Monitoring Processes Using Two Measurement Systems

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Often in industry critical quality characteristics can be measured by more than one measurement system. Typically, in such a situation, there is a fast but relatively inaccurate measurement system that may be used to provide some initial information, and a more accurate, but slower and more expensive, alternative measurement device. In such circumstances, it is desirable to determine the minimum cost procedure for monitoring the production process using some combination of the measurement systems. This article develops such a procedure. An example of its use in the automotive industry is provided.

Introduction

Metrology is an important aspect of manufacturing since measurements are necessary for monitoring and controlling production processes. However, in many situations there is more than one way to measure an important quality dimension. Frequently the choice between the different measurement systems is not clear due to tradeoffs with respect to sampling cost, sampling time, and measurement accuracy. One particular situation, that is explored in this article, occurs when there is a "quick and dirty" measurement device that is inexpensive and relatively fast, but is not the most accurate way to measure, and a slower more accurate and expensive measurement device or method. Good examples of this situation occur in many manufacturing plants. For example, in foundries the chemistry of molten iron may be checked using a quick method, called a "quick lab", or may be sent to a laboratory. In the foundry application, the quick measurement is used to monitor and control the process, since adjustments to composition are required immediately and the lab measurement takes a number of hours. The slower lab measurements are used only for after the fact confirmation. Another example is the use of in-line fixture gauges to monitor the production of engine covers. The fixture gauges provide approximate measurements for some critical dimensions. A Coordinate Measurement Machine (CMM) can be used to determine more precise values. This engine covers example is discussed in more detail later.

The current approaches to monitor production processes where two measurement devices are available is to use results from each measurement device separately and often for different purposes. However, from cost and efficiency considerations using only one of the measurement devices to monitor the process is not ideal. A procedure that monitors or controls the process through the use of both measurement devices in conjunction is the best solution. In this article a method for using both measurements in conjunction to monitor or control the process is proposed. The basic idea is straightforward. The first measurement device is inexpensive and quick, so we try initially to make a decision regarding the state of control of the process based on results from the first measurement device. If the results are not clear cut, we measure the same sample of units again using the more accurate measurement device. Notice that this procedure does not require additional sampling since the same sample is measured again if the initial results were not conclusive. Not requiring an additional independent sample is an advantage since obtaining another independent sample may be difficult and/or time consuming.

This idea of using the second measurement device only in cases where the first measurement does not yield clear cut results is motivated by earlier work by Croasdale (1974) and Daudin (1994). Croasdale and Daudin develop double sampling control charts as an alternative to traditional $\overline{X}$ control charts. Using double sampling charts warning limits are added to the traditional control charts in addition to control limits. The warning limits are used to decide when a second independent sample is needed to reach a conclusion regarding the process' stability. Double sampling charts, however, are not applicable in the two measurement devices problem since they assume that the same measurement device measures all samples and that measurement error is negligible. As a result, with double sampling charts when the initial sample does not provide a clear decision an additional independent sample taken from the production process is required in order to provide more process information.

In this article, optimal process monitoring control charts for the two measurement device case are determined. The resulting control charts are called two measurement system control charts. Two different cost models are considered, namely a sampling cost model and a production cost model. For each model the monitoring procedure that minimizes costs subject to a statistical constraint in terms of the false alarm rate and power of the resulting control chart are determined. The production cost model considers other production costs such as the cost of producing nonconformities and the cost of searching for assignable causes as well as sampling costs. The sampling cost model requires less process knowledge, and is thus appropriate when the production costs are difficult to estimate precisely.

Control Charts for Two Measurement Systems

The results from the two measurement systems are modeled as follows. Let

$$Y_{ij} = X_i + e_{ij}, \qquad i = 1, \ldots, \text{n}, \quad j = 1, 2. \tag{1}$$

where X_i is the actual dimension of the ith unit, Y_{i1} and Y_{i2} are the measured dimensions of the ith unit with measurement device one and two respectively, and e_{ij} is the measurement error. We assume e_{ij}s are normally distributed with mean zero and variance σ_j^2. Assuming that the mean of e_{ij} equals zero implies that we have compensated for any long term bias of the measurement device. The properties of the two measurement devices are assumed well known since regular gauge R&R studies for all measurement devices are required in industry. Note that for most cases of interest $\sigma_2 < \sigma_1$. We also assume that the actual dimensions of the quality characteristic of interest are normally distributed with mean μ, and, without loss of generality, have a standard deviation equal to one. Thus, $X \sim N(\mu, 1)$, and $\overline{X} \sim N(\mu, 1/n)$. Also without loss of generality, we assume that the in-control process mean has been scaled to zero. In other words, for the in-control process the X variable represents a standardized variable.

We begin by defining some terms. Measuring the n units in the sample with the first measurement device we may calculate $\overline{Y}_1 = \sum_{i=1}^{n} Y_{i1}$. If the same sample is measured with the second measurement device we may obtain $\overline{Y}_2 = \sum_{i=1}^{n} Y_{i2}$. Based on the distributional assumptions it can be shown that $\overline{Y}_1$ and $\overline{Y}_2$ are bivariate normal with

$$E(\overline{Y}_1) = E(\overline{Y}_2) = \mu,\ Var(\overline{Y}_1) = (1+\sigma_1^2)/n,\ \ Var(\overline{Y}_2) = (1+\sigma_2^2)/n, \text{ and}$$
$$Cov(\overline{Y}_1, \overline{Y}_2) = E\left(Cov(\overline{Y}_1, \overline{Y}_2 | \overline{X})\right) + Cov\left(E(\overline{Y}_1 | \overline{X}), E(\overline{Y}_2 | \overline{X})\right) = 0 + 1/n = 1/n.$$

Note $\overline{Y}_1$ and $\overline{Y}_2$ are not independent since they represent the sample average obtained by first and second measurement device respectively on the *same* sample of size n. Assuming $\sigma_2 < \sigma_1$, $\overline{Y}_2$ provides more reliable information about the true process mean than $\overline{Y}_1$. However, a weighted average of $\overline{Y}_1$ and $\overline{Y}_2$ provides even more information. Define $\overline{w}$ as the weighted sum given by (2).

$$\overline{w} \;=\; k\overline{Y}_1 + (1-k)\overline{Y}_2 \qquad (2)$$

Based on the moments of $\overline{Y}_1$ and $\overline{Y}_2$ we get:

$$E(\overline{w}) = \mu,$$
$$Var(\overline{w}) = \frac{1}{n}\left(k^2(\sigma_1^2+1) + (k-1)^2(\sigma_2^2+1) + 2k(1-k)\right),$$
$$Cov(\overline{Y}_1, \overline{w}) = (1+\sigma_1^2 k)/n$$

We obtain the most information about the true process mean when the weighting constant k is chosen so as to minimize $Var(\overline{w})$. Denoting this best value for k as k_{opt} and solving gives

$$k_{opt} \;=\; \sigma_1^2 / (\sigma_1^2 + \sigma_2^2). \qquad (3)$$

Using k_{opt}, the variance of $\overline{w}$ and the correlation coefficient relating $\overline{Y}_1$ and $\overline{w}$, denoted ρ_w, are given by (4) and (5) respectively.

$$Var\left(\overline{w} \mid k_{opt}\right) = \left(1+\frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\right)\Big/ n \tag{4}$$

$$\rho_w = \rho\left(\overline{Y}_1, \overline{w} \mid k_{opt}\right) = \frac{\left(\sigma_1^2+\sigma_2^2+\sigma_1^2\sigma_2^2\right)^{1/2}}{\left(\sigma_1^2+\sigma_2^2+\sigma_1^2\sigma_2^2+\sigma_1^4\right)^{1/2}}. \tag{5}$$

The value of k_{opt} will be close to zero if the second measurement system is much more precise than the first device. In that case, $\overline{w}$ almost equals $\overline{Y}_2$.

The proposed procedure to determine whether the process is in-control or out-of-control is as follows. Every h units of time take a rational sample of size n from the process. Measure all units with the first measurement device to obtain Y_{11}, Y_{21}, ..., Y_{n1}. Calculate $\overline{Y}_1$ and if $\overline{Y}_1$ falls outside the interval $[-c_1, c_1]$, where c_1 is the control limit for the first measurement device, we conclude the process is out-of-control. If, on the other hand, $\overline{Y}_1$ falls within the interval $[-r_1, r_1]$, where r_1 is the extra measurement limit ($r_1 \leq c_1$), we conclude the process is in-control. Otherwise, the results from the first measurement device are inconclusive, and we must measure the sample again using the second measurement device. Combining the information from the two measurements on each unit in the sample together, we base our decisions on $\overline{w}$. If $\overline{w}$ falls outside the interval $[-c_2, c_2]$, where c_2 is the control limit for the combined sample, we conclude the process is out-of-control, otherwise we conclude the process in in-control. The decision process is summarized as a flowchart in Figure 1.

In many situations it is reasonable to simplify this procedure by setting c_1 equal to infinity. The result of this restriction is that based on the results from the first measurement device we can conclude that the process is in-control or that we need more information, but not that the process is out-of-control. In applications this restriction is reasonable so long as the time delay for the second measurements is not overly great.

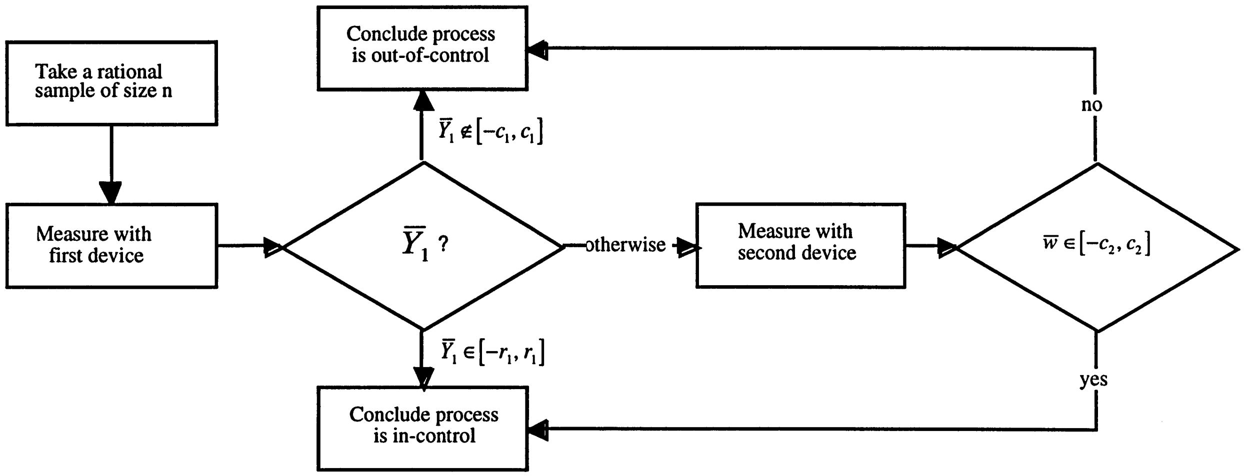

Figure 1: Decision Process for Control Charts for Two Measurement Systems

Using the assumption of normality, it is possible to determine the probabilities of making the various decisions. Let $\phi(z) = e^{-z^2/2}/\sqrt{2\pi}$ and $Q(z) = \int_z^\infty \phi(x)dx$ be the probability density function and cumulative density function of the standard normal respectively. Also, denote the probability density function of the standardized bivariate normal as $\phi(z_1, z_2, \rho) = \left(2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}\right)\exp\left(\left(z_1^2 - 2\rho z_1 z_2 + z_2^2\right)/\left(1-\rho^2\right)\right)$. Then, (6), (7) and (8) give expressions for the probabilities that the following events occur: the procedure signals the process is out-of-control based on results from the first measurement; measuring the sample with the second measurement is necessary; and the combined results from the first and second measurement devices leads to a signal.

$$\begin{aligned} p_1(\mu) &= \text{Pr(signal on first measurement)} = \Pr\left(\bar{Y}_1 > c_1 \text{ OR } \bar{Y}_1 < -c_1\right) \\ &= Q(-z_{c1}) + 1 - Q(z_{c1}) \qquad (6) \\ q_1(\mu) &= \text{Pr(second measurement needed)} = \Pr\left(r_1 < \bar{Y}_1 < c_1 \text{ OR } -\mathrm{r}_1 > \bar{Y}_1 > -c_1\right) \\ &= Q(-z_r) - Q(-z_{c1}) + Q(z_{c1}) - Q(z_r) \qquad (7) \end{aligned}$$

where $z_{c1} = (c_1 - \mu)\big/\sqrt{\left(1+\sigma_1^2\right)/n}$, and $z_r = (r_1 - \mu)\big/\sqrt{\left(1+\sigma_1^2\right)/n}$.

Similarly, we have

$$\begin{aligned} p_2(\mu) &= \text{Pr(signal on combined measurements)} \\ &= \Pr\left(\left(\bar{w} > c_2 \text{ OR } \bar{w} < -c_2\right) \& \left(r_1 < \bar{Y}_1 < c_1 \text{ OR } -\mathrm{r}_1 > \bar{Y}_1 > -c_1\right)\right) \end{aligned}$$

$$
\begin{aligned}
= \quad & \iint_{\substack{z_1 \in [z_r, z_{c1}] \\ z_2 \in [-\infty, -z_{c2}]}} \phi(z_1, z_2, \rho_w) dz_1 dz_2 + \iint_{\substack{z_1 \in [z_r, z_{c1}] \\ z_2 \in [z_{c2}, \infty]}} \phi(z_1, z_2, \rho_w) dz_1 dz_2 \\
& \iint_{\substack{z_1 \in [-z_{c1}, -z_r] \\ z_2 \in [-\infty, -z_{c2}]}} \phi(z_1, z_2, \rho_w) dz_1 dz_2 + \iint_{\substack{z_1 \in [-z_{c1}, -z_r] \\ z_2 \in [z_{c2}, \infty]}} \phi(z_1, z_2, \rho_w) dz_1 dz_2 \qquad (8)
\end{aligned}
$$

where $z_{c2} = (c_2 - \mu) \Big/ \left(\sqrt{(\sigma_1^2 + \sigma_2^2 + \sigma_1^2 \sigma_2^2) / (\sigma_1^2 + \sigma_2^2) n} \right)$. Note that p_1, p_2 and q_1 all depend on the true process mean μ.

Using the decision procedure illustrated in Figure 1, the false alarm rate α, the probability the chart signals when the process mean is in-control, and the power $1-\beta$, the probability the chart signals when the process mean shifts to μ_1, are given by (9) and (10).

$$\alpha = p_1(0) + p_2(0) \qquad (9)$$

$$1-\beta = p_1(\mu_1) + p_2(\mu_1) \qquad (10)$$

Using the simplified version where c_1 is set equal to infinity gives $p_1(\mu) = 0$ for all values of μ, and $z_{c1} = \infty$.

Design of Control Charts using Two Measurement Systems

There are five design parameters for two measurement system control charts, as outlined in the previous section. We must specify the control limits c_1 (when not set equal to infinity) and c_2, the extra measurement limit r_1, the sample size n, and the sampling interval h. Through a judicious choice of these parameters a good monitoring procedure can be defined. As pointed out by Woodall (1986 and 1987) purely economic models of control charts may yield designs that are unacceptable in terms of operating characteristics. For example, the "optimal" design from a purely cost perspective may have such a large false alarm rate that the chart is routinely ignored. For this reason, in this article, the optimal designs for two measurement system control charts are determined based on economic considerations with the addition of some constraints on the operating characteristics of the chart. In particular, two measurement system control charts are designed to closely match the operating characteristics of a Shewhart $\overline{X}$ chart with a sample of size five. Thus, the false alarm rate of the two measurement system control chart is constrained such that $\alpha \leq \alpha^* = .0027$, and the power of the chart to detect shifts in the process mean of two standard deviation units must satisfy the constraint $\beta \leq \beta^* = .0705$. The values for α^* and β^* could be changed if the objective of the chart is different than that of a standard $\overline{X}$ chart with a sample of size five.

In this article two different cost models are considered. First, a model that minimizes only the sampling costs as examined. This sampling cost only model is relatively easy to use, and is appropriate when other cost parameters can not be well estimated. The more complex cost model considers all the production costs and is based on the general framework developed by Lorenzen and Vance (1986).

Sampling Cost Model

Using the sampling cost model the goal is to minimize the sampling costs while maintaining the desired minimum error rates of the procedure. Let f_i and v_i denote the fixed and variable sampling costs for the ith measurement system respectively ($i = 1, 2$). Then the sampling cost per sample interval, $S(\mu)$, is given by (11).

$$S(\mu) = f_1 + v_1 n + (f_2 + v_2 n) q_1(\mu) \tag{11}$$

The sampling cost per interval is a function of the actual process mean through the probability the second measurement is needed $q_1(\mu)$. There are a number of ways to define an objective function using (11). Since the process will (hopefully) spend most of its time in-control we minimize the in-control sampling costs. Using this formulation, the optimal design of the control chart using two measurement devices is determined by finding the design parameters that

$$\text{minimize } S(0) \tag{12}$$
$$\text{subject to } \alpha \leq \alpha^* = .0027 \text{ and } \beta \leq \beta^* = .0705$$

where $S(0)$ is given by (11) when $\mu = 0$, and α and β are given by (9) and (10). Note that when considering only the sampling costs the sampling interval h, has no effect on the cost. As a result, the sampling interval must be determined through some other criterion, such as the production schedule. This leaves three or four design parameters c_2, r_1, n and perhaps c_1. Optimal values for these parameters that satisfy (12) can be determined using a constrained minimization approach such as applying the Kuhn-Tucker conditions. This solution approach was implemented using the routine "constr" in the optimization toolbox of MATLAB®. An alternative objective function is a weighted average of in-control and out-of-control sampling costs. However, it is not clear what weights to choose, and in most cases the optimal chart design does not change substantially unless the in-control case is given a small weight which is unrealistic.

In our analysis of the sampling cost model, without loss of generality, we may set $v_1 = 1$, since the results depend only on the relative values of the sampling costs. In addition, to restrict the possibilities somewhat, the fixed cost associated with the first measurement device is set to zero, i.e. $f_1 = 0$. This restriction is justified because typically the first measurement device is very easy and quick to use, and would not require much setup time or expense.

Figures 2-3 show the results of determining the optimal design parameters that satisfy (12) for different sampling cost parameters when setting c_1 equal to infinity. Figure 2 gives results when the second measurement device also has no fixed costs, while Figure 3 considers the situation where the fixed cost associated with the second measurement device is relatively large. Figures 2 and 3 each consist of four subplots that show contour plots of the optimal design parameters: r_1, c_2, and n as a function of σ_1 and σ_2, the variability inherent in the two measurement devices. Each subplot represents four different values of v_2, the variable sampling cost associated with the second measurement device. Optimal values for r_1, c_2, and n in the general case where c_1 is allowed to vary are very similar to those given in Figures 2 and 3. In general, the optimal value of c_1 is large and as a result does not effect the procedure much unless there is a large shift in the process mean.

Figures 2 and 3 suggest that the parameters r_1 and c_2 are the most sensitive to changes in the variability of the measurement devices. In general, when the sampling costs of the two measurement devices are comparable, as the first measurement device becomes less reliable (σ_1 increases), n increases, while r_1 decreases. This makes sense since it means we rely more on the second measurement device when the first device is less accurate. Conversely as the second measurement device becomes less reliable (σ_2 increases), c_2 and n increase while r_1 increases marginally since we rely more on the first measurement device.

Now consider the case where the second measurement device is expensive (f_2 or v_2 large). As the second measurement device becomes less reliable (σ_2 increases), again we observe that c_2 increases while n and r_1 increase marginally which makes sense. However, the pattern appears to be counterintuitive when the first measurement device becomes less reliable (σ_1 increases) since n and c_2 decrease marginally, but r_1 increases! Does this mean that we rely more heavily on the inaccurate first measurement device? Looking more closely, this apparent contradiction disappears. Although, as σ_1 increases the optimal r_1 also increases this does not mean that the decisions are more likely to be made based on the first measurement device. When the variability of a measurement device is large we expect to observe large deviations from the actual value. Thus, the observed increase in r_1 is only taking this into account. Consider Figure 4 which shows contours of the probability the second measurement is needed in the two cases: $f_1 = f_2 = 0$, $v_1 = 1$ with $v_2 = 1$ or 4. The plots in Figure 4 show clearly that as the first measurement device becomes less accurate we rely on it less even though, as shown in Figure 1, r_1 increases.

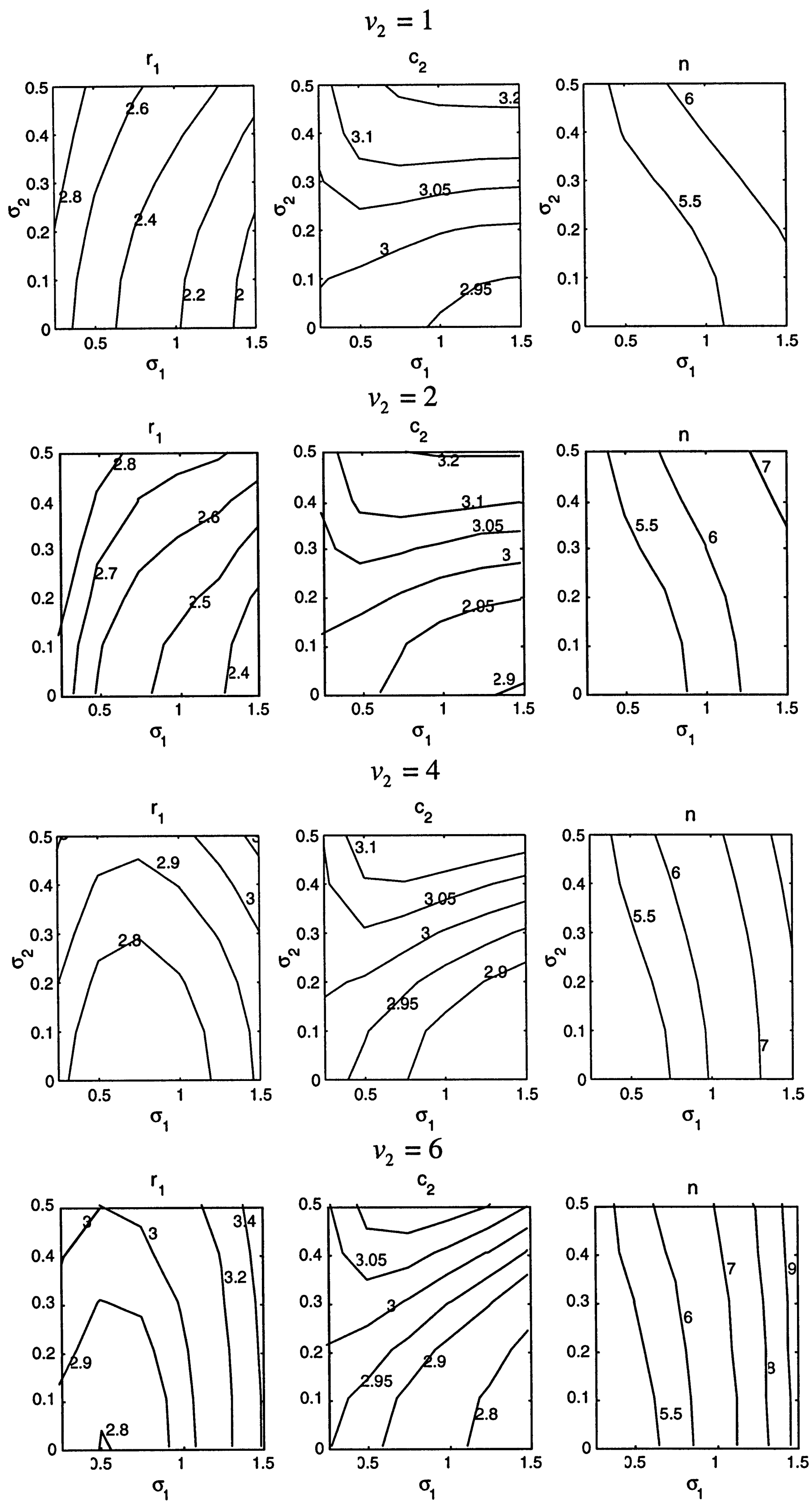

Figure 2: Contour Plots of the Design Parameters for the No Fixed Cost Case
$f_1 = 0,\ v_1 = 1,\ f_2 = 0$

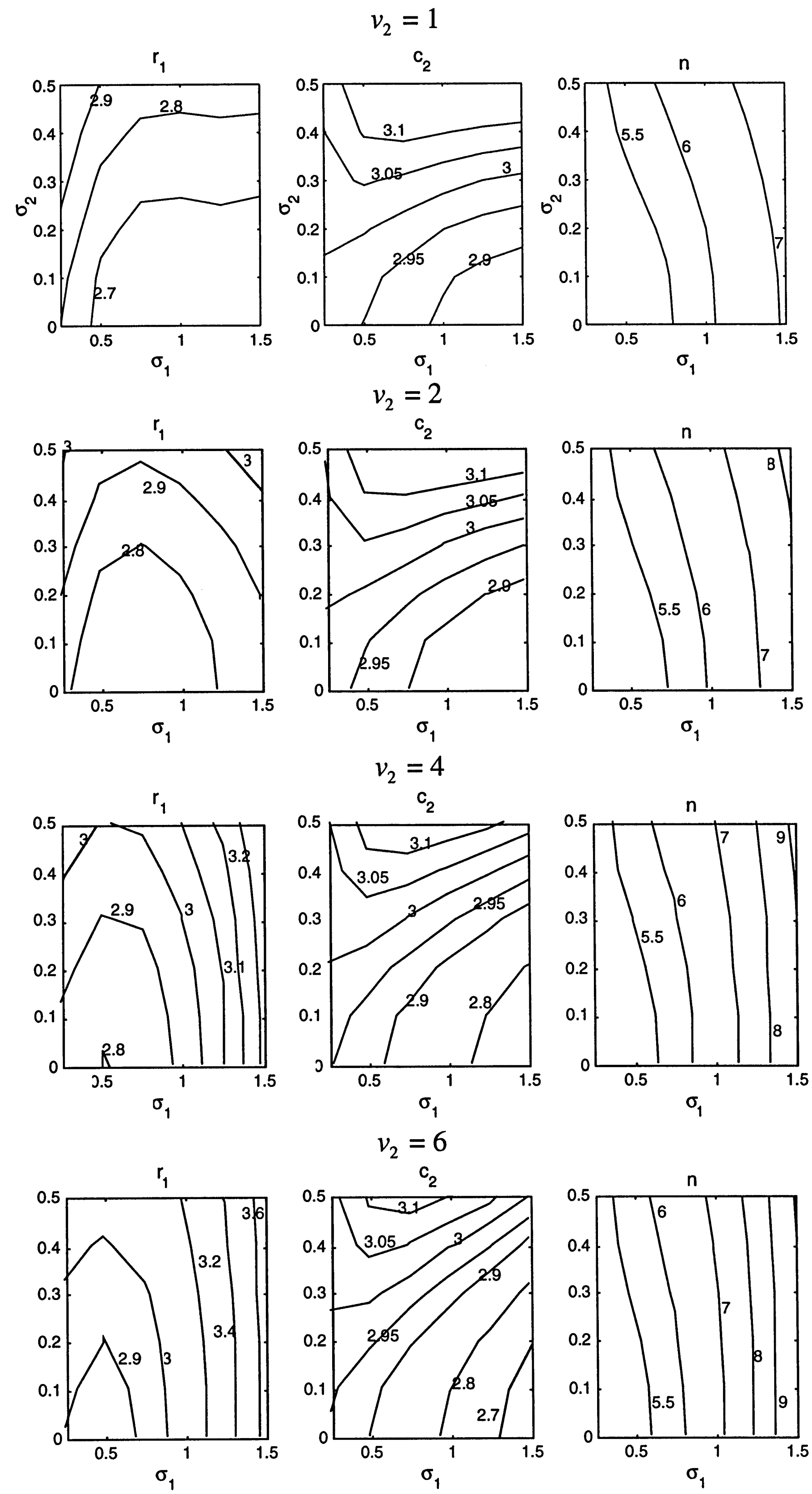

Figure 3: Contour Plots of the Design Parameters for the Large Fixed Cost Case
$f_1 = 0,\ v_1 = 1,\ f_2 = 10$

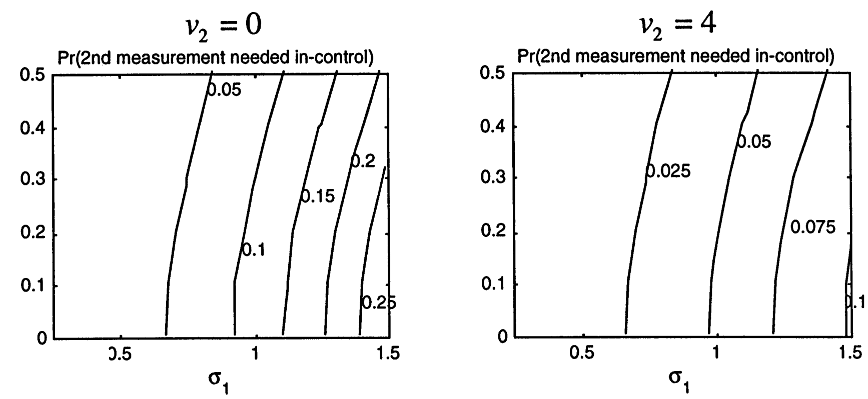

Figure 4: Contour Plots of the Probability the Second Measurement is Required
Process in-control, $f_1 = 0$, $v_1 = 1$, $f_2 = 0$

Figures 2 and 3 may be used to determine the design parameter values that are approximately optimal in terms of in-control sampling costs. For sampling cost parameters in between those given interpolation can be used to determine reasonable values for c_2, r_1, and n. In practice, the sample size, n, must be rounded off to the nearest integer value. Rounding off the sample size effects the power of the chart, but has no affect on the false alarm rate of the procedure. Of course, rounding down the sample size decreases the procedure's power, while rounding up increases the power.

Production Cost Model

Besides sampling costs many other production related costs, such as the cost of searching for assignable causes real and imagined, the cost of downtime, and the cost of producing nonconformities effect the cost efficiency of any monitoring procedure. The production cost model defined here is based on the general approach suggested by Lorenzen and Vance (1986). Using cost models to design process monitoring control charts has a fairly long history, and there is a large literature on the economic design on Shewhart type control charts starting with Duncan (1956). A good review of the early work is provided by Montgomery (1980).

Following Lorenzen and Vance (1986) we define a quality cycle as the time between the start of successive in-control periods. Repeated cycles form a renewal reward process (Ross, 1983), and thus the expected production cost per unit time can be expressed as the ratio of the expected net production cost per cycle divided by the expected length of a cycle.

The cycle time consists of the sum of the following: (a) the time until the assignable cause occurs; (b) the time until the next sample is taken; (c) the time until the first process sample showing evidence of an out-of-control situation is collected; (d) the time to analyze the sample and chart the results; and (e) the time to discover the assignable cause and repair the system. See Figure 5.

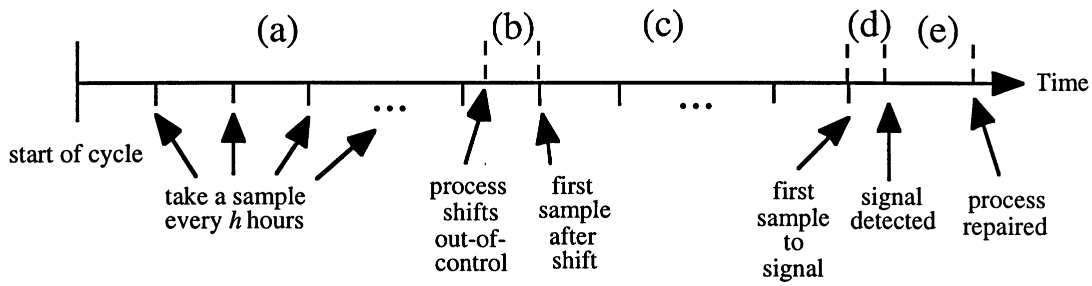

Figure 5: Process Time Line Showing the Five Periods

It is assumed that assignable causes occur according to an Poisson process with an intensity of λ. Then, it can be shown that $\tau = \left[1-(1+\lambda h)e^{-\lambda h}\right]/\left(\lambda - \lambda e^{-\lambda h}\right)$ equals the expected time of occurrence of the assignable cause within a sampling interval given that the assignable cause occurs somewhere in that interval. Also, it can be shown that the expected number of samples taken while in-control is $s = e^{-\lambda h}/\left(1-e^{-\lambda h}\right)$. Define indictor variables δ_1 and δ_2, such that δ_1 equals one if production continues during searches for assignable causes and zero otherwise, and δ_2 equals one if production continues during repair of a assignable cause and zero otherwise. The expected search time for a false alarm, the expected time to find the source of assignable cause, and the expected time to repair an assignable cause are denoted as T_0, T_1, and T_2 hours respectively.

For two measurement system control charts the expected amount of time the process spends in periods (a), (b), (c) and (e) is unchanged from the Lorenzen and Vance (1986) formulation, and are given by $1/\lambda + (1-\delta_1)s\alpha T_0$, $h-\tau$, $h\beta/(1-\beta)$, and $T_1 + T_2$ respectively.

The time in state (d), on the other hand, must be adjusted to take into account the possibility that the second measurement of the sample causes the out-of-control signal rather than the results from the first measurement device. Denote the expected time to sample and chart *one* unit with the first and second measurement devices as E_1 and E_{2v} respectively, and let the additional fixed time needed to measure the sample with the second measurement device be E_{2f}. Assuming that $0 \le nE_{2v} + E_{2f} < h$, the expected time in period (d) is given by $nE_1 + p_2\left(nE_{2v} + E_{2f}\right)/\left(p_1 + p_2\right)$, where p_1 and p_2 are determined by (6) and (8) evaluated at μ_1. The constraint $nE_{2v} + E_{2f} < h$ implies that the results of the second measurement device, if they are necessary, are known before the next independent sample is taken. This is a reasonable assumption in most production environments. If this assumption does not hold it is still possible to derive the expected time in period d), but it is more complicated since the chart may signal due to the first measurement of a sample before the results of a second measurement on a prior sample are known.

Putting this together, the expected cycle time, T, equals

$$\begin{aligned} T &= 1/\lambda + (1-\delta_1)s\alpha T_0 + h - \tau + nE_1 + \frac{p_2(nE_{2v} + E_{2f})}{p_1 + p_2} + \frac{h\beta}{(1-\beta)} + T_1 + T_2 \\ &= 1/\lambda + (1-\delta_1)s\alpha T_0 + nE_1 + \frac{h + nE_{2v}p_2 + E_{2f}p_2}{1-\beta} - \tau + T_1 + T_2. \end{aligned} \tag{13}$$

Turn now to the cycle costs. Costs per cycle are made up of three components: nonconformities, search for and repairing assignable causes or false alarms, and sampling costs. Define C_0 and C_1 ($C_1 > C_0$) as the costs per hour due to nonconformities produced while the process is in-control and out-of-control respectively. Then the expected cost per cycle due to non-conformities equals

$$C_0/\lambda + C_1\left[-\tau + nE_1 + \left(h + nE_{2v}p_2 + E_{2f}p_2\right)/(1-\beta) + \delta_1 T_1 + \delta_2 T_2\right].$$

Let W equal the cost of locating and repairing the assignable cause when one exists. Here W includes the cost of downtime while repairing the process. Also, let Y equal the expected cost per false alarm. Then the expected cost associated with false alarms and the cost of repairing the true assignable cause is $Y\alpha e^{-\lambda h}/\left(1-e^{-\lambda h}\right)+W$. Sampling costs per cycle equal the sampling costs per sample multiplied by the time producing divided by the sampling interval. Applying this rule the expected sampling costs per cycle in-control and out-of-control respectively are given by

$$\left[f_1+v_1 n+q_1(\mu_0)(f_2+v_2 n)\right]/\lambda h \text{ and}$$
$$\left[f_1+v_1 n+q_1(\mu_1)(f_2+v_2 n)\right]\left[\frac{h+nE_{2v}p_2+E_{2f}p_2}{1-\beta}+E_1 n-\tau+\delta_1 T_1+\delta_2 T_2\right]\Big/h,$$

where as defined previously f_i and v_i denote the fixed and variable sampling costs for the two measurement devices, $q_1(\mu)$ is given by (7), and $p_2 = p_2(\mu_1)$ is given by (8).

Thus, the expected net cost during a cycle, *N*, is

$$\begin{aligned} N \;=\; & C_0/\lambda+C_1\left[-\tau+\frac{h+nE_{2v}p_2+E_{2f}p_2}{1-\beta}+nE_1+\delta_1 T_1+\delta_2 T_2\right]+Y\alpha e^{-\lambda h}/\left(1-e^{-\lambda h}\right)+\mathrm{W} \\ & +\left[f_1+v_1 n+q_1(\mu_1)(f_2+v_2 n)\right]\left[\frac{h+nE_{2v}p_2+E_{2f}p_2}{1-\beta}+E_1 n-\tau+\delta_1 T_1+\delta_2 T_2\right]\Big/h \qquad (14) \\ & +\left[f_1+v_1 n+q_1(\mu_0)(f_2+v_2 n)\right]/\lambda h \end{aligned}$$

The expected cost per hour is given by the ratio N/T, and the optimization problem is to find the design parameters that

$$\begin{aligned} & \text{minimize } N/T \qquad (15) \\ & \text{subject to } \alpha \le \alpha^* = .0027 \text{ and } \beta \le \beta^* = .0705 \end{aligned}$$

where *N* and *T* are given by (14) and (13) respectively, and α and β are given by (9) and (10). Using the second measurement device results in longer times in the out-of-control state, thus leading to more nonconformities, but may reduce the sampling costs.

Due to the large number of cost and time parameters required for this model it is not possible to provide graphs or tables that would allow a practitioner to determine near optimal designs. However, as an illustration, consider the following example adapted from Lorenzen and Vance (1986). The cost and time parameters are: $\lambda = 1/500$, $\mu_1 = 2$, $E_1 = 5/60$, $E_{2v} = 10/60$, $E_{2f} = 0$, $T_0 = T_1 = 5/60$, $T_2 = 45/60$, $C_0 = 114.24$, $C_1 = 949.2$, $f_1 = 0$, $v_1 = 8.44$, $f_2 = 0$, $v_2 = 16.88$, $W = 977.4$, and $Y = 977.4$. Also, assume that production continues while searching for assignable causes, but that while repairing assignable causes production stops, i.e. $\delta_1 = 1$ and $\delta_2 = 0$. Figure 6 shows contours of the optimal design parameters for different values of σ_1 and σ_2. In this example the optimal c_1 was always large, and thus we used the simplified monitoring procedure where c_1 is set to infinity.

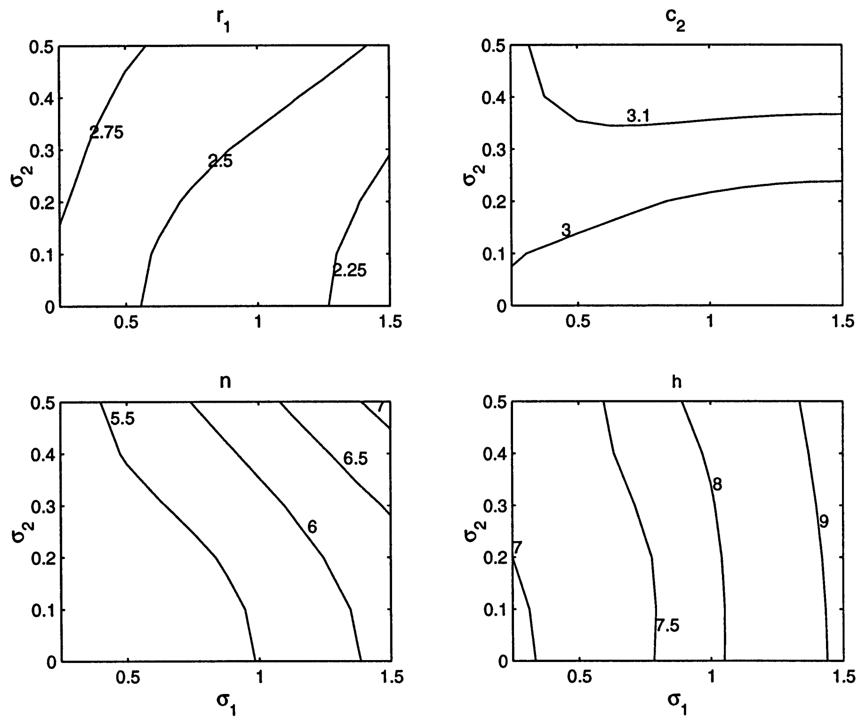

Figure 6: Contour Plots of the Design Parameters for the Production Cost Model Example

See Lorenzen and Vance (1986) for more details on sensitivity analysis for the model parameters that are in their model. To accommodate the second measurement device four additional parameters are added to the model, namely f_2, v_2, E_{2v} and E_{2f}. The effect of the additional sampling cost parameters: f_2, v_2 are fairly straightforward. In the cost model they influence only the cycle cost. Increasing (decreasing) either f_2 or v_2 results in lower (higher) levels for the optimal r_1 value, and slightly larger (smaller) values for n and h. In other words, as the cost of the second measurement increases (decreases) we try to rely more (less) on the results from the fist measurement device. The effect of the sampling time parameters E_{2v} and E_{2f} is more complicated since changes in the sampling time effects both the cycle cost and cycle time. As a result, the effect of increases or decreases in the sampling times is not consistent. In general, changes to the variable sampling costs or times have more influence than changes to the fixed sampling costs or times.

Example

In the manufacture of engine front covers there are many critical dimensions. One such critical dimension is the distance between two bolt holes in the engine cover used to attach the cover to the engine block. This distance may be accurately measured using a coordinate measurement machine (CMM). However, using a CMM is expensive and time consuming. An easier, but less accurate measurement method involves the use of a fixture gauge. The fixture gauge clamps the engine cover in a fixed position while measuring hole diameters and relative distances.

In this example, denote the fixture gauge as the first measurement device, and the CMM as the second measurement device. Based on previous measurement system studies it has been determined that $\sigma_1 = .5$ and $\sigma_2 = .05$. As expected, the CMM measurement has less variability than the fixture gauge results. We also know that on a relative cost basis the using the CMM is six times as expensive as the fixture gauge in terms of personnel time. We shall assume that the fixed costs associated with the two measurement methods is zero. Thus, in terms of the notation from the sample cost model we have: $f_1 = f_2 = 0$, $v_1 = 1$, and $v_2 = 6$.

Additional information about the production costs was felt to be difficult to estimate. As a result, we use the sampling costs only model. Solving (12) with the additional simplification that $c_1 = \infty$ gives: $r_1 = 2.80$, $c_2 = 2.92$, with $n = 5.26$ for a relative cost of 5.65. In this optimal solution the values for r_1 and c_2 are almost equal. From an implementation perspective setting r_1 and c_2 equal is desirable since it simplifies the resulting control chart as will be shown. For this example the optimal solution to (12) with the additional constraint that $r_1 = c_2$ results in the constrained solution: $r_1 = c_2 = 2.89$, $n = 5.36$ with a corresponding cost of 5.67. For implementation the sample size is rounded off to five. These costs are around 10% less than the sampling costs associated with a similar plan that uses only the first measurement device.

Figure 7 gives an example of the resulting two measurement control chart. On the chart the average of the measurements obtained with the first measurement device are shown on the plot with an "o", while the average of the combined first and second measurement sample (if it is necessary) are shown with a "x"s. The extra measurement limit ($\pm r_1$) for the results from the first measurement device and control limit ($\pm c_2$) for the combined sample are given by the solid horizontal lines on the chart. If the initial measurement average plots between the extra measurement limits the chart concludes that the process is in-control. Otherwise, if the initial point lies outside the extra measurement limits a second measurement of the sample is required. Using the second measurement we calculate the combined sample weighted average $\overline{w}$ = $.01\overline{Y}_1 + .99\overline{Y}_2$. If $\overline{w}$ falls outside the control limits we conclude the process shows evidence of an assignable cause, otherwise the process appears to be in-control. The dashed/dotted line gives the center line of the chart. In creating the time sequence plot if two samples were drawn at a time period the midpoint of the two plotted points is used to connect that paired sample with the adjacent time periods. In this example, for illustration, after the 19th observation one was added to all the measurements to simulate a one sigma shift in the process mean. Figure 7 shows that in the 25 measurements a second sample was required five times, at sample numbers 3 and 20, 21, 24 and 25. However, only samples 21 and 25 yield an out-of-control signal, in the other cases the second measurement of the sample suggested the process was still in-control. Of course the number of times the second measurement was needed after observation 19 is also an indication that the process has shifted.

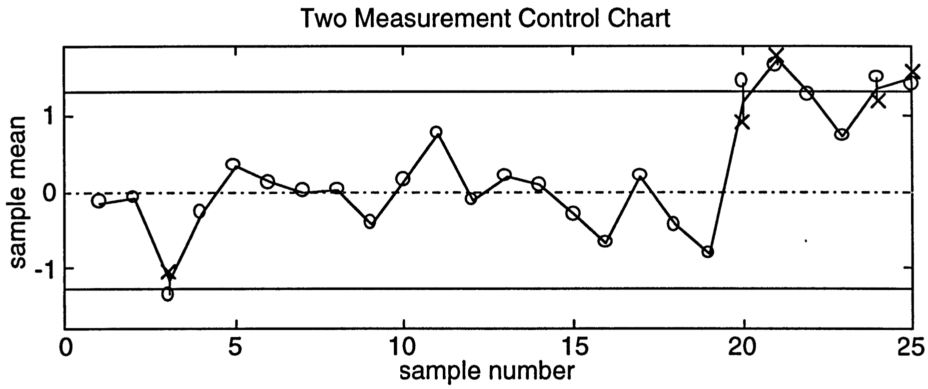

Figure 7: Two Measurement Control Chart

Other Issues

An alternative approach to process monitoring in this context is to use a second sample that is different than first. So rather than measuring the first sample again we take a completely new sample. However, it may be difficult to get a new independent sample in a timely manner since much of the measurement variability may be due to factors that are difficult to alter, such as the operator, the setup, the environmental factors, etc. However, if these sampling concerns can be overcome, the advantage of using an additional sample is that more information about the true level of the process is available in two independent samples than measuring the same sample twice.

In a similar vain, we may consider situations where repeated measurements with a single measurement systems are allowed. If independent measurements are possible then by averaging the results we would be able to reduce the measurement error by a factor of $\sqrt{n}$. We could consider the second measurement to be simply the results of repeated measurements on the units with the first measurement device. However, using repeated measurements from the same measurement device will only work if we can obtain independent measurements of the units which is often not the case.

Additional applications of this monitoring procedure occur in the medical field. In medical testing there is often a choice between a screening type test that is cheap, but is relatively inaccurate, and another more expensive test that is much more precise. This problem is very similar to the previously described industrial monitoring problem. Using the proposed methodology it is possible to determine a testing scheme that minimizes the sampling (testing) costs while making few misclassifications of the patients.

Summary

This article develops a sampling cost model and a total production cost model that can be used to determine an optimal process monitoring control chart that utilizes two measurement devices. It is assumed that the first measurement device is fast and cheap, but relatively inaccurate, while the second measurement device is more accurate, but also more costly. Applications of this methodology occur in industry and medical situations. The proposed monitoring procedure may be thought of as an adaptive monitoring method that provides a reasonable way to compromise between sampling cost and sampling accuracy.

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Appendix - Glossary of Symbols

Design Parameters

c_1 control limit for results from first measurement

c_2 control limit for the results from the second measurement

r_1 limit used to determine if a second measurement is needed

h sampling interval time

n sample size

Decision Variables

X actual dimension of unit

$\overline{Y}_1$ subgroup average using first measurement device

$\overline{Y}_2$ subgroup average using second measurement device

$\overline{w}$ weighted average of $\overline{Y}_1$ and $\overline{Y}_2$

Sample and Production Cost Model Parameters

f_i fixed cost associated with measurement device i

v_i variable cost per unit associated with measurement device i

C_0 cost per hour due to nonconformities while process in-control

C_1 cost per hour due to nonconformities while process out-of-control

W cost of locating and repairing the assignable cause when one exists

Y expected cost per false alarm

T_0 expected time to searching for a false alarms

T_1 expected time to find an assignable cause

T_2 expected time to repair an assignable cause

E_1 time required to obtain one measurement using the first measurement device

E_{2f} fixed time required to obtain measurements using the second measurement device

E_{2v} time required to obtain one measurement using the second measurement device

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Key Words: Control Chart; Production Costs; Sampling Costs

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