Estimation of Limiting Availability for a Stationary Bivariate Process

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Abstract

We estimate the limiting availability of a system when the operating and repair time form a stationary bivariate sequence. These estimators are shown to be consistent and asymptotically normal under certain conditions. In particular, we estimate the limiting availability for a bivariate exponential autoregressive process.

Key Words: Bivariate geometric distribution, consistent and asymptotically normal, ergodicity, exponential autoregressive model, ϕ-mixing.

1. INTRODUCTION

Suppose that we have a repairable system and let $\{X_n\}$ and $\{Y_n\}$ denote the sequences of operating and repair times, respectively. The first operating time and repair time constitute the first cycle of the system. One of the important characteristics of such a system is the measure of instantaneous availability denoted by $A(t)$, which is the probability that the system is in working state at time t. If we define

$$\xi(t) = \begin{cases} 1 & \text{if the system is operating} \\ 0 & \text{otherwise} \end{cases} \tag{1.1}$$

then $A(t) = E[\xi(t)] = P[\xi(t) = 1]$.

Note that the properties of $A(t)$ depend on the distribution of $\{X(n) = (X_n, Y_n),\ \ n \geq 1\}$ and the exact expression of $A(t)$ is difficult to obtain. However, one can compute $A = \lim A(t)$ as $t \to \infty$, under certain conditions and A is called the limiting availability. It is important to study the properties of A because one may be interested in knowing the

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extent to which the system will be available after it has been run for a long time. The properties of A and the problem of its estimation are discussed by different authors under various conditions.

Let $\{X_n\}$ and $\{Y_n\}$ be independent sequences of independent and identically distributed (iid) non-negative random variables (r.v.'s) with $E(X_1) = \mu$ and $E(Y_1) = \theta$. Under this set up it is proved by using the theory of renewal process that as $t \to \infty$ $A(t) \to A = \mu/(\mu + \theta)$. See eg. Hoyland and Rausand (1994). Similar results are also proved by Gut and Janson (1983) for the case when $\{X(n)\}$ is a sequence of iid bivariate random vectors and Jie Mi (1995) for a sequence of independent non-identically distributed bivariate random vectors.

There are two estimators proposed for A. They are

$$\begin{aligned} \bar{A}_n &= \sum_{i=1}^{n} X_i / \{\sum_{i=1}^{n} (X_i + Y_i)\} \\ &= \text{Average availability in the first } n \text{ cycles} \end{aligned} \tag{1.2}$$

and

$$\bar{A}(t) = \alpha(t)/t = \{\text{Total operating time in } (0,t)\}/t. \tag{1.3}$$

These estimators are proved to be consistent and asymptotically normal (CAN) for A under different conditions. In particular, if $\{X(n)\}$ is a sequence of iid bivariate random vectors with $E(X_1) = \mu$, $E(Y_1) = \theta$, $\text{Var}(X_1) = \sigma_x$, $\text{Var}(Y_1) = \sigma_y$ and $\text{Corr}(X_1, Y_1) = \rho$, then

$$\sqrt{n}(\bar{A}_n - \mu/(\mu + \theta)) \xrightarrow{\mathcal{L}} N(0, \frac{\Delta^2}{\mu + \theta}) \quad \text{as } n \to \infty \tag{1.4}$$

and

$$\sqrt{t}(\bar{A}(t) - \mu/(\mu + \theta)) \xrightarrow{\mathcal{L}} N(0, \Delta^2) \text{ as } t \to \infty, \tag{1.5}$$

where $\xrightarrow{\mathcal{L}}$ denotes the convergence in distribution and

$$\Delta^2 = (\theta^2 \sigma_x^2 + \mu^2 \sigma_y^2 - 2\mu\theta\sigma_x\sigma_y\rho)/(\mu + \theta)^3. \tag{1.6}$$

If (1.4) and (1.5) hold then the asymptotic variance of $\bar{A}_n$ and $\bar{A}(t)$ are respectively given by

$$AV(\bar{A}_n) = \frac{\Delta^2}{(\mu + \theta)} \text{ and } AV(\bar{A}(t)) = \Delta^2/t.$$

The non-parametric confidence estimation and point estimation of A are discussed by Baxter and Li (1994) and Baxter and Li (1996) respectively.

Our interest in this paper is to estimate A by $\bar{A}_n$ and $\bar{A}(t)$ when $\{X(n)\}$ is a stationary, ϕ-mixing bivariate sequence. This is done in Section 2. In particular, we take $\{X(n)\}$ as a first order bivariate exponential autoregressive (BEAR(1)) process defined by Block, Langberg and Stoffer (1988). The Section 3 of the paper describes the BEAR(1) model and discusses its useful properties. A comparison of asymptotic variances of our estimators for BEAR(1) sequence with those corresponding to iid sequences is made in Section 4. Section 5 is the concluding remarks.

2. PROPERTIES OF THE ESTIMATORS

In this section, we study the properties of the estimators $\bar{A}n$ and $\bar{A}(t)$ of the limiting availability A introduced in Section 1, when $\{X(n)\}$ is a stationary ϕ-mixing sequence. The results are summarised in the following theorem.

Theorem 2.1: Let $\{X(n) = (X_n, Y_n), n \geq 0\}$ be a stationary, ergodic, ϕ-mixing sequence of bivariate random vectors on $R_2^+ = \{(x, y) : 0 \leq x < \infty, 0 < y < \infty\}$ and let $\bar{A}n$ and $\bar{A}(t)$ be as defined by (1.2) and (1.3) respectively. For some $\delta > 0$, if $E(X_1^{2+\delta}) < \infty, E(Y_1^{2+\delta}) < \infty$ and $\sum_{h=1}^{\infty} \phi^{1/2}(h) < \infty$ then both $\bar{A}n$ and $\bar{A}(t)$ are CAN estimators for $A = \mu/(\mu + \theta)$, where $\mu = E(X_1), \theta = E(Y_1)$ and $\phi(h), h = 1, 2, ...$ are mixing coefficients.

Proof:

Since $\{X(n)\}$ is stationary and ϕ-mixing with mixing coefficients $\phi(h)$ and $\sum_{h=1}^{\infty} \phi^{1/2}(h) < \infty$, by the central limit theorem for such sequences (cf. Theorem 20.1, Billingsley (1968)) we have as $n \to \infty$,

$$\sqrt{n}(\bar{X}_n - \mu, \quad \bar{Y}_n - \theta) \xrightarrow{\mathcal{L}} N_2(0, \Sigma_2), \tag{2.1}$$

where $N_2(0, \sum_2)$ is a bivariate normal vector with mean $0 = (o, o)$ and dispersion matrix

$$\sum_2 = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix}, \tag{2.2}$$

$$\begin{aligned} \sigma_{xx} &= Var(X_0) + 2 \sum_{h=1}^{\infty} Cov(X_0, X_h) \\ \sigma_{xy} &= Cov(X_0, Y_0) + \sum_{h=1}^{\infty} Cov(X_0, Y_h) + \sum_{h=1}^{\infty} Cov(X_h, Y_0) \\ \sigma_{yx} &= Cov(X_0, Y_0) + \sum_{h=1}^{\infty} Cov(X_h, Y_0) + \sum_{h=1}^{\infty} Cov(X_0, Y_h) \\ \sigma_{yy} &= Var(Y_0) + 2 \sum_{h=1}^{\infty} Cov(Y_0, Y_h). \end{aligned} \tag{2.3}$$

The expressions of the covariance terms in these summations may be obtained once we specify the distribution of $\{X(n), n \geq 0\}$.

Let us consider the estimator,

$$\bar{A}_n = \sum_{i=1}^{n} X_i / \sum_{i=1}^{n} (X_i + Y_i).$$

As $\{X(n)\}$ is stationary and ergodic, we have $\bar{X}_n \longrightarrow \mu$ a.s., $\bar{Y}_n \longrightarrow \theta$ a.s. and hence

$$\bar{A}_n \longrightarrow \mu/(\mu + \theta) = A, \text{a.s.}, \tag{2.4}$$

where a.s. stands for almost sure.

If we define $g(x, y) = x/(x + y)$ then $g(\bar{X}_n, \bar{Y}_n) = \bar{A}_n$ and hence by using results from Serfling (1981, p. 122) we can show that

$$\sqrt{n} \left(\bar{A}_n - \frac{\mu}{\mu + \theta} \right) \xrightarrow{\mathcal{L}} N(0, \tau^2), \tag{2.5}$$

where τ^2 is a positive constant given by

$$\tau^2 = \{\theta^2 \sigma_{xx} + \mu^2 \sigma_{yy} - \mu\theta(\sigma_{xy} + \sigma_{yx})\}/(\mu + \theta)^4. \tag{2.6}$$

That is $\bar{A}_n$ is a CAN estimator of $A = \mu/(\mu + \theta)$.

For studying the properties of $\bar{A}(t)$ let $Z_n = X_n + Y_n$ and $S_n = Z_1 + Z_2 + \cdots + Z_n$, $n = 1, 2, \cdots$. If we define $N(t) = inf\{n : S_n \leq t\}$, then $N(t)$ counts the number of cycles completed in the interval $[0,t]$. Clearly as $t \to \infty$, $N(t) \to \infty$ a.s. and $S_n/n \to \mu+\theta$ a.s. as $n \to \infty$. This in turn implies that $\frac{N(t)}{t} \to \frac{1}{\mu+\theta}$ a.s. as $t \to \infty$. Now following the notations of Jie Mi (1995) the total up time $\alpha(t)$ in the interval $[o,t]$ may be represented as

$$\alpha(t) = \lambda(t) \sum_{j=1}^{N(t)+1} X_j + (1-\lambda(t))\{\sum_{j=1}^{N(t)} X_j + t - S_{N(t)}\}, \tag{2.7}$$

where

$$\lambda(t) = I(S_{N(t)} + X_{N(t)+1} \leq t < S_{N(t)}). \tag{2.8}$$

We have $Z_{N(t)+1} = S_{N(t)+1} - S_{N(t)}$ and hence $\frac{Z_{N(t)+1}}{t} \to 0$ a.s. as $t \to \infty$. Moreover, it is also clear that $0 \leq t - S_{N(t)} \leq Z_{N(t)+1}$ so that $\frac{t-S_{N(t)}}{t} \to 0$ a.s. as $t \to \infty$. Using these results in (2.7) we can show that as $t \to \infty$,

$$\frac{\alpha(t)}{t} = \bar{A}(t) \to \frac{\mu}{\mu+\theta} \text{a.s.}$$

Hence $\bar{A}(t)$ is strongly consistent for A.

Now let us define

$W_j = \theta X_j - \mu Y_j$, $j = 0,1,2,\cdots$. Note that the sequence $\{W_n, n \geq 0\}$ is also stationary, ergodic and ϕ-mixing with $E(W_n) = 0$. From (2.1) and Serfling (1981, p. 122) it follows that as $n \to \infty$,

$$\sqrt{n}(\theta\bar{X}_n - \mu\bar{Y}_n) \xrightarrow{\mathcal{L}} N(0,\gamma^2), \tag{2.9}$$

where

$$\gamma^2 = \sigma_{xx}\theta^2 + \sigma_{yy}\mu^2 - (\sigma_{xy} + \sigma_{yx})\mu\theta$$

.

Now using the central limit theorem for random sum of ϕ-mixing r.v's (cf. Billingsley (1968), p. 180) we have as $t \to \infty$,

$$\sum_{j=1}^{N(t)+1} W_j/\sqrt{N(t)+1} = \frac{1}{\sqrt{N(t)+1}} \sum_{j=1}^{N(t)+1} (\theta X_j - \mu Y_j) \xrightarrow{\mathcal{L}} N(0,\gamma^2). \tag{2.10}$$

Let us write

$$(\mu+\theta)\sum_{j=1}^{N(t)+1} X_j - \mu t = \sum_{j=1}^{N(t)+1} (\theta X_j - \mu Y_j) + \left[\sum_{j=1}^{N(t)+1} (X_j + Y_j) - t\right] \mu \qquad (2.11)$$

For $\in > 0$, consider

$$P[\frac{X_n + Y_n}{\sqrt{n}} > \in] \le P[X_n > \frac{\in \sqrt{n}}{2}] + P[Y_n > \frac{\in \sqrt{n}}{2}] \le \{E(X_1^{2+\delta}) + E(Y_1^{2+\delta})\}\frac{1}{n^{1+\delta/2}}.$$

The last inequality is obtained using Markov inequality and the hypothesis of the theorem. As the expectations on the right hand side are finite, it follows that

$$\sum_{n=1}^{\infty} P\left[\frac{X_n + Y_n}{\sqrt{n}} > \in\right] < \infty.$$

This shows that $\frac{X_n+Y_n}{\sqrt{n}} \to 0$ a.s. as $n \to \infty$, and $\frac{X_{N(t)}+Y_{N(t)}}{\sqrt{N(t)}} \to 0$ a.s. as $t \to \infty$.

Hence

$$0 \le \frac{S_{N(t)+1} - t}{\sqrt{N(t)+1}} \le \frac{X_{N(t)+1} + Y_{N(t)+1}}{\sqrt{N(t)+1}} \to 0 \quad \text{a.s. as} \quad t \to \infty$$

.

That is,

$$\{\sum_{j=1}^{N(t)+1} (X_j + Y_j) - t\}/\sqrt{N(t)+1} \to 0 \quad \text{a.s.} \quad \text{as} \quad t \to \infty$$

.

Hence from (2.10) and (2.11) we get as $t \to \infty$,

$$\frac{(\mu+\theta)\sum_{j=1}^{N(t)+1} X_j - t\mu}{\sqrt{N(t)+1}} \xrightarrow{\mathcal{L}} N(0, \gamma^2). \qquad (2.12)$$

Now by rewriting (2.7) in a convenient form and using the above convergence result it can be shown that as $t \to \infty$,

$$\frac{\bar{A}(t) - \mu/(\mu+\theta)}{(\mu+\theta)^{-1}[\sqrt{\{N(t)+1\}/t}]} \xrightarrow{\mathcal{L}} N(0, \gamma^2)$$

.

After simplification it becomes as $t \to \infty$,

$$\sqrt{t}[\bar{A}(t) - \mu/(\mu+\theta)] \xrightarrow{\mathcal{L}} N(0, \frac{\gamma^2}{(\mu+\theta)^3}). \tag{2.13}$$

Thus $\bar{A}(t)$ is also CAN for A. This completes the proof.

3. AVAILABILITY ESTIMATION FOR A BEAR (1) PROCESS

In this section we discuss the applications of the results in Section 2 for a BEAR(1) model. Before defining the BEAR(1) model we need to describe the following. The random vector (N_1, N_2) has a bivariate geometric distribution defined by Block (cf. Block, Langberg and Stoffer (1988)) if

$$P(N_1 > n_1, N_2 > n_2) = \begin{cases} p_{11}^{n_1}(p_{01}+p_{11})^{n_2-n_1}, & n_2 \geq n_1 \\ p_{11}^{n_2}(p_{10}+p_{11})^{n_1-n_2}, & n_2 \leq n_1 \\ \quad n_1, n_2 = 1, 2, \ldots\ , \end{cases} \tag{3.1}$$

where $0 \leq p_{ij} \leq 1$, $i, j = 0, 1$ such that $p_{00}+p_{10}+p_{01}+p_{11} = 1$, $0 < p_{01}+p_{11} < 1$, $0 < p_{10}+p_{11} < 1$.

A random vector (E_1, E_2) is said to have a bivariate exponential distribution if each of its marginal distribution is univariate exponential. We denote a bivariate exponential random vector with mean $(1/\lambda_1, 1/\lambda_2)$ and correlation coefficient ρ by BVE $(\lambda_1, \lambda_2, \rho)$. Now let us define the BEAR(1) model.

Let $\{E(n) = (E_{1n}, E_{2n}),\ n = 0, \pm1, \pm2, \ldots\}$ be a sequence of iid BVE $(\lambda_1, \lambda_2, \rho)$ and (N_1, N_2) be a bivariate geometric random vector specified by (3.1) which is independent of $E(n)$ for all n. Let $\{(I_1(n), I_2(n))\}$ be a sequence of iid bivariate Bernoulli random vectors with $Pr[I_1(n) = i, I_2(n) = j] = p_{ij}$, $i, j = 0, 1$, where p_{ij}'s are as in (3.1). Define

$$X(n) = \begin{cases} E(0), & n = 0 \\ A(n)X(n-1) + BE(n), & n = 1, 2, \ldots, \end{cases} \tag{3.2}$$

where

$$E(0) = \left(\pi_1 \sum_{j=1}^{N_1} E_{1,-j},\ \pi_2 \sum_{j=1}^{N_2} E_{2,-j} \right), \tag{3.3}$$

$$A(n) = \begin{pmatrix} I_1(n) & 0 \\ 0 & I_2(n) \end{pmatrix}, \quad B = \begin{pmatrix} \pi_1 & 0 \\ 0 & \pi_2 \end{pmatrix},$$

$$P_{01} + P_{00} = \pi_1, \quad P_{01} + P_{00} = \pi_2$$

such that $\{A(n)\}$ and $\{E(n)\}$ are mutually independent iid sequences of random vectors. Then for each $n \geq 0$, $X(n)$ has BVE $(\lambda_1, \lambda_2, \rho)$ distribution. It can be shown that $E(0)$ defined by (3.3) has BVE $(\lambda_1, \lambda_2, \rho)$ distribution. The sequence $\{X(n), n \geq 0\}$ defined by (3.2) is referred to as a BEAR(1) process. The common dispersion matrix of $\{X(n)\}$ is given by

$$\Sigma_X = \begin{pmatrix} \lambda_1^{-2} & \rho(\lambda_1\lambda_2)^{-1} \\ \rho(\lambda_1\lambda_2)^{-1} & \lambda_2^{-2} \end{pmatrix}. \tag{3.4}$$

The autocovariance matrix $\Gamma_X(k)$ of $\{X(n)\}$ becomes

$$\Gamma_X(k) = \text{Cov}(X(n), X(n+k)) = \begin{pmatrix} (1-\pi_1)^k/\lambda_1^2 & \rho(1-\pi_1)^k/\lambda_1\lambda_2 \\ \rho(1-\pi_2)^k/\lambda_1\lambda_2 & (1-\pi_2)^k/\lambda_2^2 \end{pmatrix}, \quad k = 0, 1, 2, \dots \, . \tag{3.5}$$

In the following theorem we prove an important property of the BEAR(1) sequence.

Theorem 3.1: The BEAR(1) sequence $\{X(n)\}$ defined by (3.2) is stationary, ergodic and ϕ-mixing with mixing parameters

$$\phi(h) = (p_{10} + p_{11})^{h-1} + (p_{01} + p_{11})^{h-1}, \quad h = 1, 2, \dots \, . \tag{3.6}$$

Proof: Recursive use of the model (3.2) tells us that $X(n)$ is a function of the mutually independent r.v.'s $X(0)$, $A(1)$, $A(2), \dots, A(n)$, $E(1)$, $E(2), \dots, E(n)$. Then by using the arguments given in Nicholls and Quinn (1982), it follows that $\{X(n)\}$ is stationary and ergodic.

For proving mixing property let $A \in \sigma\{X(1), X(2), \dots, X(n)\}$ and $B \in \sigma\{X(n+h), X(n+h+1), \dots\}$, where $\sigma\{X(1), X(2), \dots\}$ denotes the minimal sigma field induced by $X(1), X(2), \dots$. The sequence $\{X(n)\}$ is ϕ-mixing if

$$|P(A \cap B) - P(A)P(B)| \leq P(A) \cdot \phi(h)$$

such that $\phi(h) \to 0$ as $h \to \infty$ for every A and B defined above. Note that mixing means asymptotic independence of A and B as $h \to \infty$. If we write the model (3.2) as

$$X(n) = \begin{pmatrix} X_n \\ Y_n \end{pmatrix} = \begin{pmatrix} I_1(n)X_{n-1} + \pi_1 E_{1n} \\ I_2(n)Y_{n-1} + \pi_2 E_{2n} \end{pmatrix}, \tag{3.7}$$

then it follows that the independence of A and B is decided by the r.v.'s $(I_1(n+j), I_2(n+j))$, $j = 1, 2, ..., h-1$. Let $M_i = \sum_{j=1}^{h-1} I_i(n+j)$, $i = 1, 2$ then M_i has a binomial distribution with parameters $h-1$ and $1-\pi_i$. Since the random vectors $(I_1(n), I_2(n))$, $n \geq 1$ are independent, the events A and B are independent if $[M_1 < h-1, M_2 < h-1]$. That is, A and B are conditionally independent given $M_1 < h-1$ and $M_2 < h-1$. Further the r.v.'s $I(A)$ and (M_1, M_2) are independent, where $I(A)$ denotes the indicator function of A. Using these observations we can write,

$$\begin{aligned}
&|P(A \cap B) - P(A) \cdot P(B)| \\
= \; & P(A)|P(M_1 = h-1, M_2 = h-1)\{P(B|A, M_1 = h-1, M_2 = h-1) - P(B|M_1 = h-1, M_2 = h-1)\} \\
& + P(M_1 = h-1, M_2 < h-1)\{P(B|A, M_1 = h-1, M_2 < h-1) - P(B|M_1 = h-1, M_2 < h-1)\} \\
& + P(M_1 < h-1, M_2 = h-1)\{P(B|A, M_1 < h-1, M_2 = h-1) - P(B|M_1 < h-1, M_2 = h-1)\}| \\
& \leq P(A)[P(M_1 = h-1, M_2 = h-1) + P(M_1 = h-1, M_2 < h-1) + P(M_1 < h-1, M_2 = h-1)] \\
& \leq P(A) \cdot \phi(h),
\end{aligned}$$

where $\phi(h) = (p_{11} + p_{10})^{h-1} + (p_{01} + p_{11})^{h-1} \to 0$ as $h \to \infty$. This completes the proof.

Note that if $\phi(h)$ is given by (3.6) then $\sum_{h=1}^{\infty} \phi^{1/2}(h) < \infty$. Further for BEAR(1) sequence all the moments of X_n and Y_n are finite. In this case $\mu = 1/\lambda_1, \theta = 1/\lambda_2$ and the auto-covariance matrix of X_n is given by (3.5). Thus we have proved that all the conditions of Theorem 2.1 hold for the BEAR(1) sequence.

Hence,

$$\sqrt{n}(\bar{X}_n - \lambda_1^{-1}, \;\; \bar{Y}_n - \lambda_2^{-1}) \xrightarrow{\mathcal{L}} N_2(o, \Sigma_2)$$

,

where the elements of $\sum_2$ can be simplified using (2.2), (2.3) and (3.5). Thus we get

$$\Sigma_2 = \begin{pmatrix} \frac{2-\pi_1}{\pi_1\lambda_1^2} & \frac{\rho}{\lambda_1\lambda_2}(\frac{\pi_1+\pi_2-\pi_1\pi_2}{\pi_1\pi_2}) \\ \frac{\rho}{\lambda_1\lambda_2}(\frac{\pi_1+\pi_2-\pi_1\pi_2}{\pi_1\pi_2}) & \frac{2-\pi_2}{\pi_2\lambda_2^2} \end{pmatrix} \tag{3.8}$$

This in turn implies that, for a BEAR(1) sequence the estimators $\bar{A}_n$ and $\bar{A}(t)$ are CAN for the limiting availability $A = \lambda_2/(\lambda_1+\lambda_2)$. In this case

$$\sqrt{n}\left(\bar{A}_n - \frac{\lambda_2}{\lambda_1+\lambda_2}\right) \xrightarrow{\mathcal{L}} N\left(0, \frac{2(1-\rho)(\lambda_1\lambda_2)^2}{\pi_1\pi_2(\lambda_1+\lambda_2)^4}(\pi_1+\pi_2-\pi_1\pi_2)\right) \tag{3.9}$$

as $n \to \infty$

and

$$\sqrt{t}\left[\bar{A}(t) - \frac{\lambda_2}{\lambda_1+\lambda_2}\right] \xrightarrow{\mathcal{L}} N\left(0, \frac{2\lambda_1\lambda_2(1-\rho)(\pi_1+\pi_2-\pi_{1\pi_2})}{(\lambda_1+\lambda_2)^3\pi_1\pi_2}\right) \tag{3.10}$$

as $t \to \infty$.

4. SENSITIVITY OF THE ESTIMATORS FOR BEAR(1) PROCESS.

We have already shown that the asymptotic variances of the estimators are given in (3.9) and (3.10) when the process is BEAR(1). It is interesting to see how sensitive these variances are for different values of the marginal autocorrelations of $\{X_n\}$ and $\{Y_n\}$. It can be shown that if $\{(X_n, Y_n),\ n \geq 0\}$ is a sequence of iid BVE $(\lambda_1, \lambda_2, \rho)$, then

$$\gamma_n^* = AV(\bar{A}_n) = 2(1-\rho)(\lambda_1\lambda_2)^2/n(\lambda_1+\lambda_2)^4$$

and $\gamma_t^* = AV(\bar{A}(t)) = 2(1-\rho)\lambda_1\lambda_2/t(\lambda_1+\lambda_2)^3$.

Let γ_n and γ_t be the asymptotic variances of $\bar{A}_n$ and $\bar{A}(t)$ (see (3.9) and (3.10)) under BEAR(1) model. Thus we consider the ratio

$$\begin{aligned} \frac{\gamma_n}{\gamma_n^*} &= \frac{\text{Asymptotic variance under BEAR(1) model}}{\text{Asymptotic variance under iid } BVE(\lambda_1,\lambda_2,\rho) \text{ set up}} \\ &= \frac{\pi_1+\pi_2-\pi_1\pi_2}{\pi_1\pi_2} = \frac{1}{1-\beta_1} + \frac{1}{1-\beta_2} - 1 = \frac{\gamma_t}{\gamma_t^*} \end{aligned}$$

where $\beta_1 = 1 - \pi_1$ and $\beta_2 = 1 - \pi_2$ are the marginal lag 1 autocorrelations of the $\{X_n\}$ and $\{Y_n\}$ sequences respectively. Note that the ratio is always greater than unity and we show this for few values of π_1 and π_2.

β_1 \ β_2	0.2	0.5	0.7	0.9
0.2	1.5	2.25	3.58	10.25
0.5	-	3	4.33	11
0.7	-	-	5.66	12.33
0.9	-	-	-	19

We see that as the marginal auto-correlations β_1 and β_2 increase, the ratio increases. In fact if $\beta_1 = \beta_2 = 0.5$ the ratio is 3. This means that under the assumption of independence the variance is drastically under estimated if the true process is BEAR(1). This could lead to eroneous conclusions. Even when the correlation is small $(\beta_1 = \beta_2 = 0.2)$ the ratio is 1.5 indicating under estimation of 50%.

5. CONCLUDING REMARKS

We have discussed the estimation of limiting availability when the operating and repair times of a system form a stationary ϕ-mixing bivariate sequence of random vectors. The proposed estimators of limiting availability are proved to be consistent and asymptotically normal. The general theory is applied for a stationary BEAR(1) sequence. These are compared with estimators under the iid set up in terms of their asymptotic variances. It is observed that when the true model is BEAR(1), the assumption of iid sequence under estimates the variance of the estimators significantly.

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