+	97
22	+	+	+	+	−	+	+	+	−	−	−	+	−	+	+	−	+	−	+	−	+	−	−	+	53
23	−	+	−	+	+	−	−	+	+	−	+	−	−	+	−	−	−	+	+	−	−	−	+	+	276
24	+	−	−	−	+	+	+	−	+	+	+	+	+	−	−	+	−	−	+	−	+	+	+	+	145
25	+	+	+	+	+	−	+	−	+	−	−	+	−	−	−	−	−	+	−	+	+	−	+	−	130
26	−	+	−	−	+	+	+	+	−	−	+	−	+	+	+	+	−	−	−	+	+	−	+	−	55
27	+	−	−	+	−	+	−	+	+	+	+	+	−	+	−	−	+	−	−	+	+	−	+	−	160
28	−	−	+	−	−	−	−	−	−	−	+	+	−	+	−	−	−	−	−	+	−	+	−	−	127
CP	−365	−281	−95	511	175	−165	−189	415	213	199	−51	−221	−321	−433	−1209	−321	−599	−191	−49	−683	−117	−451	−167	−173	3065
EF	−26	−20	−7	37	13	−12	−14	30	15	14	−4	−16	−23	−31	−86	−23	−43	−14	−4	−49	−8	−32	−12	−12	109

*The level of run 8, factor 20 is here shown as +. In the source reference, the lower level appears: that seems to be a typographical error.

Table A.2

14 Run Supersaturated Designs
from a Hadamard Matrix of order 28.

Design number	Rows from the PB design													
1	1	2	4	5	7	9	11	14	18	22	23	24	26	28
2	3	6	8	10	12	13	15	16	17	19	20	21	25	27
3	1	3	4	5	7	9	13	15	16	17	18	21	23	27
4	2	6	8	10	11	12	14	19	20	22	24	25	26	28
5	1	3	4	6	8	9	10	13	17	22	23	24	25	28
6	2	5	7	11	12	14	15	16	18	19	20	21	26	27
7	1	4	9	10	11	13	14	17	18	19	20	21	25	26
8	2	3	5	6	7	8	12	15	16	22	23	24	27	28

Table A.3

Covariance matrix for the design in Lin (1993)

	c1	c2	c3	c4	c5	c6	c7	c8	c9	c10	c11	c12	c13	c14	c15	c17	c18	c19	c20	c21	c22	c23	c24
c1	14	6	-2	2	-2	-2	6	-2	2	2	-2	2	6	-2	2	-2	-2	-2	-2	2	-2	-2	-2
c2	6	14	2	6	-2	-2	-2	2	-2	2	-2	-2	-2	-2	-2	-2	-2	2	-2	-2	-6	2	-2
c3	-2	2	14	2	-2	2	2	2	2	-2	-2	-2	-6	-2	-2	2	-6	-2	6	2	2	-2	-2
c4	2	6	2	14	6	2	-2	-2	2	-2	-2	-2	-2	2	6	2	2	2	2	-2	-2	2	-2
c5	-2	-2	-2	6	14	-2	-2	-2	6	2	-2	2	2	2	-2	2	2	2	2	2	2	6	2
c6	-2	-2	2	2	-2	14	2	-2	-2	2	2	-2	-2	-6	6	2	-2	6	-2	2	2	2	2
c7	6	-2	2	-2	-2	2	14	-2	6	-2	2	2	-2	-2	2	2	2	-2	-2	6	-2	-2	2
c8	-2	2	2	-2	-2	-2	-2	14	2	-2	-6	-6	2	2	-2	2	-2	2	-2	2	-2	-2	6
c9	2	-2	2	2	6	-2	6	2	14	-2	2	-6	2	-2	-2	-2	2	-2	2	2	2	2	2
c10	2	2	-2	-2	2	2	-2	-2	-2	14	2	-2	6	-2	-2	2	-2	6	2	2	2	-2	-2
c11	-2	-2	-2	-2	-2	2	2	-6	2	2	14	-2	-2	-2	-2	-2	2	2	2	-6	-2	-2	2
c12	2	-2	-2	-2	2	-2	2	-6	-6	-2	-2	14	-2	2	-2	2	-2	-2	-2	2	-2	2	2
c13	6	-2	-6	-2	2	-2	-2	2	2	6	-2	-2	14	-2	2	2	-2	-2	2	-2	2	-2	2
c14	-2	-2	-2	2	2	-6	-2	2	-2	-2	-2	2	-2	14	2	2	2	2	-2	2	2	-6	-2
c15	2	-2	-2	6	-2	6	2	-2	-2	-2	-2	-2	2	2	14	6	2	-2	-2	-2	2	-2	-2
c17	-2	-2	2	2	2	2	2	2	-2	2	-2	2	2	2	6	14	-2	-2	2	-2	-2	-2	6
c18	-2	-2	-6	2	2	-2	2	-2	2	-2	2	-2	-2	2	2	-2	14	2	2	2	-6	-2	-2
c19	-2	2	-2	2	2	6	-2	2	-2	6	2	-2	-2	2	-2	-2	2	14	-2	6	-2	-2	2
c20	-2	-2	6	2	2	-2	-2	-2	2	2	2	-2	2	-2	-2	2	2	-2	14	-2	-2	-6	-2
c21	2	-2	2	-2	2	2	6	2	2	2	-6	2	-2	2	-2	-2	2	6	-2	14	2	-2	-2
c22	-2	-6	2	-2	2	2	-2	-2	2	2	-2	-2	2	2	2	-2	-6	-2	-2	2	14	2	-6
c23	-2	2	-2	2	6	2	-2	-2	2	-2	-2	2	-2	-6	-2	-2	-2	-2	-6	-2	2	14	2
c24	-2	-2	-2	-2	2	2	2	6	2	-2	2	2	2	-2	-2	6	-2	2	-2	-2	-6	2	14

Table A.4

Design from Wu (1993)

Runs	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	−1	1	1	1	−1	−1	−1	1	−1	1	−1
2	−1	1	1	−1	1	1	1	−1	−1	−1	1	−1	−1
3	1	−1	1	1	−1	1	1	1	−1	−1	−1	−1	1
4	−1	1	−1	1	1	−1	1	1	1	−1	−1	−1	1
5	−1	−1	1	−1	1	1	−1	1	1	1	−1	1	−1
6	−1	−1	−1	1	−1	1	1	−1	1	1	1	1	1
7	1	−1	−1	−1	1	−1	1	1	−1	1	1	−1	−1
8	1	1	−1	−1	−1	1	−1	1	1	−1	1	1	−1
9	1	1	1	−1	−1	−1	1	−1	1	1	−1	1	1
10	−1	1	1	1	−1	−1	−1	1	−1	1	1	−1	−1
11	1	−1	1	1	1	−1	−1	−1	1	−1	1	−1	1
12	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	1	1

Table A.5

New 12 Run Design

Run	Factors												
	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
2	1	1	−1	−1	−1	−1	1	1	−1	−1	1	1	1
3	−1	−1	1	1	1	1	−1	−1	−1	−1	1	1	1
4	−1	−1	1	1	−1	−1	1	1	1	1	−1	−1	1
5	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	−1
6	1	−1	1	−1	−1	1	−1	1	−1	1	−1	1	−1
7	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1	−1
8	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	−1
9	1	−1	−1	1	1	−1	−1	1	1	−1	−1	1	1
10	1	−1	−1	1	−1	1	1	−1	−1	1	1	−1	−1
11	−1	1	1	−1	1	−1	−1	1	−1	1	1	−1	−1
12	−1	1	1	−1	−1	1	1	−1	1	−1	−1	1	1