Analysis of Variation Transmission in Manufacturing Processes

J.F. Lawless, R.J. MacKay, and J.A. Robinson
University of Waterloo

I.I.Q.P. Research Report
RR-96-08
December 1996

Analysis of Variation Transmission in Manufacturing Processes

J.F. Lawless[1], R.J. MacKay[1] and J.A. Robinson[2]

[1]University of Waterloo, Waterloo, Ontario, Canada N2L 3G1

[2]General Motors R&D Center, Warren, MI 48090

SUMMARY

This article discusses methods of studying variation in quality characteristics of products that move through a multi-stage manufacturing process. The ideas are based on tracking and measuring the characteristics of individual parts through sequences of stages. If this is done then simple regression and analysis of variance tools may be used to study the amount of variation that is attributable to different stages of the process, and opportunities for variation reduction may be identified.

KEY WORDS AND PHRASES: autoregressive models, repeated measurements, variation reduction

1 Introduction

A fundamental strategy to reduce variation in manufacturing processes is to first identify the sources of the variation and then to take remedial action. The identification of variation sources is useful, both for improving the current process and for designing more robust future processes. In processes consisting of discrete stages there may be certain stages at which considerable variation originates and other stages that effectively absorb variation introduced upstream (i.e. at previous stages) in the process. In order to target variation reduction activities it is therefore important to understand how variation is added and transmitted across the stages of a process. This article discusses methodology for doing this and, in particular, for attributing the variation in key process or quality characteristics to the various stages in a process.

A pair of examples taken from automobile manufacturing will illustrate the main ideas. Following a description of the examples, the second section presents formal models for variation transmission and the third discusses the two examples further. A fourth section discusses the effect of measurement error, and a fifth concludes with some remarks on extensions to the methodology.

Example 1: Crankshaft Machining

We consider the final two stages in the machining of automobile crankshafts, portrayed in Figure 1. Crankshafts are ground by one of four grinders and then passed to a lapper which removes some additional metal (approximately 25×10^{-4} inches). The key characteristic we focus on here is a particular journal diameter y, measured from nominal, in units of 10^{-4} inches. The objective is to reduce variation in y.

We assume that a study can be carried out whereby the journal diameter on a crankshaft may be measured before and after being processed by the lapper; we denote these measurements as x and y respectively. In addition, we define a covariate z such that $z = j$ if a part is ground by grinder j.

We may partition variation in y into components by using the conditional variance formula

$$Var(y) = Var\left\{E(y|x,z)\right\} + E\left\{Var(y|x,z)\right\}. \tag{1}$$

The expectations on the right side of (1) are with respect to x and z. Data described in Section 3 suggest the relationships

$$E(y|x, z = j) = \alpha + \beta x, \quad Var(y|x,z) = \sigma_a^2.$$

Then (1) gives

$$\sigma_y^2 = \beta^2\sigma_x^2 + \sigma_a^2. \tag{2}$$

The first term on the right hand side represents variation that is transmitted from upstream, through the lapper; the second term represents variation added by the lapper. If β were close to zero then the lapper would effectively screen out variation from upstream sources, represented by $\sigma_x^2 = Var(x)$. Note that, if desired, σ_x^2 can be partitioned into within and between - grinder components:

$$\sigma_x^2 = \sum_{j=1}^{4}\frac{1}{4}\sigma_j^2 + \sum_{j=1}^{4}\frac{1}{4}\left(\mu_j - \mu_x\right)^2, \tag{3}$$

where $\mu_j = E(x|z=j), \sigma_j^2 = Var(x|z=j)$, $\mu_x = \sum_{j=1}^4 \mu_j/4$, and we assume that each grinder processes 1/4 of the crankshafts.

Example 2: Hood Fits

This example is taken from the assembly of automobiles. We consider four operations widely spaced in the assembly process that relate to the installation of hoods. The four operations are: 1) install or "hang" the hood, 2) paint the hood (and the rest of the car), 3) install hardware such as the hood latch, and 4) adjust or "finesse" the hood for better fit. These operations will simply be denoted as HANG, PAINT, HARDWARE, and FINESSE. There can be physical movement of the hood or adjacent fenders at all of these operations, including PAINT, where thermal effects from the bake ovens are possible.

The quality characteristic is flushness of the hood to the surrounding fenders. We will consider four measurements, two along each side of the hood, one near the front and one near the rear. By convention, a plus sign on the deviation indicates that the hood is high relative to the fender, and a negative sign indicates that the hood is low. The nominal condition is zero deviation at all locations, corresponding to a perfectly flush hood.

We are interested in changes in mean levels from one operation to the next since such changes will affect the tolerances at upstream locations required to attain the desired results at the end of the line. If mean changes were the only concern, then one could simply sample cars (not necessarily the same ones) following each of the four operations. In addition to mean changes, we will also be interested in determining where variation is added to the process and the degree to which it is transmitted through the process. Such information can be useful in directing resources for problem-solving.

For this type of analysis, it is necessary to "track" vehicles through the plant, taking measurements on the same vehicles at each operation, in order to estimate the statistical relationships from one operation to the next. This requires following a sample of cars, making sure that each car is met and measured at the designated operations. These measurements were taken with a special hand-held tool. Variation due to the measurement system is a concern that we will discuss in more detail later.

As in the first example, let y denote the measure of interest, in this case, the flushness from nominal in millimeters at one of the locations. We have four operations here so y_t will denote the flushness at the t^{th} operation, $t = 1, \ldots, 4$, corresponding to HANG, PAINT, HARDWARE, and FINESSE. Ignoring covariates other than the corresponding flushness measurement at the previous operation, the linear model discussed in the next section has

$$E(y_t|y_{t-1}) = \alpha_t + \beta_t y_{t-1}, \;\; Var(y_t|y_{t-1}) = \sigma^2_{e_t}$$

and

$$\sigma_t^2 = \beta_t^2 \sigma_{t-1}^2 + \sigma_{e_t}^2, \tag{4}$$

for $t = 2, 3, 4$, where σ_t^2 denotes the variance of y_t. Here, $\sigma^2_{e_t}$ represents the variation added at the t^{th} operation. The amount of variation observed at operation t that is transmitted from the previous operation is $\beta_t^2 \sigma_{t-1}^2$. Hence, the regression coefficient, β_t, governs the degree of transmission of variation.

2 A Variation Transmission Model

We assume to start that a quality characteristic y is measured at each of T process stages, and denote the measurement at stage t as y_t. The following simple model is adequate in many situations:

$$y_1 \sim N\left(\mu_1, \sigma_1^2\right) \tag{5}$$

$$y_t = \alpha_t + \beta_t y_{t-1} + e_t, \quad t = 2, \cdots, T \tag{6}$$

where $e_t \sim N\left(0, \sigma_{e_t}^2\right)$. More specifically, this is a first order autoregressive model in which the distribution of y_t given $y_{t-1}, y_{t-2}, \cdots, y_1$ depends only on y_{t-1}. This assumption can often be justified in the context of a sequential manufacturing process. The process at stage t only "knows" what is presented to it from stage $t-1$ in the form of work in process. Its only "memory" of previous stages is through that work in process, which must pass through stage $t-1$. The process is assumed stable in the sense that the model (5) and (6) is valid over time.

Let us denote $E(y_t)$ by μ_t and $Var(y_t)$ by σ_t^2. It follows from (1) and (2) that for $t = 2, \cdots, T$

$$\begin{aligned} \mu_t &= \alpha_t + \beta_t \mu_{t-1} \\ \sigma_t^2 &= \beta_t^2 \sigma_{t-1}^2 + \sigma_{e_t}^2. \end{aligned} \tag{7}$$

This corresponds to formula (1) in Section 1. The first term on the right hand side of (7) represents variation transmitted to y_t from stage $t-1$, and the second term represents variation added at stage t. As such, they indicate possibilities for reducing variation in y_t by (i) reducing σ_{e_t}, (ii) reducing σ_{t-1}, or (iii) making β_t closer to zero.

We are interested in making σ_T^2 small, since it represents variation in y at the final stage. We can consider σ_T^2 in terms of variation added at stage T plus variation transmitted from stage $T-1$, via (7). In addition, by using (7) recursively we get

$$\sigma_T^2 = \beta_T^2 \beta_{T-1}^2 \sigma_{T-2}^2 + \beta_T^2 \sigma_{e_{T-1}}^2 + \sigma_{e_T}^2 \tag{8}$$

and, continuing back to stage 1 (and writing $\sigma_1^2 = \sigma_{e_1}^2$),

$$\sigma_T^2 = (\beta_T \beta_{T-1} \cdots \beta_2)^2 \sigma_{e_1}^2 + (\beta_T \beta_{T-1} \cdots \beta_3)^2 \sigma_{e_2}^2 + \cdots + \beta_T^2 \sigma_{e_{T-1}}^2 + \sigma_{e_T}^2. \tag{9}$$

This decomposes the variation in y_t into components attributable to each stage $t = 1, \cdots, T$, and may be used to suggest which stages we might concentrate on in order to reduce σ_T^2.

It should be noted that the utility of (9) is dependent on the validity of (5) and (6), especially when there are several stages in the process. In section 4 we discuss this point further and make suggestions concerning the use of (9).

2.1 Estimation of Variation Components

It is assumed that a study can be carried out in which $(y_1, y_2, \cdots, y_T)$ may be measured on a representative set of n parts or units. That is, we assume that a random sample $(y_{i1}, \cdots, y_{iT})\ i = 1, \cdots, n$, of measurements from the model (5) and (6) are available. In that case it is easy to estimate the parameters μ_1, σ_1 and $\alpha_t, \beta_t, \sigma_{e_t} (t = 2, \cdots, T)$: the maximum likelihood estimates are

$$\begin{aligned}
\hat{\mu}_1 &= \overline{y}_1 \qquad \hat{\sigma}_1^2 = \frac{1}{n}\sum_{i=1}^{n}(y_{i1}-\overline{y}_1)^2 \\
\hat{\beta}_t &= \frac{S_{t-1,t}}{S_{t-1,t-1}}, \quad \hat{\alpha}_t = \overline{y}_t - \hat{\beta}_t\overline{y}_{t-1} \qquad t = 2,\cdots,T \\
\hat{\sigma}_{e_t}^2 &= S_{tt} - \hat{\beta}_t S_{t-1,t} \qquad t = 2,\cdots,T
\end{aligned}$$

where $\overline{y}_t = \sum_{i=1}^{n} y_{it}/n$, $S_{tt} = \sum_{i=1}^{n}(y_{it}-\overline{y}_t)^2/n$ and $S_{t-1,t} = \sum_{i=1}^{n}\left(y_{i,t-1}-\overline{y}_{t-1}\right)(y_{it}-\overline{y}_t)/n$. The estimated version of (2.3) is then

$$\hat{\sigma}_t^2 = \hat{\beta}_t^2\hat{\sigma}_{t-1}^2 + \hat{\sigma}_{e_t}^2 \tag{10}$$

where $\hat{\sigma}_t^2 = S_{tt}$. We use the maximum likelihood estimates $\hat{\sigma}_t^2$ and $\hat{\sigma}_{e_t}^2$ so inserting estimates into (7), (8) or (9) gives an exact partition of the total observed variation.

2.2 Extensions to Include Covariates

Covariates can be introduced into models (5) and (6) for several reasons.

Suppose at stage t, an input covariate x_t is measured that perhaps contributes to the added variation at that stage. The model can be modified to

$$y_t = \alpha_t + \beta_t y_{t-1} + \gamma_t x_t + e_t^*$$

where if γ_t is relatively large, the standard deviation on e_t^* should be much less than that of e_t. In this instance, the added variation at stage t can be reduced by reducing variation in the input x_t.

A second application of covariates is to model the effect of parallel processing streams. Suppose at stage t there are two parallel operations. Consider the model

$$y_t = z_t(\alpha_t + \beta_t y_{t-1} + e_t) + (1 - z_t)(\alpha_t^* + \beta_t^* y_{t-1} + e_t^*)$$

where z_t indicates the processing stream ($z_t = 0, 1$) and e_t, e_t^* have standard deviations $\sigma_{e_t}, \sigma_{e_t}^*$ respectively. In this model, the transmitted and added variation may be stream dependent. We can write

$$Var(y_t) = E(Var(y_t|z_t)) + Var(E(y_t|z_t)). \tag{11}$$

The second term on the right side of (11) is due to the variation in targeting of the two streams. If this component is found to be large, then it can be reduced by ensuring the two streams have the same mean value given y_{t-1}. The first term can be partitioned as usual for each z_t into added and transmitted components.

Aside from discussing example 1 further in the next section, we do not consider covariates in detail, but it is clear that they can be employed usefully.

3 Examples

We illustrate the ideas of the preceding section by considering the two examples introduced in Section 1.

Example 1. Crankshaft Machining

Over a period of several days 96 crankshafts were randomly selected from production and their journal diameters measured before and after passing through the lapper; 24 crankshafts came from each of Grinders 1, 2, 3 and 4. The data are portrayed in Figure 2; x is the diameter before lapping and y the diameter after lapping, measured in units of 10^{-4} inches. The line for each plot is a least squares line, discussed below.

The scatter plots in Figure 2 are reasonably consistent with the model introduced in Section 1, where z ($= 1, 2, 3$ or 4) indicates which grinder a crankshaft was processed by, and

$$E(y|x, z) = \alpha + \beta x, \quad Var(y|x, z) = \sigma_a^2. \tag{12}$$

The fit of the model is discussed briefly later. Let x_{ji} and y_{ji} denote the diameters for crankshaft i from grinder j ($j = 1, 2, 3, 4;\ i = 1, ..., 24$) before and after lapping and define $\bar{x} = \sum_j \sum_i x_{ji}/96$, $\bar{y} = \sum_j \sum_i y_{ji}/96$. Then we have the normal distribution maximum likelihood estimates $\hat{\sigma}_y^2 = \sum_j \sum_i (y_{ji} - \bar{y})^2/96$, $\hat{\sigma}_x^2 = \sum_j \sum_i (x_{ji} - \bar{x})^2/96$, $\hat{\beta} = \hat{\sigma}_{xy}/\hat{\sigma}_x^2$ and $\hat{\sigma}_a^2 = \hat{\sigma}_y^2 - \hat{\beta}^2 \hat{\sigma}_x^2$, where $\hat{\sigma}_{xy} = \sum_j \sum_i (x_{ji} - \bar{x})(y_{ji} - \bar{y})/96$. These estimates lead to the partition of variation in (2) as

$$\hat{\sigma}_y^2 = \hat{\beta}^2 \hat{\sigma}_x^2 + \hat{\sigma}_a^2,$$

which here gives

$$11.68^2 = (1.026^2)(10.68^2) + 4.04^2. \tag{13}$$

The partition (13) indicates that little of the variation in the final diameter y is added by the lapper: most of it is due to variation in the pre-lapper diameters x that is transmitted through the lapper. If we break $\hat{\sigma}_x^2$ into within and between grinder components we find, analogous to (3), that $10.68^2 = 9.22^2 + 5.39^2$. Most of the variation in y is therefore due to variation in the crankshafts that come out of each grinder. Note that if we had not measured the diameter x of each crankshaft before lapping but had observed which grinder the crankshaft came from, then we would be able to determine that not much of the variation in y was due to differences between crankshafts processed by the four grinders. However, we would not know that most of the variation was transmitted from upstream, as opposed to being added by the lapper.

Figure 2 shows the least squares (maximum likelihood) line $y = \bar{y} + \hat{\beta}(x - \bar{x})$ superimposed on the scatter plot for each grinder. The data are reasonably consistent with the model (12); there is evidence of very mild departures, especially for grinder 3. However, if we fit models that accommodate these small departures, for example by allowing separate regression lines for each grinder, the partition of variation changes very little and the overall conclusions are unchanged. Hence, we prefer to retain (12) as a useful and quite accurate summary of variation in the data.

Example 2. Hood Fits

This section contains an analysis of data from the hood fits example. The discussion will center around three sets of graphs given in Figures 3-7.

The sequence or line charts in Figure 3 contain plots of the flushness versus plant operation for each of the four measurement locations. Each connected set of points represents one car. The most obvious observation is that the hood is moving up relative to the fenders at the rear during processing. The statistics indicate that the movement is approximately

1.5mm on each side, most of it accounted for at FINESSE. The end result is that near nominal fits are attained on average on the left side, but hoods remain 1.66mm low on the right. While seemingly trivial, we have found this type of finding to be extremely important in practice. It would indicate here, for example, that to remove the FINESSE operation, it would be necessary to change tolerances so that hoods are installed higher at the rear than is the current practice.

As mentioned earlier, it is not necessary to track vehicles to determine this type of change in mean. There are, however, more complex questions that require tracking vehicles. Where is the variation coming from? Is it being passed through the system or is it being added at a few key operations? The line charts provide a glimpse. Parallel lines between stages indicate transmission of variation. With parallel lines the mean may change, but the variation at the end is the same as at the beginning. Lines that splay out indicate a magnification of variation, while converging or crisscrossing lines indicates some degree of adjustment. When there is adjustment at an operation, large values are made smaller, and small values are made larger. With these general guidelines in mind, it seems clear in this example that there is a great deal of transmitted variation at the rear of the hood and a lesser amount at the front. However, this can be displayed more clearly with scatter plots.

Figure 4 contains scatter plots of the deviations following the PAINT operation against the corresponding deviations following the HANG operation. Three of the four regression slopes, including the two corresponding to the rear of the hood, are near one, indicating a high degree of transmitted variation. The residual standard deviations can be interpreted as indicating the amount of variation added at PAINT. Figure 5 displays the scatter plots for the next pair of consecutive operations, PAINT and HARDWARE. The results are similar except the added variation (scatter) is larger at the front than before. Figure 6 shows the scatter plots for the final two operations, HARDWARE and FINESSE. Again, the regression slopes are near one at the rear. These scatter plots bear out the general conclusion of a high degree of transmitted variation at the rear of the hood. This means that variation that exists after the hood is hung is being passed on to the end of the line. We conclude that end-of-line variation at the rear of the hood could be reduced if variation at HANG were reduced.

But the front of the hood is a different story. There is strong transmission of variation through HARDWARE on the left side, but it is much weaker on the right side (Figures 4 and 5). Also, there is essentially no transmission of variation through FINESSE on the left side (Figure 6). It is operating as a pure adjuster – the output being completely independent of the input. For all operations, the added variation, as measured by the residual standard deviation, is greater at the front than at the rear.

It helps to quantify the results when they are mixed, as these are. The variance decomposition (9) into components corresponding to individual operations can be used for this purpose. The variance components for the hood data were calculated according to the procedure outlined in Section 2.1. A stacked bar chart is a convenient way to display the results. The height of each bar is the total variation immediately following that operation. The components of variation make up the elements of the stack. The convention followed here is to place the component corresponding to the first operation at the bottom, and then proceed upward in order, so that the top component corresponds to the variation "added" at that operation. One should bear in mind that the chart portrays variances, but that the effects on the standard deviation of y_T are of primary interest.

Figure 7 displays the bar charts for the hood data. It confirms and quantifies the previous conclusions. Variation is being passed through at the rear, but there is no corresponding transmission of variation at the front beyond HARDWARE. At the front nearly all of the variation following FINESSE is due to that operation (left = 100%, right = 80%). This means that to benefit in an overall way from reducing variation at HANG, we would need to eliminate (or change) the FINESSE operation. As noted previously, targets would have to be adjusted to change mean levels.

We can focus on the third bars in Figure 7 to understand the impact of removing the FINESSE operation. We might conclude that to reduce variation below current levels at the front, we would need to improve both the HANG and HARDWARE operations. Obviously, in practice this would have to be discussed in terms of the engineering understanding and impact on the process.

4 Important Points Related to the Assumptions

The analysis described above depends on the approximate validity of the model (5) - (6). Diagnostic checks based on regression fits and residuals should therefore be part of the analysis. In this section we mention two important points related to the assumed model.

The first point concerns measurement error. So far we have assumed that y_t is measured exactly, but this is typically not the case. To see the effect of measurement error, suppose that instead of y_t we observe

$$\tilde{y}_t = y_t + \delta_t, \tag{14}$$

where the δ_t's $(t = 1, \cdots, T)$ are independent $N\left(0, \sigma_{\delta_t}^2\right)$ variables, and are independent of the y_t's. If we proceed with our analysis by treating the $\tilde{y}_t$'s as the y_t's then we are in effect using the incorrect model, and it may easily be shown that (e.g. Fuller 1987) in large samples

$$\hat{\sigma}_t^2 \quad \rightarrow \quad \sigma_t^2 + \sigma_{\delta_t}^2 = var\left(\tilde{y}_t\right) \tag{15}$$

$$\hat{\beta}_t \quad \rightarrow \quad \frac{\beta_t \sigma_{t-1}^2}{\sigma_{t-1}^2 + \sigma_{\delta_{t-1}}^2} = \frac{cov\left(\tilde{y}_{t-1}, \tilde{y}_t\right)}{var\left(\tilde{y}_{t-1}\right)}, \tag{16}$$

where "$\rightarrow$" denotes convergence in probability. Thus $\hat{\beta}_t$ tends to underestimate β_t; this is the well known measurement error attenuation effect.

We need to consider (7) and (10) along with (15) and (16) in order to see the effect of the measurement error on the variance decomposition, and hence on our assessment of the possibilities for reducing variation.

Table 1 summarizes the effects of ignoring measurement variation on the estimates of the three components of variation at a given stage: variation transmitted from the previous stage, variation added at the stage, and measurement variation. Transmitted variation is underestimated by the same proportional factor affecting the regression slope. The estimate of added variation is biased upward by measurement error, as we would expect. But there is an additional positive bias equal in magnitude to the negative bias for transmitted variation.

If $\sigma_{\delta_{t-1}}$ is sufficiently small that $\sigma_{t-1}^2 / \left(\sigma_{t-1}^2 + \sigma_{\delta_{t-1}}^2\right)$ exceeds, say, .9 then the effect of measurement error may be deemed relatively unimportant. However, if there are several stages in the process and we use (9) to attribute variation to the different stages, then for earlier stages the underestimation is much more severe, because of the presence of terms involving products of the β_t's. For example, the proportional bias factor associated with the amount of variance transmitted from stage s, or earlier, to stage T, is $\prod_{u=s+1}^{T-1} \sigma_u^2/(\sigma_u^2 + \sigma_{\delta_u}^2)$, which becomes smaller as s becomes smaller, indicating a greater degree of underestimation for "far upstream" stages. Consequently, (9) should be used with caution if T is very large. If the measurement error variances $\sigma_{\delta_t}^2$ are known then it is possible to make adjustments to the analysis, the simplest approach being to employ (15) and (16); we will not pursue this here.

A second caveat concerns the validity of the model (5) and (6) more generally. Occasionally there may be situations where, even if measurement error is insignificant, the distribution of y_t may depend on not only y_{t-1} but also earlier measurements. In this case the variance decomposition (7) is still valid when the measurements are approximately jointly normally distributed, and it provides insight into the transfer of variation from stage $t-1$ to stage t. However, multi-stage formulas (8) and (9) do not usually provide much insight and it may even be difficult to assess the effect of changes to stage $t-1$ on the stage t measurements, given the complex relationship between the different measurements. Diagnostic checks on the dependence of y_t on only y_{t-1} may be made by regressing y_t on y_{t-1} and upstream measurements. We recommend that when checks indicate that the first order autoregressive model is unsatisfactory, the joint distribution of $(y_1, \cdots, y_T)$ be examined carefully.

5 Concluding Remarks

This article has discussed how to study variation in key quality characteristics as discrete parts move through a multi-stage process. The idea is to track and measure the characteristics of individual parts through the stages of the process. In many situations this allows us to determine how much variation is added at different stages, and how much of that variation is transmitted downstream. To use the methodology it is necessary to identify a characteristic for study, to have a capable process for measuring the characteristic, and to have the ability to follow individual parts through the process. It is important that measurement error be small and that the first order autoregressive model described in Section 2 provide a reasonably accurate representation of the measurements. Checks of these assumptions should therefore be included as part of the application of these methods.

Whether measurement error is suitably "small" can be assessed through the results in Section 4. If $\sigma_{\delta_t}/\sigma_t$ is less than say, .1, then there will be relatively little harm in ignoring the measurement error when the number of stages is only two or three, for example. If desired, estimates of β_t's and σ_t's and corresponding partitions of variance can be adjusted using formulas (15) and (16). More formal methods of dealing with measurement error are discussed elsewhere (Agrawal, Lawless and MacKay 1996). These authors also present methods for obtaining confidence limits on components of variation, such as the terms in (9).

For most processes there will be several characteristics of interest; for example, the hood fitting process discussed earlier had many key measurements, of which we considered four. The characteristics can be studied individually, as in Example 2 of Section 3. However, this does not take account of the way that the multiple characteristics interact and in some cases this may lead to important opportunities for variation reduction being missed. Correlated measurements may also make separate first order autoregressive models for single characteristics inappropriate. Methodology for variation analysis based on multivariate autoregressive models will be discussed in a separate article. Model fitting techniques that accommodate both measurement error and missing observations with autoregressive models (Fong and Lawless 1996) may be applied.

The discussion in this article has emphasized methods for studying the transmission of variation across stages in a process. The reason for doing this is to identify opportunities for reducing variation. We have not specifically discussed targeting of the characteristics, but this is easily done. Similar ideas may be used more generally to study the relationship of upstream variables $x_1, ..., x_k$ to one or more quality characteristics y on a finished part. The idea is to assess variation in $(x_1, ..., x_k)$ and how it affects y. In general, the variables may be of various types (categorical, continuous...) and relationships may be non-normal and nonlinear, so rather different models than the ones used in this paper may be needed. These ideas will be considered elsewhere, but some analogies with the work of Taguchi and others (e.g. Taguchi, Elsayed and Hsiang 1984) on the analysis of variation in products and systems are apparent.

Acknowledgement

This research was supported by General Motors and by grants to the first two authors from the Natural Sciences and Engineering Research Council of Canada.

References

Agrawal, R., Lawless, J.F. and MacKay, R.J. (1996). Analysis of variation transmission for manufacturing processes in the presence of measurement error. Submitted for publication.

Fong, D.Y.T.,and Lawless, J.F. (1996). The analysis of process variation transmission with multivariate measurements. Submitted for publication.

Fuller, W.A. (1987). *Measurement Error Models.* New York: John Wiley and Sons.

Taguchi, G., Elsayed, E. and Hsiang, T. (1989). *Quality Engineering in Production Systems.* New York: McGraw Hill.

Source	Actual	Estimated	Bias
Transmitted	$\beta_t^2\sigma_{t-1}^2$	$\beta_t^2\sigma_{t-1}^2 R_t$	$-\beta_t^2\sigma_{t-1}^2(1-R_t)$
Added	$\sigma_{e_t}^2$	$\sigma_{e_t}^2 + \sigma_{\delta_t}^2 + \beta_t^2\sigma_{t-1}^2(1-R_t)$	$\sigma_{\delta_t}^2 + \beta_t^2\sigma_{t-1}^2(1-R_t)$
Measurement	$\sigma_{\delta_t}^2$	0	$-\sigma_{\delta_t}^2$

where $R_t = \sigma_{t-1}^2/(\sigma_{t-1}^2 + \sigma_{\delta_{t-1}}^2)$

Table 1: Bias of Components of $var(y_t)$ when Ignoring Measurement Variation

FIGURE LEGENDS

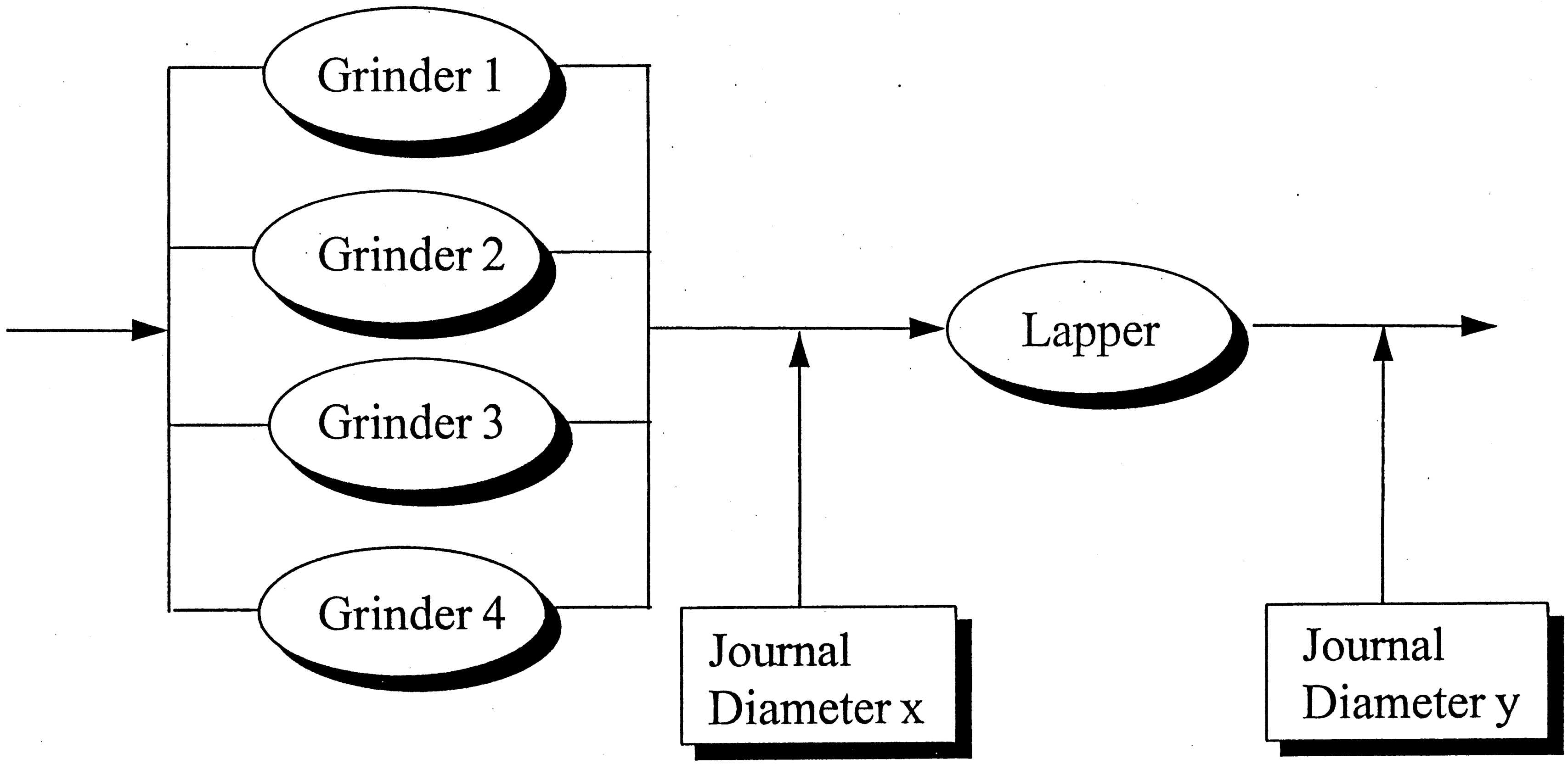

Figure 1: Crankshaft Machining Process

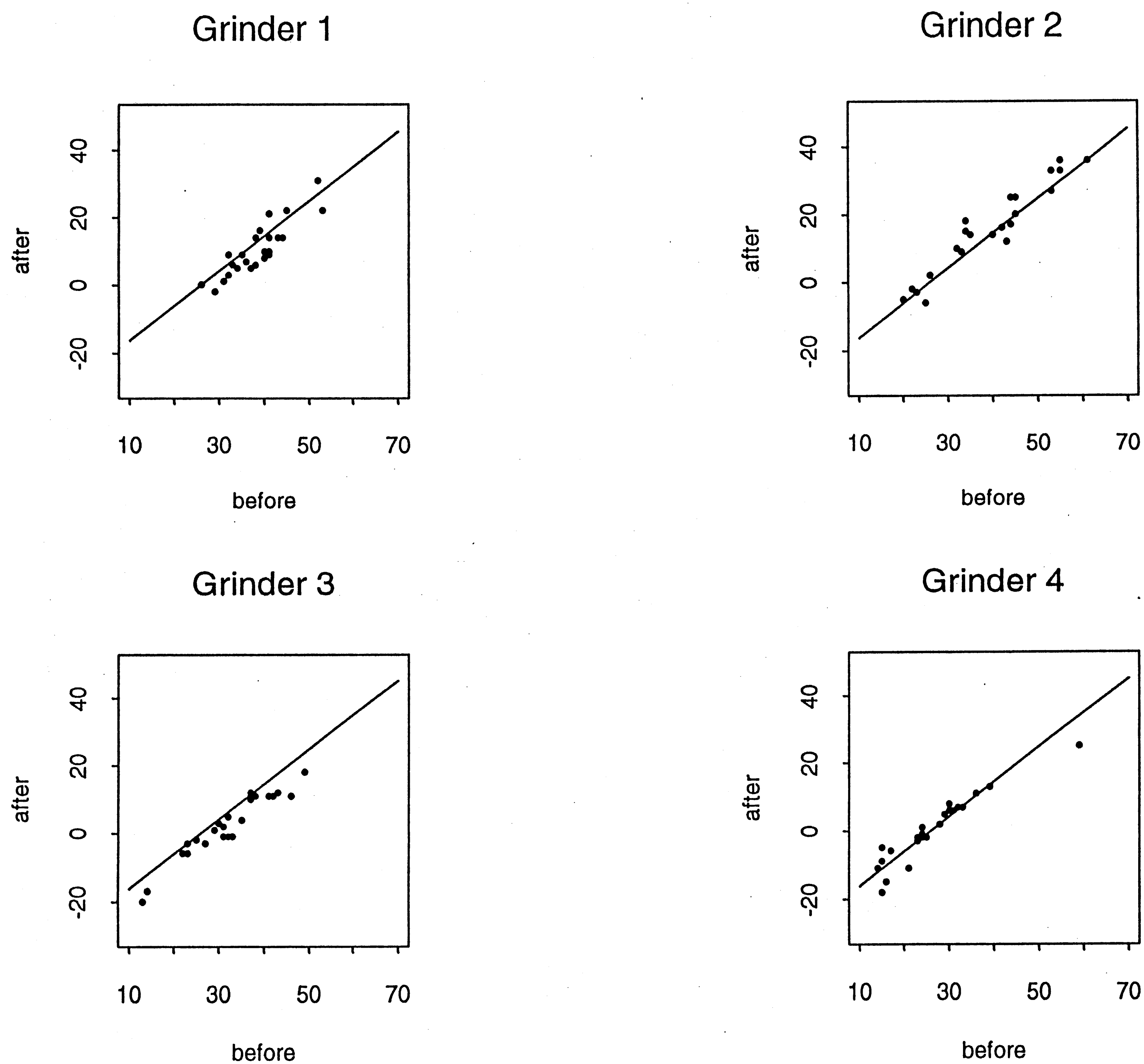
Grinder 1
Grinder 2
Grinder 3
Grinder 4
after
before
-20
0
20
40
10
30
50
70

Figure 2

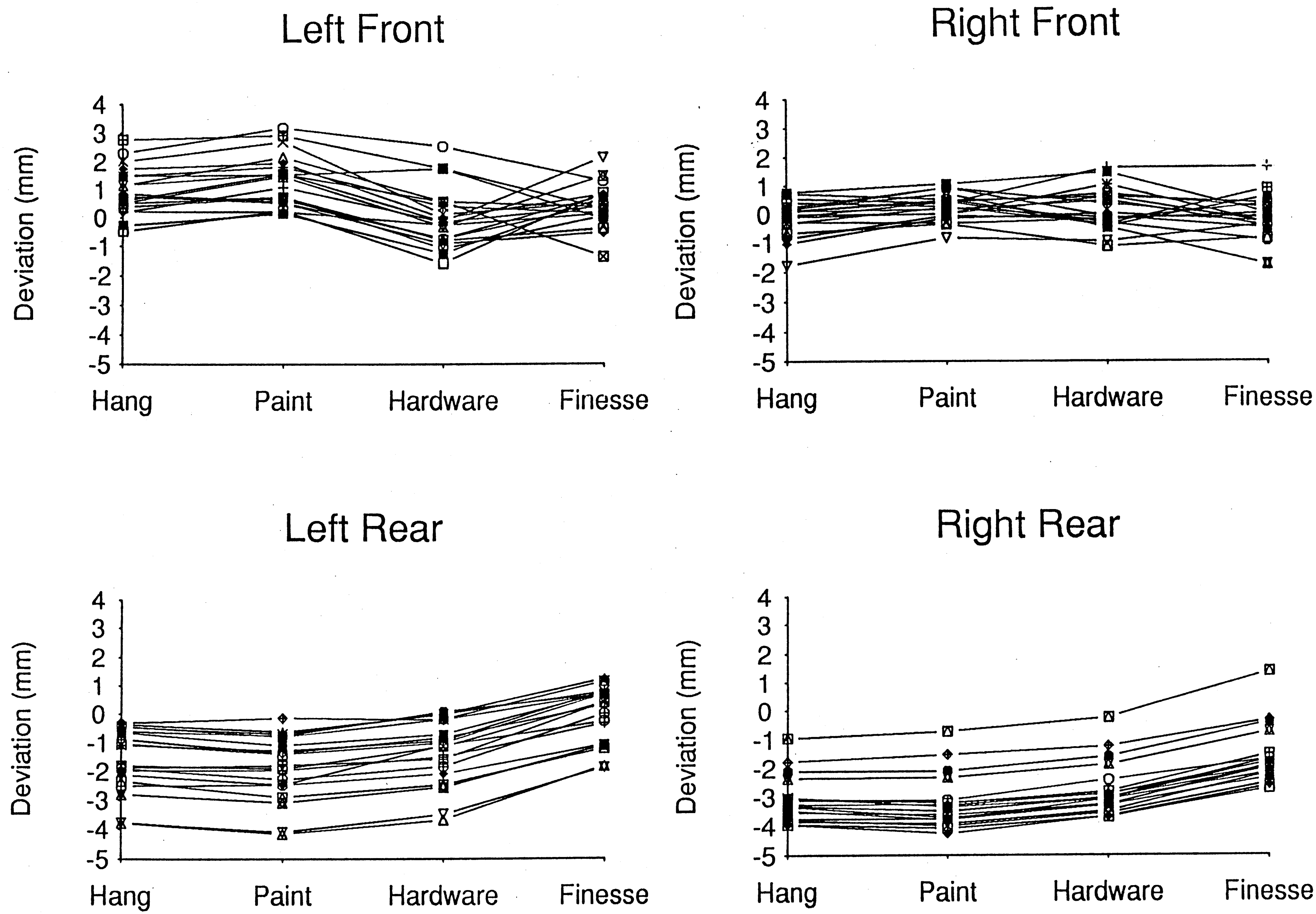

Figure 3: Flushness at Four Plant Operations

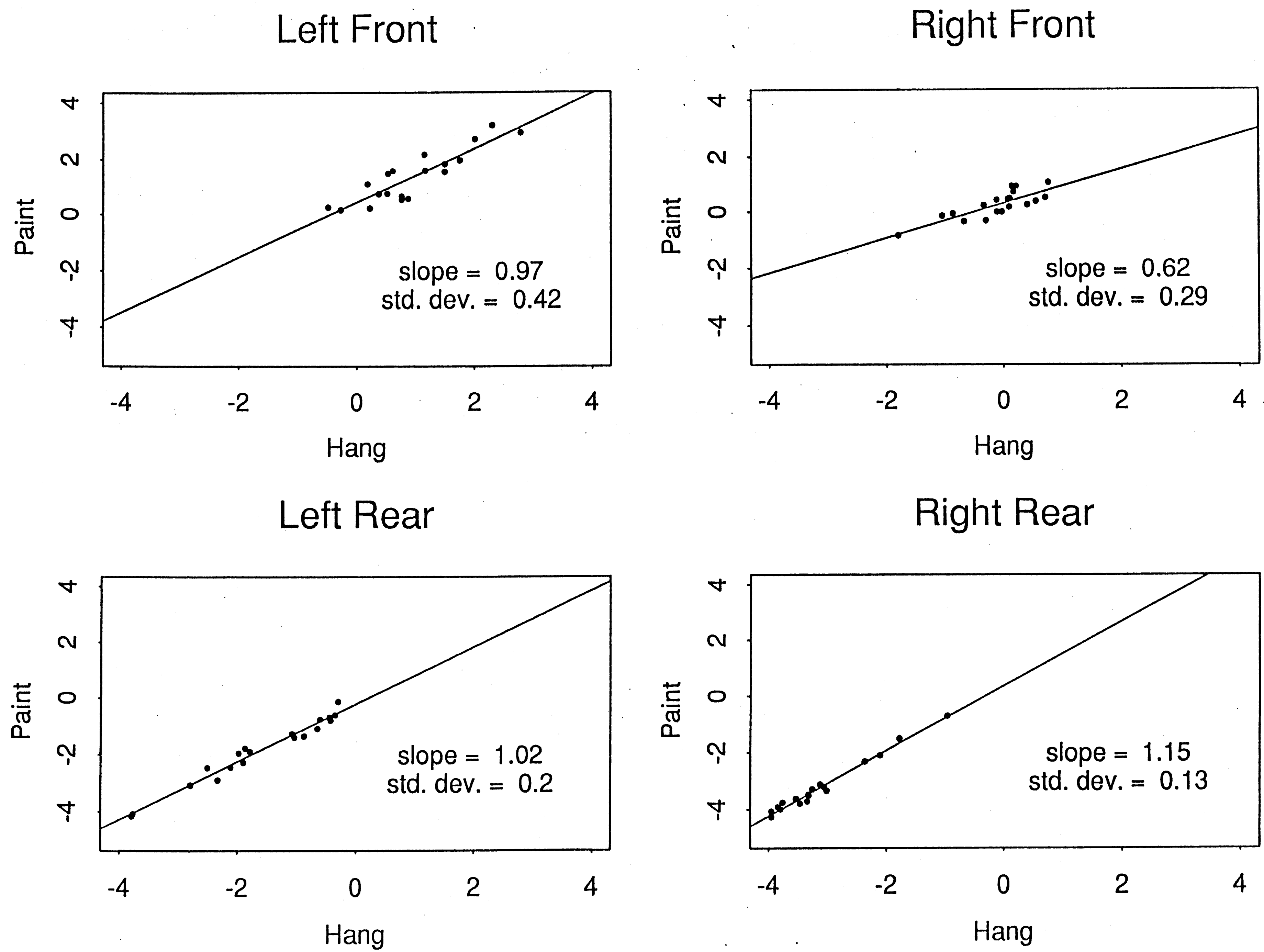

Figure 4: Flushness after HANG and PAINT Operations

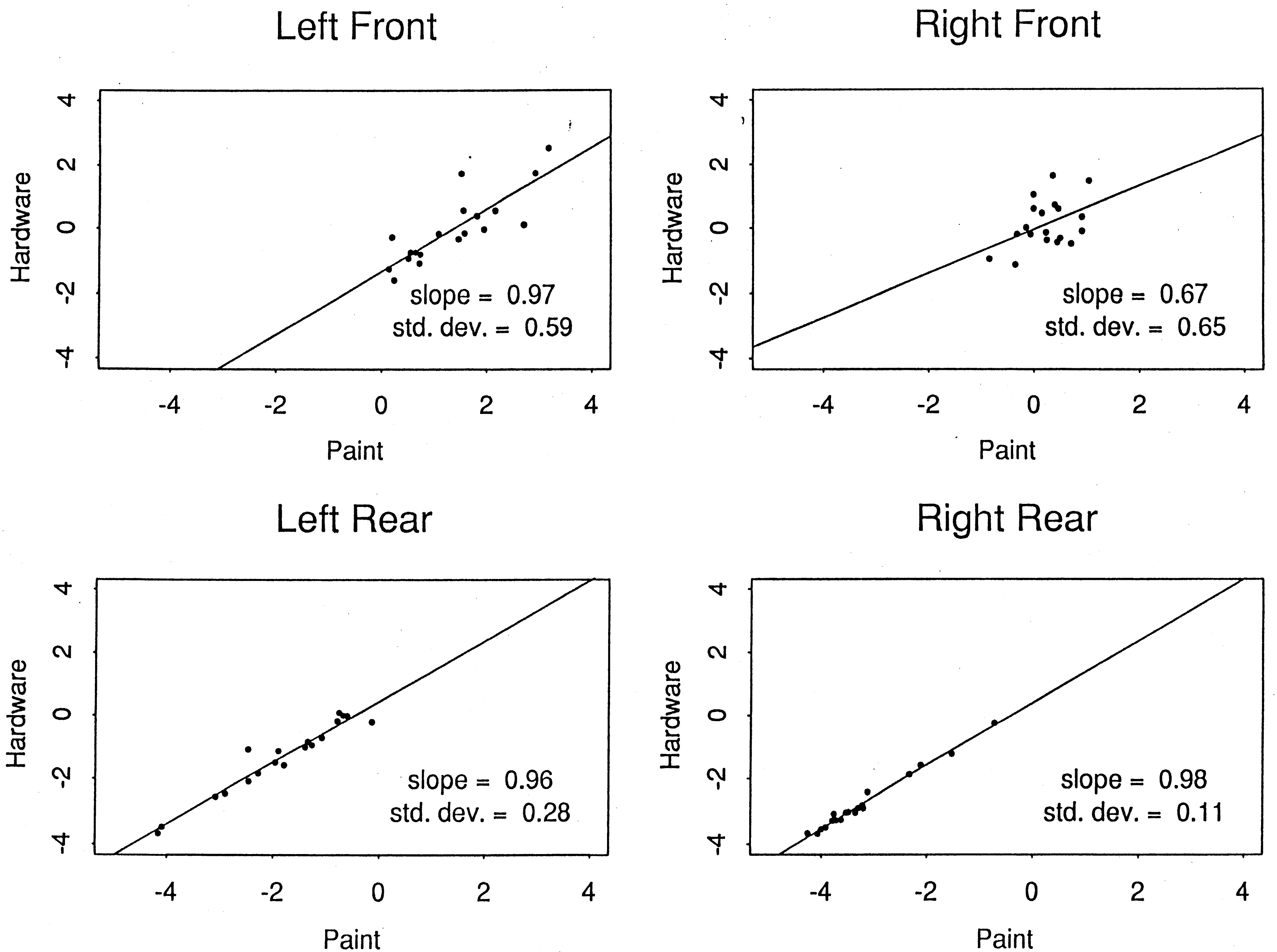

Figure 5: Flushness after PAINT and HARDWARE Operations

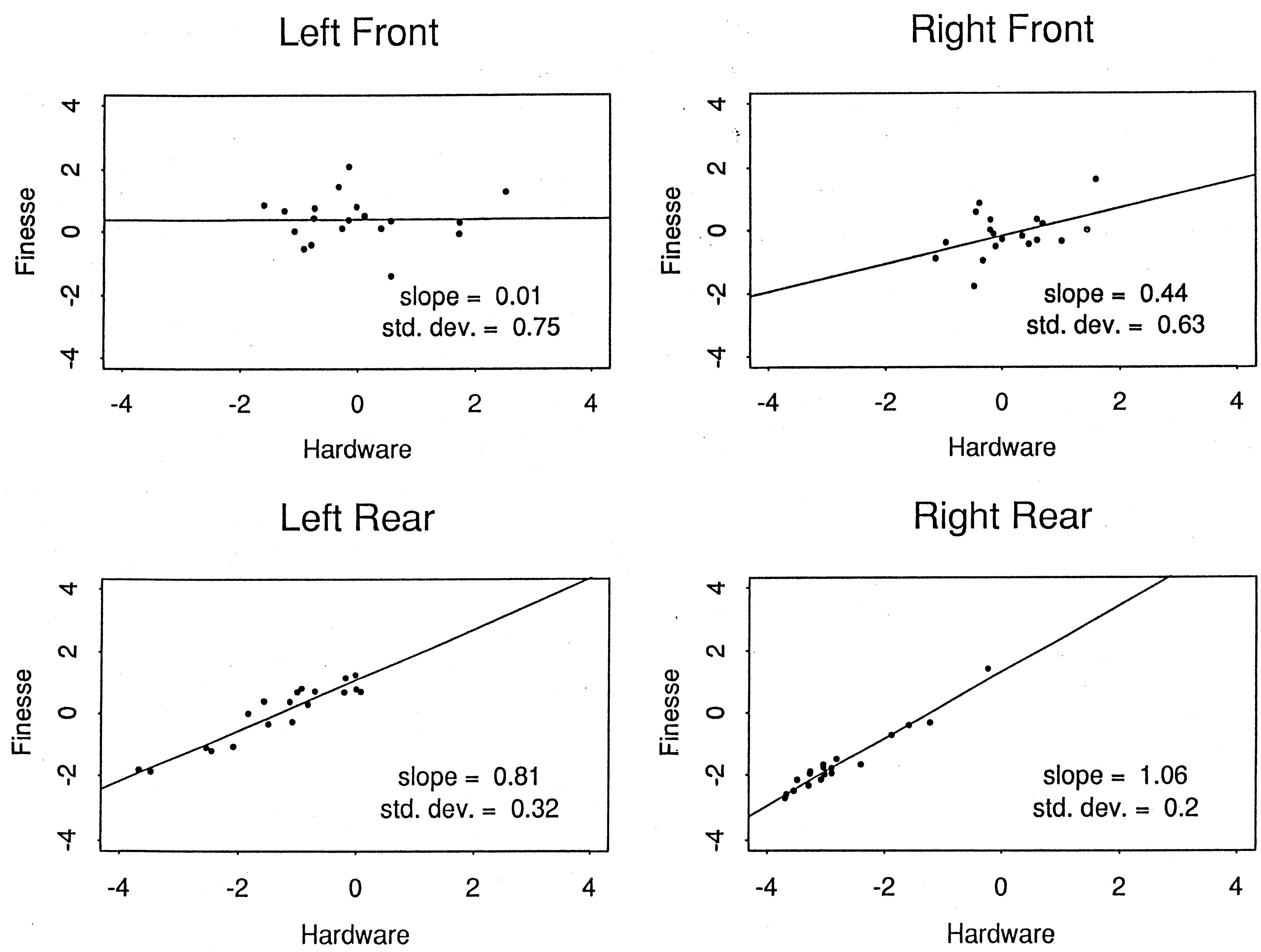

Figure 6: Flushness after HARDWARE and FINESSE Operations

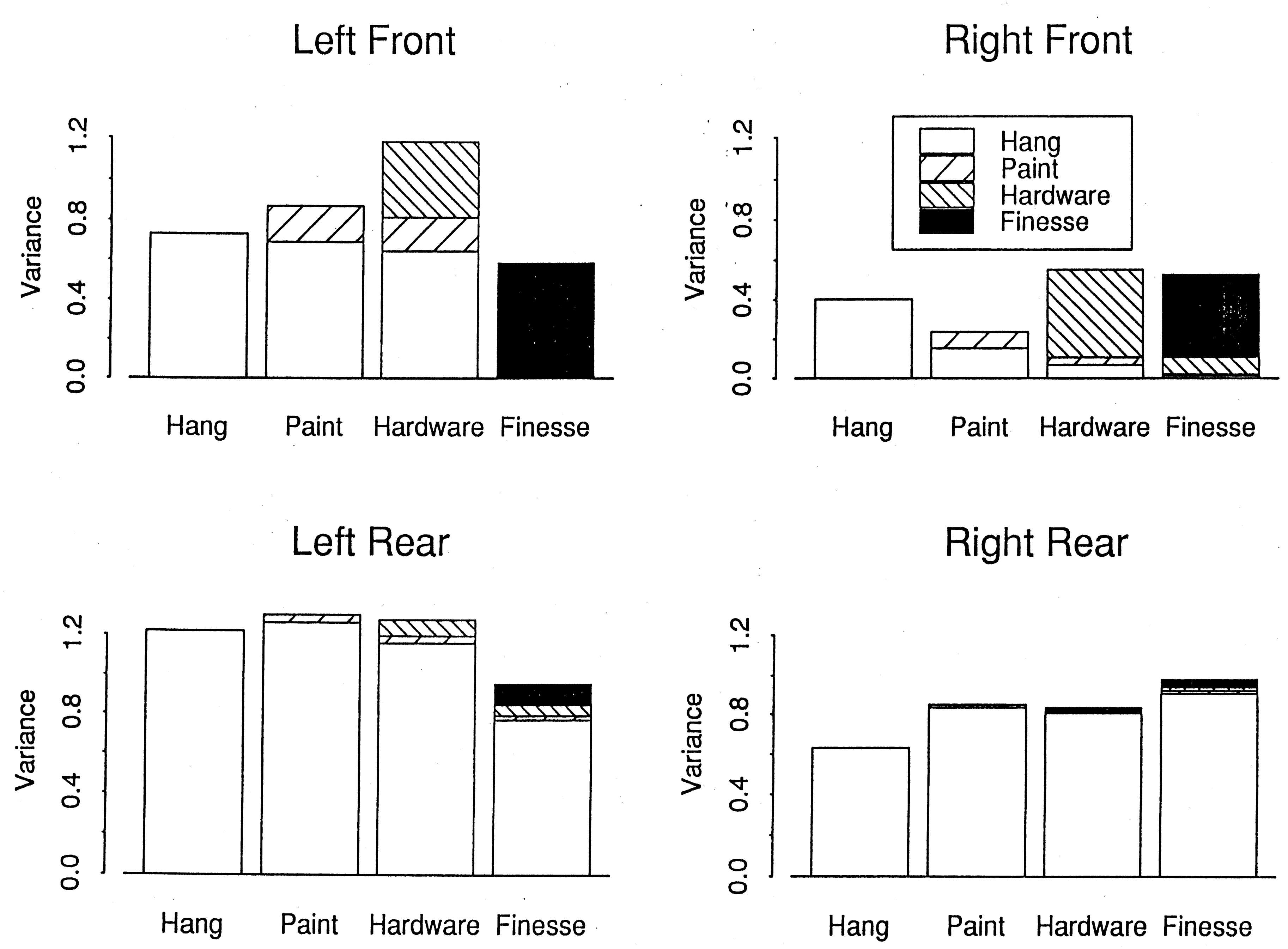

Figure 7: Flushness Variation by Source at Four Plant Operations