Worst-Case Copulas, Mass Transportation and Wrong-Way Risk in Counterparty Credit Risk Management

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1 Abstract

We study the problem of finding the worst-case joint distribution of a set of risk factors given prescribed multivariate marginals with nonlinear loss function.

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The method has applications to any situation where marginals are provided, and bounds need to be determined on total portfolio risk. This arises in many financial contexts, including pricing and risk management of exotic options, analysis of structured finance instruments, and aggregation of portfolio risk across risk types. Applications to counterparty credit risk are highlighted, and include assessing wrong-way risk in the credit valuation adjustment, and counterparty credit risk measurement.

2 Introduction

In recent years counterparty credit risk management has become an increasingly important topic for both regulators and participants in over-the-counter derivatives markets. Indeed, even before the financial crisis, the Counterparty Risk Management Policy Group noted that counterparty risk is “probably the single most important variable in determining whether and with what speed financial disturbances become financial shocks, with potential systemic traits” (CRMPG [2005]). This concern over counterparty credit risk as a source of systemic stress has been reflected in developments in the Basel Capital Accords (BCBS [2006], BCBS [2011], see also Section 3 below).

Counterparty Credit Risk (CCR) is defined as the risk owing to default or the change in creditworthiness of a counterparty before the final settlement of the cash flows of a contract. An examination of the problems of measuring and managing this risk reveals a number of key features. First, the risk is bilateral, and current exposure can lie either with the institution or its counterparty. Second, the evaluation of exposure must be done at the portfolio level, and must take into
account relevant credit mitigation arrangements, such as netting and the posting of collateral, which may be in place. Third, the exposure is stochastic, and is contingent on current market risk factors, as well as the creditworthiness of the counterparty, and credit mitigation.\footnote{Since in general the risk is bilateral, in the case of pricing contracts subject to counterparty credit risk (i.e. computing the credit valuation adjustment), the creditworthiness of both parties to the contract is relevant. Thus the possible dependence between credit risk and exposure, known as wrong-way risk, is an important modelling consideration. In this paper, we take a unilateral perspective, focusing exclusively on the creditworthiness of the counterparty.} Finally, the problem is horrendously complex. To calculate risk measures for counterparty credit risk, one requires the joint distribution of all market risk factors affecting the portfolio of (possibly tens of thousands of) contracts with the counterparty, as well as the creditworthiness of both counterparties, and values of collateral instruments posted. It is nearly impossible to estimate this joint distribution accurately. Consequently, we are faced with a problem of risk management under uncertainty, where at least part of the probability distribution needed to evaluate the risk measure is unknown.

Fortunately, we do have partial information to aid in the calculation of counterparty credit risk. Most financial institutions have in place models for simulating the joint distribution of counterparty exposures, created (for example) for the purpose of enforcing exposure limits. Additionally, internal models for assessing default probabilities, and credit models (both internal and regulatory) for assessing the joint distribution of counterparty defaults are available. We can regard the situation as one where we are given the (multi-dimensional) marginal distributions of certain risk factors, and need to evaluate portfolio risk for a loss variable that depends on their joint distribution. The Basel Accord (BCBS [2006]) has employed a simple adjustment based on the “alpha multiplier” to address this problem. A stress-testing approach, employing different copulas and financially
relevant “directions” for dependence between the market and credit factors is presented in Garcia-Cespedes et al. [2010] and Rosen and Saunders [2010]. This method allows for a computationally efficient evaluation of counterparty credit risk, as it leverages pre-computed portfolio exposure simulations.\(^2\)

In this paper, we investigate the problem of determining the worst-case joint distribution, i.e. the distribution that has the given marginals, and produces the highest risk measure. Since the marginals are specified, this is equivalent to finding the worst-case copula with the prescribed (possibly multi-variate) marginals. This approach is motivated by a desire to have conservative measures of risk, as well as to provide a standard of comparison against which other methods may be evaluated. While in this paper we focus on the application to counterparty credit risk, the problem formulation is completely general, and can be applied to other situations in which marginals for risk factors are known, but the joint distribution is unknown. Finally, we note that we work with Conditional Value-at-Risk (CVaR), rather than Value-at-Risk (VaR), which is the risk measure that currently determines regulatory capital charges for counterparty credit risk in the Basel Accords (BCBS [2006], BCBS [2011]). The motivation for this choice is twofold. First, it yields a computationally tractable optimization problem for the worst-case joint distribution, which can be solved using linear programming. Second, the Basel Committee is considering replacing VaR with CVaR as the risk measure for determining capital requirements for the trading book (BCBS [2012]).

Model uncertainty, and problems with given marginal distributions or partial in-

\(^2\)Generally, the computational cost of the algorithms is dominated by the time required for evaluating portfolio exposures - which involves pricing thousands of derivative contracts under at least a few thousand scenarios at multiple time points - rather than from the simulation of portfolio credit risk models.
formation have been studied in many financial contexts. One example is the pricing of exotic options, where no-arbitrage bounds may be derived based on observed prices of liquid instruments. Related studies include Bertsimas and Popescu [2002], Hobson et al. [2005a], Hobson et al. [2005b], Laurence and Wang [2004], Laurence and Wang [2005], and Chen et al. [2008], for exotic options written on multiple assets \((S_1, \ldots, S_T)\) observed at the same time \(T\). The approach closest to the one we take in this paper is that of Beiglbock et al. [2011], in which the marginals \((\Psi(S_T^1), \ldots, \Psi_T^k)\) are assumed to be given, and infinite-dimensional linear programming is employed to derive price bounds. There is also a large literature on deriving bounds for joint distributions with given marginals, and corresponding VaR bounds. For a recent survey, see Puccetti and Rüschendorf [2012]. The problems considered in this paper are distinguished by the combination of the facts that we use an alternative risk measure (CVaR), which are provided with multivariate (non-overlapping) marginal distributions, and have losses that are a non-linear (and non-standard) function of the underlying risk factors.

Haase et al. [2010] propose a model-free method for bilateral credit valuation adjustment; their proposed approach does not rely on any specific model for the joint evolution of the underlying risk factors. Talay and Zhang [2002] treat model risk as a stochastic differential game between the trader and the market, and prove that the value function is the viscosity solution of the corresponding Isaacs equation. Avellaneda et al. [1995], Denis et al. [2011] and Denis and Martini [2006] consider pricing under model uncertainty in a diffusion context. Recent works on risk measures under model uncertainty include Kervarec [2008] and Bion-Nadal and Kervarec [2012].
The remainder of the paper is structured as follows. Section Two presents the problem of finding worst-case joint distributions for risk factors with given marginals, and illustrates how this reduces to a linear programming problem when the risk measure is given by CVaR and the distributions are discrete. Section Three presents the application of this general approach to counterparty credit risk in the context of the model underlying the CCR capital charge in the Basel Accord. Section Four presents a numerical example using a real portfolio, and Section Five presents conclusions and directions for future research.

3 Worst-Case Joint Distributions and Mass Transportation Problems

Let \( Y \) and \( Z \) be two vectors of risk factors. We assume that the multi-dimensional marginals of \( Y \) and \( Z \), denoted by \( P_Y \) and \( P_Z \) respectively, are known, but that the joint distribution of \((Y, Z)\) is unknown (Note: in the context of counterparty credit risk management discussed in the next section, \( Y \) and \( Z \) will be vectors of market and credit factors respectively). Portfolio losses are defined to be \( L = L(Y, Z) \), where in general this function may be non-linear. We are interested in determining the joint distribution of \((Y, Z)\) that maximizes a given risk measure \( \rho \):

\[
\max_{\pi(Y, Z)} \rho (L(Y, Z)) \tag{1}
\]

where \( \pi(Y, Z) \) is the set of all possible joint distributions of \( Y \) and \( Z \), matching their previously defined marginal distributions. More explicitly, \( \pi(Y, Z) \) is the set of all probability distributions such that \( \Pi_y Q = P_Y \) and \( \Pi_z Q = P_Z \), where
Π_{y,z} denote the projections that take the joint distribution to its (multi-variate) marginals. While we are mainly interested in other applications, we note that bounds on instrument prices can be derived within the above formulation by taking the risk measure to be the expectation operator.

It is well known that given a time horizon and confidence level \( \alpha \), Value-at-Risk (VaR) is defined as the \( \alpha \)-percentile of the loss distribution over the specified time horizon. The shortcomings of VaR as a risk measure are well known. An alternative measure that addresses many of these shortcomings is \textit{Conditional Value at Risk (CVaR)}, also known as \textit{tail VaR} or \textit{Expected Shortfall}. If the loss distribution is continuous, CVaR is the expected loss given that losses exceed VaR. More formally, we have the following.

\textbf{Definition 3.1.} For the confidence level \( \alpha \) and loss random variable \( L \), the Conditional Value at Risk at level \( \alpha \) is defined by

\[
\text{CVaR}_\alpha(L) = \frac{1}{1 - \alpha} \int_0^1 \text{VaR}_\xi(L) \, d\xi
\]

We will use the following result, which is part of a Theorem from Schied [2008] (translated into our notation). Here \( L \) is regarded as a random variable defined on a probability space \( (\Omega, \mathcal{F}, \mathbb{P}) \).

\textbf{Theorem 3.1.} \textit{CVaR}_\alpha(L) can be represented as

\[
\text{CVaR}_\alpha = \sup_{Q \in \mathcal{Q}_\alpha} E_Q[L]
\]

where \( \mathcal{Q}_\alpha \) is the set of all probability measures \( Q \ll \mathbb{P} \) whose density \( dQ/d\mathbb{P} \) is \( \mathbb{P} \)-a.s. bounded by \( 1/(1 - \alpha) \).
Applying the above result, with $\rho = \text{CVaR}_\alpha$, the worst case copula problem stated in (1) can be conveniently reformulated as:

$$\max_{Q, P \in \mathcal{P}} \mathbb{E}_Q[L]$$  

$$\Pi_y P = P_Y$$

$$\Pi_z P = P_Z$$

$$\frac{dQ}{dP} \leq \frac{1}{1 - \alpha} \text{ a.s.}$$

where $\mathcal{P}$ is the set of all probability measures, and the final constraint includes the stipulation that the corresponding density exists.

In many practical cases the marginal distributions will be discrete, either due to a modelling choice, or because they arise from the simulation of separate continuous models for $Y$ and $Z$. In this case, the marginal distributions can be represented by $p_m = \mathbb{P}_Y(Y = y_m), m = 1, \ldots, M$, and $q_n = \mathbb{P}_Z(Z = z_n), n = 1, \ldots, N$. Any joint distribution of $(Y, Z)$ is then specified by the quantities $p_{nm} = \mathbb{P}(Z = Z_n, Y = Y_m)$, and the worst-case CVaR optimization problem above can be further simplified to:

$$\max_{\mu, p} \frac{1}{1 - \alpha} \sum_{n,m} L_{nm} \cdot \mu_{nm}$$

$$\sum_n p_{nm} = p_m \quad m = 1, \ldots, M$$

$$\sum_m p_{nm} = q_n \quad n = 1, \ldots, N$$

$$\sum_{n,m} \mu_{nm} = 1 - \alpha$$

$$p_{nm} \geq \mu_{nm} \geq 0$$
This is a linear programming problem, and has the general form of a mass transportation problem. Note that since the sum of each marginal distribution is equal to one, we do not have to include the additional constraint that the total mass of \( p \) equal one. Excluding the bounds, this LP has \( 2mn \) variables and \( m + n + 1 + nm \) constraints. Consequently, the above formulation can lead to large linear programs. For example, in the numerical examples presented later in this paper, we employ marginal distributions with 2,000 market scenarios and 1,000 credit scenarios, yielding \( mn = 2 \times 10^6 \). Specialized algorithms for linear programs that take advantage of the structure of the transportation problem may well be required for problems defined by marginal distributions with larger numbers of scenarios.

\[ \text{Capital} = \text{EAD} \cdot \text{LGD} \cdot \left[ \Phi \left( \frac{\Phi^{-1}(PD) + \sqrt{\rho} \cdot \Phi^{-1}(0.999)}{\sqrt{1-\rho}} \right) \right] \cdot MA(M, PD) \] (4)

Here \( \Phi \) is the cumulative distribution function of a standard normal random variable, and \( MA \) is a maturity adjustment (see BCBS [2006]).\(^3\) The probability of default is estimated based on an internal rating system, while the LGD is the

\(^3\)In the most recent version of the charge, exposure at default may be reduced by current CVA, and the maturity adjustment may be omitted, if migration is accounted for in the CVA capital charge. See BCBS [2006] for details.
estimate of a downturn LGD for the counterparty based on an internal model. Another parameter appearing in the formula, the correlation ($\rho$), is essentially determined as a function of the probability of default.

The exposure at default in the above formula is a constant. However, as noted above, counterparty exposures are in fact stochastic, and potentially correlated with counterparty defaults (wrong-way risk). The Basel accord circumvents this issue by setting $EAD = \alpha \cdot \text{Effective EPE}$, where Effective EPE is a functional of a given simulation of potential future exposures (see BCBS [2006], De Prisco and Rosen [2005] or Garcia-Cespedes et al. [2010] for detailed discussions). The multiplier $\alpha$ defaults to a value of 1.4, however it can be reduced through the use of internal models (subject to a floor of 1.2). Using internal models, a portfolio’s alpha is defined as the ratio of CCR economic capital from a joint simulation of market and credit risk factors ($EC^{Total}$) and the economic capital when counterparty exposures are deterministic and equal to expected positive exposure.\footnote{Expected positive exposure (EPE) is the average of potential future exposure, where averaging is done over time and across all exposure scenarios. See BCBS [2006], De Prisco and Rosen [2005] or Garcia-Cespedes et al. [2010] for detailed formulas.}

$$\alpha = \frac{EC^{Total}}{EC^{EPE}}$$

The numerator of $\alpha$ is economic capital based on a full joint simulation of all market and credit risk factors (i.e. exposures are taken to be stochastic, and they are not treated as independent of the credit factors). The denominator is economic capital calculated using the Basel credit model with all counterparty exposures treated as constant and equal to EPE. For infinitely granular portfolios in which PFEs are independent of each other and of default events, one can assume that exposures are deterministic and given by the EPEs; Calculating $\alpha$ tells us how far
4.1 Worst-Case Copulas in the Basel Credit Model

In this section, we demonstrate how the worst-case copula problem can be applied in the context of the Basel portfolio credit risk model for the purpose of calculating the worst-case alpha multiplier.

In order to calculate the total portfolio loss, we have to determine whether each of the counterparties in the portfolio has defaulted or not. To do so, we define the creditworthiness index of each counterparty $k, 1 \leq k \leq K$, using a single factor Gaussian copula as:

$$CWI_k = \sqrt{\rho_k} \cdot Z + \sqrt{1 - \rho_k} \cdot \epsilon_k$$  \hspace{1cm} (6)

where $Z$ and $\epsilon_k$ are independent standard normal random variables and $\rho_k$ is the factor loading giving the sensitivity of counterparty $k$ to the systematic factor $Z$. If $PD_k$ is the default probability of counterparty $k$, then that counterparty will default if:

$$CWI_k \leq \Phi^{-1}(PD_k)$$  \hspace{1cm} (7)

Assuming that we have $M < \infty$ market scenarios in total, if $y_{km}$ is the exposure to counterparty $k$ under market scenario $m$, the total loss under each market scenario is:

$$L_m = \sum_{k=1}^{K} y_{km} \cdot 1 \{ CWI_k \leq \Phi^{-1}(PD_k) \}$$  \hspace{1cm} (8)

Below we focus on the co-dependence between the market factors $Y$ and the systematic credit factor $Z$. In particular, we assume that the market factors
Y and the idiosyncratic credit risk factors \( \varepsilon_k \) are independent. This amounts to assuming that there is systematic wrong-way risk, but no idiosyncratic wrong-way risk (see Garcia-Cespedes et al. [2010] for a discussion). Define the systematic losses under market scenario \( m \) to be:

\[
L_m(Z) = \mathbb{E}[L_m | Z] = \sum_{k=1}^{K} y_{km} \Phi \left( \frac{\Phi^{-1}(PD_k) - \sqrt{\rho_k} \cdot Z}{\sqrt{1-\rho_k}} \right)
\]

(9)

with probability \( \mathbb{P}(Y = y_m) = p_m \). In the example that follows, \( M = 2000 \). If we discretize the systematic credit factor \( Z \) using \( N \) points and define \( L_{mn} \) as:

\[
L_{mn}(Z) = \mathbb{E}[L_m | Z_n] = \sum_{k=1}^{K} y_{km} \Phi \left( \frac{\Phi^{-1}(PD_k) - \sqrt{\rho_k} \cdot Z_n}{\sqrt{1-\rho_k}} \right)
\]

(10)

\[
\mathbb{P}(Z = z_n) = q_n \quad \text{for} \quad 1 \leq n \leq N
\]

(11)

where \( L_{mn} \) represents the losses under market scenario \( m \), \( 1 \leq m \leq M \), and credit scenario \( n \), \( 1 \leq n \leq N \).

In finding the worst-case joint distribution, we focus on systematic losses, and systematic wrong-way risk, and consequently we need only discretize the systematic credit factor \( Z \). We employ a naive discretization of its standard normal marginal:

\[
\mathbb{P}_Z(Z = z_n) = q_n = \Phi(z_n) - \Phi(z_{n-1}) \quad j = 1, \ldots, N
\]

(12)

where \( z_0 = -\infty \) and \( z_{N+1} = \infty \). In the implementation stage in this paper, we set \( N = 1000 \), and take \( z_j \) to be equally spaced points in the interval \([-5, 5] \).

This enables us to consider the entire portfolio loss distribution under the worst-case copula. In a production implementation focusing on calculating risk at a particular confidence level, there is potentially still much scope for improvement.
over our strategy by choosing a finer discretization of $Z$ in the left tail.

For a given confidence level $\alpha$, the worst-case joint distribution of market and credit factors, $p_{mn}, m = 1, \ldots, M, n = 1, \ldots, N$ can be obtained by solving the LP stated in (3). Having found the discretized worst case joint distribution, we can simulate from the full (not just systematic) credit loss distribution using the following algorithm in order to generate portfolio losses:

1. Simulate a random market scenario $m$ and credit state $N$ from the discrete worst-case joint distribution $p_{nm}$.

2. Simulate the creditworthiness index of each counterparty. Supposing that $z_n$ is the credit state for the systematic credit factor from Step 1, simulate $Z$ from the distribution of a standard normal random variable conditioned to be in $(z_{n-1}, z_n)$. Then generate $K$ i.i.d. standard normal random variables $\varepsilon_k$, and determine the creditworthiness indicators for each counterparty using equation (6).

3. Calculate the portfolio loss for the current market/credit scenario: using the above simulated creditworthiness indices and the given default probabilities and asset correlations, calculate either systematic credit losses using (9) or total credit losses using (8).

5 Example

In this section, we illustrate the use of the worst-case copula problem to calculate an upper bound on the alpha multiplier for counterparty credit risk using a real-
world portfolio of a large financial institution. The portfolio consists of over-the-counter derivatives with a wide range of counterparties, and is sensitive to many risk factors, including interest rates and exchange rates. Results calculated using the worst-case joint distribution are compared to those using the stress-testing algorithm correlating the systematic credit factor to total portfolio exposure, as described in Garcia-Cespedes et al. [2010] and Rosen and Saunders [2010]. More specifically we begin by solving the worst-case CVaR linear program (3) for a given, pre-computed set of exposure scenarios, and the discretization of the (systematic) credit factor in the single factor Gaussian copula credit model described above. We then simulate the full model based on the resulting joint distribution, under the assumption of no idiosyncratic wrong-way risk (so that the market factors and the idiosyncratic credit risk factors remain independent).

The market scenarios are derived from a standard Monte-Carlo simulation of portfolio exposures, so that we have:

\[ p_Y(Y = y_m) = p_m = \frac{1}{M} \quad i = 1, \ldots, M \]  

(13)

### 5.1 Portfolio Characteristics

The analysis that we present in this section is based on a large portfolio of over-the-counter derivatives including positions in interest rate swaps and credit default swaps with approximately 4,800 counterparties. We focus on two cases, the largest 220 and largest 410 counterparties as ranked by exposure (EPE); these two cases account for more than 95% and 99% of total portfolio exposure respectively.

\[ \text{Exposures are single-step EPEs based on a multi-step simulation using a model that assumes mean reversion for the underlying stochastic factors.} \]
Figures 1 and 2 present exposure concentration reports, giving the number of effective counterparties among the largest 220 and 410 counterparties respectively.\textsuperscript{1} The effective number of counterparties for the entire portfolio is shown in Figure 3. As can be seen in these figures the choice of largest 220 and 410 counterparties is justified as the number of effective counterparties for the entire portfolio is 31.

\textsuperscript{1}Counterparty exposures (EPEs) are sorted in decreasing order. Let \( w_n \) be the \( n^{th} \) largest exposure; then the Herfindahl index of the \( N \) largest exposures is defined as:

\[
H_n = \frac{\sum_{n=1}^{N} w_n^2}{(\sum_{n=1}^{N} w_n)^2}
\]

The effective number of counterparties among the \( N \) largest counterparties with respect to total portfolio exposures is \( H_n^{-1} \).
Figure 3: Effective number of counterparties for the entire portfolio.

The exposure simulation uses $M = 2000$, while the systematic credit risk factor is discretized with $N = 1000$ using the method described above. For CVaR calculations, we employ the 99.9% confidence level used for the Basel Capital charge (with the VaR risk measure).

The ranges of individual counterparty exposures are plotted in Figure 4. The 95th and 5th percentiles of the exposure distribution are given as a percentage of the mean exposure for each counterparty. The volatility of the counterparty exposure tends to increase as the mean exposure of the respective counterparties decreases. In other words, counterparties with higher mean exposure tend to be less volatile compared to counterparties with lower mean exposure. Given the above characteristics, we would expect that wrong-way risk could have an important impact on portfolio risk, and that the contribution of idiosyncratic risk will also be significant. The distribution of the total portfolio exposures from the exposure simulation is given in Figure 5. The histogram shows that the portfolio exposure distribution is both leptokurtic and highly skewed.
Figure 4: 5% and 95% percentiles of the exposure distributions of individual counterparties, expressed as a percentage of counterparty mean exposure (counterparties are sorted in order of decreasing mean exposure).

Figure 5: Histogram of total portfolio exposures from the exposure simulation.

### 5.2 Results

To assess the severity of the worst-case joint distribution, and to determine the degree of conservativeness in earlier methods, we compare risk measures calculated using the worst-case joint distribution to those computed based on the stress-testing algorithm presented in Garcia-Cespedes et al. [2010] and Rosen and Saunders [2010]. In this method, exposure scenarios are sorted in an economically meaningful way, and then a two-dimensional copula is applied to simulate
the joint distribution of exposures (from the discrete distribution defined by the exposure scenarios) and the systematic credit factor. The algorithm is efficient, and preserves the (simulated) joint distribution of the exposures. Here, we apply a Gaussian copula, and sort exposure scenarios by the value of total portfolio exposure (this intuitive sorting method has proved conservative in many tests, see Rosen and Saunders [2010] for details). For each level of correlation in the Gaussian copula, we calculate the ratio of risk (as measured by 99.9% CVaR) estimated using the sorting method to risk estimated using the worst-case loss distribution.

![Figure 6: Ratio of systematic CVaR using the Gaussian copula algorithm to systematic CVaR using the worst-case copula for the largest 220 counterparties.](image)

Figures 6 and 7 show the results for the largest 220 and 410 counterparties respectively. Each graph shows the ratio of the CVaR of the systematic portfolio loss to the CVaR calculated using worst-case copula across various levels of market-credit correlation in the Gaussian copula used in the stress testing algorithm of Garcia-Cespedes et al. [2010]. The distribution simulated using the worst-case copula has a higher CVaR s compared to previous simulation methods for the largest 220 and 410 counterparties by 4.9% and 5.2% respectively when the systematic risk factor and market risk factor are fully correlated. The difference is larger for lower levels
of market-credit correlation in the stress testing algorithm. The sorting methods do indeed produce relatively conservative numbers (at high levels of market-credit correlation) for this portfolio. Similar results for calculating CVaR ratios using the total portfolio loss are shown in Figures 8 and 9.
6 Conclusion and future work

In this paper, we studied the problem of finding the worst-case joint distribution of a set of risk factors given prescribed multivariate marginals and a nonlinear loss function. We showed that when the risk measure is CVaR, and the distributions are discretized, the problem can be solved conveniently using linear programming. The method has applications to any situation where marginals are provided, and bounds need to be determined on total portfolio risk. This arises in many financial contexts, including pricing and risk management of exotic options, analysis of structured finance instruments, and aggregation of portfolio risk across risk types. Applications to counterparty credit risk were highlighted, and include assessing wrong-way risk in the credit valuation adjustment, and counterparty credit risk measurement. A detailed example illustrating the use of the algorithm for counterparty risk measurement on a real portfolio was subsequently presented and discussed.

The method presented in this paper will be of great interest to regulators, who...
seek to assess how conservative dependence structures estimated (or assumed) by risk managers in industry actually are. It will also be of interest to risk managers, who can employ it to stress test their assumptions regarding dependence in risk measurement calculations.

There are a number of possible directions that may be pursued in future research. These include assessing the impact of discretization on the final results by addressing the following questions. Can one prove that the discretized problems converge to the ‘true’ worst-case joint distribution; if so, how quickly does convergence occur? Can the speed of convergence be increased by employing importance sampling? Other questions include sensitivity of the worst-case distribution to the choice of risk measure (e.g. CVaR as opposed to VaR, and the particular confidence level chosen for CVaR), and the extension of the algorithm developed in the current paper to other risk measures (e.g. spectral risk measures).

References


