Optimal Reinsurance Analysis from a Crop Insurer’s Perspective

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Purpose - The primary objective of this paper is to analyze the optimal reinsurance contract structure from the crop insurer’s perspective.

Design/methodology/approach - A very powerful and flexible empirical-based reinsurance model is used to analyze the optimal form of the reinsurance treaty. The reinsurance model is calibrated to unique data sets including private reinsurance experience for Manitoba, and loss cost ratio experience for all of Canada, under the assumption of the standard deviation premium principle and conditional tail expectation risk measure.

Findings - The Vasicek distribution is found to provide the best statistical fit for the Canadian LCR data, and the empirical reinsurance model stipulates that a layer reinsurance contract structure is optimal, which is consistent with market practice.

Research limitations/implications - While the empirical reinsurance model is able to reproduce the optimal shape of the reinsurance treaty, the model produces some inconsistencies between the implied and observed attachment points. Future research will continue to explore the reinsurance model that will best recover the observed market practice.

Practical implications - Private reinsurance premiums can account for a significant portion of a crop insurer’s budget, therefore, this study should be useful for crop insurance companies to achieve efficiencies and improve their risk management.

Originality/value - To the best of our knowledge, this is the first paper to show how a crop insurance firm can optimally select a reinsurance contract structure that minimizes its total risk exposure, considering the total losses retained by the insurer, as well as the reinsurance premium paid to private reinsurers.

Keywords: Crop insurance; optimal reinsurance; conditional tail expectation risk measure; premium principle; loss cost ratio.

Article Type: Research paper.
1 Introduction

Crop insurance plays a vital role in the stability and growth of the agriculture sector, helping to ensure a more productive and stable food supply for countries around the world. Crop insurance programs can be found in most countries, including established programs that date back to the 1930's in such countries as Canada and the US, as well as more novice, but rapidly expanding programs in countries such as China and India.

The primary hazard in crop insurance is due to adverse weather, such as a flood, drought, or frost, at times destroying entire crops in a region. While more typical types of insurance with independent risk exposures have losses that tend to be small, frequent, and uncorrelated, crop losses are often quite different and have loss characteristics that resemble catastrophe insurance. For instance, crop losses tend to be less frequent, but larger, and at times also exhibit spatial dependence within regions. In a study by Miranda and Glauber (1997), the level of risk associated with a portfolio of risks was measured to be roughly 20 times larger than the portfolio risk faced in conventional insurance with independent risk exposures. As a result, crop insurance has the considerable challenge of managing a portfolio of risk with unique tail behavior. See also Goodwin (2001), Okhrin et al. (2012), Quiggin (1994), Skees and Barnett (1999), Wang and Zhang (2003).

The aim of crop insurance is to help stabilize farm incomes against production related risks, which stem from the systemic risk associated with adverse weather. One way in which crop insurance helps to stabilize farm incomes, is by assisting farmers in obtaining loans and financing by serving as a form of loan collateral. Additionally, crop insurance encourages farmers to use a sufficient amount of farm inputs on their crops (e.g. fertilizer, pesticides, herbicides), in order to improve yields and increase agricultural productivity. The reason for this is intuitive, for in the event of crop failure, farmers have offset risk and recovered their input costs. Through such mechanisms, crop insurance encourages farmers to take greater risks, which ultimately helps to boost agricultural production, and attain efficiencies in yield output, which helps meet the growing food demands of the world.

For crop insurance in Canada, Porth (2011) shows that risk management methods such
as private reinsurance are needed in order to sufficiently diversify risk and operate a stable crop insurance program. Reinsurance serves as a form of “insurance” for provincial crop insurance organizations, and transfers risk outside of the organization without a promise to repay, which helps to stabilize reserves and premium rates from year to year (Duncan and Meyers, 2000). Reinsurance has been shown to enhance shareholder value, and in some cases reinsurance may be viewed as a less expensive substitute to holding capital (Scordis and Steinorth, 2012). Therefore, reinsurers are often considered to be efficient in spreading losses that are large, infrequent, and spatially dependent, such as in crop insurance. This can be explained by the large portfolios of diversified risk exposures that private reinsurers hold, comprised of many different product types (e.g. aviation, agriculture, earthquake, hurricane, etc.), and across many different geographic regions (e.g. outside of Canada). This is in comparison to the crop insurance program in Canada which is confined to a portfolio of relatively undiversified risk exposures limited to one line of business (various crop types), and geographically to one country.

While private reinsurance may improve diversification for provincial crop insurance organizations, hedging risk through private reinsurance comes at a tradeoff between the cost of the reinsurance premium and the benefits of diversification. This tradeoff demonstrates the importance of optimal reinsurance design. The pioneering work on optimal reinsurance is attributed to Borch (1960). Recent advances on optimal reinsurance that exploits explicit risk measures such as value at risk (VaR) or conditional tail expectation (CTE) can be found in Cai and Tan (2007), Cai et al. (2008), Tan and Weng (2012), Tan et al. (2009, 2011).

The objective of this paper, therefore, is to analyze the optimal reinsurance contract structure from the crop insurer’s perspective, using a very flexible and powerful novel empirical reinsurance model developed by Weng (2009). This is the first paper to show how a crop insurance firm can optimally select a reinsurance contract structure that minimizes their total risk exposure, considering the total losses retained by the insurer, as well as the reinsurance premium paid to private reinsurers. This has substantial implications, as depending on the reinsurance contract structure selected by the insurance firm, the under-
lying risk profile of the crop insurance firm can vary substantially (Porth, 2011). Beyond crop insurers, this investigation is also of great importance to government and agricultural producers (e.g. farmers). This is because inefficiencies can lead to higher costs which may decrease the program competitiveness or participation (Cox and Lin, 2007), and this could lead to information asymmetry, including adverse selection and moral hazard. Asymmetric information has often been cited as one of the major contributors to crop insurance market difficulties (Chambers, 1989; Cohen and Siegelman, 2010; Esuda et al., 2007; Holstrom, 1979; Quiggin, 1994; Nelson and Loehman, 1997; Serra, Goodwin and Featherstone, 2003; Skees and Reed, 1986), therefore, this could also increase the liability to the federal government.

The remainder of this study is organized as follows. Section 2 provides an overview of the crop insurance operation in Canada and the US. Following this, the data set utilized in this study is described in Section 3, and the CTE minimization optimal reinsurance framework for parametric and empirical models under the standard deviation premium principle is presented in Section 4. Section 5 describes the methodology to calibrate the reinsurance premium principle, including estimates of the loading factor, as well as distribution modeling for the loss cost ratio (LCR). Section 6 analyzes the structures of the empirical optimal reinsurance treaties and Section 7 concludes the paper.

2 Crop Insurance Operation

In Canada, federal legislation establishes the national framework for the crop insurance program, however, each provincial crop insurance company operates its own crop insurance program independently of the other nine provinces. This helps to ensure that there is sufficient flexibility for individual provinces to modify the program to meet the needs of their producers (farmers).

Agricultural producers participate in the program voluntarily, and the costs are substantially subsidized by government. The federal government pays 36% of premiums, the provincial government pays 24% of premiums, and producers are responsible for 40% of costs. In addition, each provincial crop insurance company also has the option of participating
in a reinsurance arrangement offered by the federal government under the Farm Income Protection Act (FIPA). If funds in the federal reinsurance account fall short of the required reinsurance payment, the Minister of Finance is responsible for advancing the funds to the reinsurance account. Unlike private reinsurance, there is a promise to repay, and outstanding advances are repaid from future reinsurance premiums. In 2012, five of the ten Canadian provinces participated in the federal-provincial reinsurance arrangement, including Alberta, Saskatchewan, Manitoba, New Brunswick, and Nova Scotia.

In addition to having the option of participating in the federal reinsurance program, provincial crop insurance companies can also purchase reinsurance from the private reinsurance market. In 2012, five of the ten Canadian provinces, British Columbia, Alberta, Manitoba, Ontario, and PEI, purchased private reinsurance. As an example of how crop insurance operates, the AgriInsurance program in Manitoba (MASC, 2012) is considered. AgriInsurance offers a crop production guarantee (in tonnes), including a quality adjustment for most crop types, caused by natural perils (e.g. drought, excessive heat, excessive moisture, excessive rainfall, fire, flood, frost, hail, wind, winterkill, disease, pests, big game, and waterfowl). Producers are able to select crop coverage levels between 50% and 80% of their probable yields, for over 60 different crop types. In the event that actual harvests fall short of the projected production guarantee, a claim is paid on the production shortfall (multiplied by the insurers’ dollar value for the specific crop). See Boyd et al. (2011) for a detailed example of the calculated indemnity payment.

Crop insurance operates very similarly in the US as in Canada, with the exception that it is delivered by private crop insurance firms. In the US, private insurance companies are central to the delivery and risk-sharing of the federal crop insurance program (sharing in the underwriting risks in over $110 billion of insured value, the largest crop insurance program in the world). To ensure that crop insurance is offered in all regions of the country, including high-risk regions, the government provides reinsurance to encourage delivery where it otherwise might not be profitable for the private sector (i.e. where gains are low or exposure is high). The Standard Reinsurance Agreement (SRA) governs the relationship between the
Federal Crop Insurance Corporation (FCIC), and the authorized private insurance companies (16 in 2012), and establishes the terms and conditions under which the FCIC provides subsidy and reinsurance.

The SRA also provides options for reinsurance companies to decrease their risk exposure by ceding a portion of liability to the FCIC and selectively allocating the retained portions of their portfolios among several reinsurance funds including the Assigned Risk Fund (ARF), the Developmental Fund (DF) and Commercial Fund (CF) which are further segregated according to insurance product class (CAT, Revere, and All Other Plans Funds) (Coble et al., 2007; Pai and Boyd, 2010). This structure reflects that a company electing to write crop insurance policies in a state must offer coverage to any farmer in that state, in addition to accepting all rates and provisions set by the FCIC. A detailed breakdown of companies shares in underwriting gains and losses under the SRA can be found in Vedenov et al. (2004).

3 Data

A comprehensive data set representing the entire crop insurance program in Canada was obtained from Agriculture and Agri-Food Canada’s (AAFC) Production Insurance National Statistical System (PINSS). Actual indemnities and liabilities are considered for 279 crop types, across 10 regions, for 32 years over the periods 1978-2009. The ten regions considered include British Columbia, Manitoba, New Brunswick, Newfoundland, Nova Scotia, Ontario, Prince Edward Island, Quebec and Saskatchewan. The LCR was calculated as the ratio of indemnities to coverage (liabilities), and was aggregated across all crop types, for each of the 10 provinces.

4 CTE Minimization Optimal Reinsurance Model

In this section, we first describe the general risk measure based optimal reinsurance model. Next the empirical reinsurance model of Weng (2009) is reviewed as well as the adaption of the model for the investigation of the optimal reinsurance for the Canadian crop data.
4.1 Parametric Reinsurance Models

This subsection discusses the optimal risk measure based reinsurance model that is pertinent to this study. Let $X$ denote the loss random variable initially assumed by an insurer. Assume that the insurer cedes part of its losses, say $f(X)$ satisfying $0 \leq f(X) \leq X$, to a reinsurer, in the reinsurance contract. The insurer thus retains loss $I_f(X) = X - f(X)$, where the function $f(x)$ is known as the ceded loss function. By transferring part of its losses to a reinsurer, the insurer incurs a cost in the form of a reinsurance premium, which is payable to the reinsurer. The reinsurance premium is usually calculated according to some premium principle $\Pi$ so that the premium is $\Pi(f(X))$. Consequently, the sum $T_f(X) := I_f(X) + \Pi(f(X))$ can be interpreted as the total risk of the insurer in the presence of reinsurance. We note that the total risk depends on the characteristics of the loss $X$, the structure of the ceded loss function $f$, as well as the premium principle $\Pi$.

From an insurer’s perspective, it is prudent to select a reinsurance contract so that the resulting total risk exposure $T_f(X)$ is as “small” as possible. Hence, a plausible optimal reinsurance model could be formulated as follows:

$$
\begin{array}{ll}
\min_{f} & \rho(T_f(X)) \\
\text{s.t.} & \Pi(f(X)) \leq \pi, \\
& 0 \leq f(X) \leq X,
\end{array}
$$

(4.1)

where $\rho$ denotes an appropriately chosen risk measure, and $\Pi(f(X)) \leq \pi$ is a reinsurance premium budget constraint, implying that the reinsurance premium is not allowed to exceed a preset budget $\pi$. Risk measures such as VaR, CTE, expected shortfall, among others could be reasonable choices. We emphasize that by a careful choice of the risk measure, the tail risk of the insurer can be managed effectively via the above optimal reinsurance model.

In this paper, we focus on CTE, given that it has the desirable property of coherence (Artner et al., 1997) and that is widely utilized among financial institutions and insurance companies. The precise definition of CTE is as follows.

**Definition 4.1** The CTE of a risk $X$ at the confidence level $(1 - \alpha)$, where $0 < \alpha < 1$, is
defined as

\[ \text{CTE}_\alpha(X) = \frac{1}{\alpha} \int_0^\alpha \text{VaR}_s(X) ds, \]

where \( \text{VaR}_\alpha(X) = \inf\{x \geq 0 : \mathbb{P}(X > x) \leq \alpha\} \).

When the CTE is employed as the risk measure to be minimized, the optimal reinsurance model (4.1) becomes

\[
\begin{align*}
\min & \left\{ \text{CTE}_\alpha(T_f) \equiv \text{CTE}_\alpha(X - f(X) + \Pi(f)) \right\} \\
\text{s.t.} & \quad 0 \leq f(x) \leq x \quad \text{for all } x \geq 0, \\
& \quad \Pi(f) \leq \pi,
\end{align*}
\]

where \( 0 < \alpha < 1 \) with \( 1 - \alpha \) representing the confidence level associated with the risk measure.

The optimal solution to the above CTE minimization problem depends on the selected premium principle \( \Pi \) in addition to the distribution of the underlying risk \( X \). In this paper, we focus on the standard deviation premium principle, given that in addition to the pure expectation of the indemnity it also captures variability. This is particularly important when considering various “layers” in the reinsurance contract structure, since different layers have different risk characteristics (i.e. higher layers will have a less frequent, but more severe probability of loss, compared to lower layers which will have a more frequent, but less severe probability of loss). Thus, the standard deviation premium principle may be more appropriate in capturing the variability of the layers while premium principles such as the expectation principle focus only on the pure expectation of the indemnity.

Under the expectation premium principle, Tan et al. (2011) and Chi and Tan (2011) show that a pure stop-loss reinsurance contract is optimal for the above CTE minimization. However, if the premium is changed to a more complicated premium principle, it can be more challenging to derive the optimal solutions. To resolve this issue, an “empirical reinsurance model” that is based solely on the empirical data is proposed in Weng (2009). This approach is innovative and flexible. It provides a way of investigating optimal reinsurance designs for a wide class of reinsurance models by formulating the reinsurance models directly
based on empirical data. Furthermore, most of the resulting empirical models can be cast as second-order-conic programs so that the solutions can be solved efficiently. Another appealing feature of this method is that we do not need to impose any explicit assumptions about the distribution of the underlying risk. In this paper, we use the empirical-based reinsurance model with the standard deviation premium principle to gain additional insight to the optimal reinsurance structure for the Canadian crop data.

4.2 Empirical Reinsurance Models

By introducing the following auxiliary model

\[
\begin{align*}
\min_{(\xi,f)} & \left\{ G_\alpha(\xi,f) \equiv \xi + \frac{1}{\alpha} \mathbb{E} \left[ \left( X - f(X) + \Pi(f(X)) - \xi \right)_+ \right] \right\} \\
\text{s.t.} & \quad \Pi(f(X)) \leq \pi, \ 0 \leq f(x) \leq x,
\end{align*}
\]

(4.3)

and using Theorem 14 of Rockafellar and Uryasev (2002), Weng (2009) argues that the above formulation is equivalent to (4.2) in the sense that \((\xi^*, f^*)\) solves (4.3) if and only if \(f^*\) solves (4.2) and \(\xi^*\) minimizes \(G_\alpha(\xi, f^*)\). Moreover, the optimal values of both models are equal, which means that the minimum optimal value of model (4.3) is the minimum CTE under the optimal reinsurance contract.

In what follows, we will establish a connection between the above model and the empirical reinsurance model of Weng (2009). Let \(x^T = (x_1, \ldots, x_n)\), where \(x_i\) represents the empirically observed indemnity (before application of any reinsurance) in year \(i\). The key to the empirical approach is to re-formulate the problem of optimal reinsurance by determining an optimal reinsurance coverage \(f_i\) for each observed \(x_i\). This implies that \(f_i\) is the decision variable so that the vector \(f^T := (f_1, f_2, \cdots, f_n)\) represents all of the \(n\) optimization variables. Furthermore, the empirical analog of the theoretical model (4.3) can be formulated as:

\[
\begin{align*}
\min_{(\xi,f)} & \left\{ \hat{G}_\alpha(\xi, f; x) = \xi + \frac{1}{\alpha n} \sum_{i=1}^{n} \left[ (x_i - f_i + \hat{\Pi}(f) - \xi)_+ \right] \right\} \\
\text{s.t.} & \quad \hat{\Pi}(f) \leq \pi, \ 0 \leq f_i \leq x_i \text{ for } i = 1, 2, \cdots, n.
\end{align*}
\]

(4.4)
Note that the “sample average” \( \sum_{i=1}^{n} \left[ \left( x_i - f_i + \hat{\Pi}(f) - \xi \right) + \right] \) embedded in the objective function of the above model is used to approximate \( \mathbb{E} \left[ \left( X - f(X) + \Pi(f(X)) - \xi \right) + \right] \) in the theoretical model (4.3). The objective value \( \hat{G}_a(\xi; f; x) \) can be interpreted as the empirical estimate of \( G_a(\xi; f) \). Note that the former notation has an extra argument to emphasize its explicit dependence on the empirical data \( x \). Similarly, \( \hat{\Pi}(f) \) can be interpreted as the empirical estimate of \( \Pi(f(X)) \) given the decision vector \( f \). Because the above formulation of the reinsurance model is based explicitly on the empirical data \( x \), we refer to the model as the empirical reinsurance model. Suppose \( f^* = (f_1^*, \ldots, f_n^*)^T \) is the solution to the empirical reinsurance model (4.4), then the optimal reinsurance structure is represented by a finite set of points \( (x_i, f_i^*), i = 1, \ldots, n \).

The empirical-based reinsurance model is powerful because the optimization problem (4.4) can be readily solved using existing numerical methods. In particular, Weng (2009) shows that (4.4) can equivalently be cast as a Second-Order-Conic Programming (SOCP) for many premium principles including the standard deviation principle. SOCP is a wide class of common convex optimization problems. Linear programming, convex quadratic programming and quadratically constrained convex quadratic programming can all be regarded as special cases of SOCP, as can many other problems that do not fall into these categories. Detailed discussions on SOCP can be found in Alizadeh and Goldfarb (2003), Ben-Tal and Nemirovski (2001), and Lobo et al. (1998). The SOCP problems can be solved efficiently using several available solvers based on the interior-point method. These softwares include SeDuMi (Sturm, 1999), SDPT3 (Tütüncü et al., 2003) and CVX (Grant and Boyd, 2008). In our research, we use CVX, which is a Matlab based package and is a freeware.

We now demonstrate the applicability of the empirical reinsurance model by assuming the standard deviation premium principle as the underlying premium principle. Theoretically, the standard deviation principle is computed by

\[
\Pi(f) = \mathbb{E}[f] + \theta \sqrt{\text{Var}[f]},
\]

where the loading amount depends on the standard deviation of the ceded loss through
a factor $\theta > 0$. This premium principle is often perceived to be more appropriate than the commonly used expectation premium principle as its premium takes into account the variability of the losses.

The corresponding empirical estimate of (4.5) is given by

$$\hat{\Pi}(f) = \bar{f} + \theta \sqrt{\frac{1}{n} \sum_{i=1}^{n} (f_i - \bar{f})^2}$$

where $\bar{f} = \frac{1}{n} \sum_{i=1}^{n} f_i$. Hence the empirical version of the budget constraint $\hat{\Pi}(f) \leq \pi$ becomes

$$\bar{f} + \theta \sqrt{\frac{1}{n} \sum_{i=1}^{n} (f_i - \bar{f})^2} \leq \pi.$$

(4.6)

By defining $e$ as the $n$-dimensional column vector of ones and

$$Q = \begin{pmatrix} 1 - \frac{1}{n} & -\frac{1}{n} & \cdots & -\frac{1}{n} \\ -\frac{1}{n} & 1 - \frac{1}{n} & \cdots & -\frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{n} & -\frac{1}{n} & \cdots & 1 - \frac{1}{n} \end{pmatrix},$$

(4.7)

it is easy to show that the constraint (4.6) is equivalent to

$$\|Qf\| \leq -\frac{1}{\sqrt{n}} e^T f + \sqrt{n}.$$

Finally the CTE-minimization problem (4.4) can be re-formulated as

$$\min_{(\xi, f,z)} \begin{cases} \xi + \frac{1}{\alpha n} \sum_{i=1}^{n} z_i \\ \text{s.t.} \quad \|Qf\| \leq -\frac{1}{\theta \sqrt{n}} e^T f + \frac{\sqrt{n}}{\theta} \pi, \\ 0 \leq f_i \leq x_i, i = 1, 2, \cdots, n, \\ z_i \geq 0 \text{ and } z_i \geq x_i - f_i + \left[ \bar{f} + \theta \sqrt{\frac{1}{n} \sum_{i=1}^{n} (f_i - \bar{f})^2} \right] - \xi, \ i = 1, 2, \cdots, n. \end{cases}$$

(4.8)

As shown in Weng (2009), the above formulation is a SOCP and hence it can be solved efficiently using software such as CVX.
4.3 Normalized Empirical Reinsurance Models

In the context of crop insurance, it is customary to model the loss cost ratio instead of the loss itself. The loss cost ratio (LCR) is defined as the ratio of the loss to the liability exposure; i.e. $\text{LCR} = \frac{X}{L}$ where $L > 0$ is the assumed liability and $X$, as previously defined, is the loss random variable. Since the incurred loss cannot be larger than the liability exposure $L$, this implies $\text{LCR} \in [0, 1]$. Also, the liability $L$ is predetermined in a crop insurance contract so that once LCR is known, so is the indemnity. For this reason, the reinsurance is typically expressed in term of LCR. The actual reinsurance premium and the reinsurance indemnity is then adjusted appropriately depending on the assumed liability $L$.

To adapt the empirical reinsurance model (4.4) to crop insurance, it is convenient to rewrite the empirical data $x_i$ and the decision variable $f_i$ as

$$x_i = Ly_i \quad \text{and} \quad f_i = Lg_i, \quad i = 1, \ldots, n,$$

where $y_i \in [0, 1]$ and $g_i \in [0, 1]$. If $L$ is the liability exposure, then $y_i$ can be interpreted as the percentage of loss per dollar of liability. Similarly, $g_i$ can be interpreted as the percentage of loss per dollar of liability that is ceded to a reinsurer.

Suppose $y^T = (y_1, \ldots, y_n), g^T = (g_1, \ldots, g_n), \xi = L\tilde{\xi}, \pi = L\tilde{\pi}$ and assume $\hat{\Pi}(g)$ is a positive homogeneous function; i.e. $\hat{\Pi}(cg) = c\hat{\Pi}(g)$ for any constant $c > 0$. Now consider the following re-parameterized formulation of the optimal empirical reinsurance model:

$$\begin{align*}
\min_{(\tilde{\xi}, g)} & \left\{ \hat{G}_\alpha(\tilde{\xi}, g; y) = \tilde{\xi} + \frac{1}{\alpha n} \sum_{i=1}^{n} \left[ (y_i - g_i + \hat{\Pi}(g) - \tilde{\xi})_+ \right] \right\} \\
\text{s.t.} & \quad \hat{\Pi}(g) \leq \tilde{\pi}, \quad 0 \leq g_i \leq y_i \quad \text{for} \quad i = 1, 2, \ldots, n.
\end{align*}$$

It can easily be verified that the above reinsurance model is related to (4.4) in the sense that if $(\tilde{\xi}^*, g^*)$ is a solution to (4.10) with minimum optimal value $\hat{G}_\alpha(\tilde{\xi}^*, g^*; y)$, then $(L\tilde{\xi}^*, Lg^*)$ is also a solution to (4.4) with corresponding minimum optimal value $\hat{G}_\alpha(L\tilde{\xi}^*, Lg^*; x)$. The advantage of analyzing the reinsurance model (4.10) instead of (4.4) is that the former model is normalized to a liability of one dollar. Once we obtain the solutions to (4.10), the optimal reinsurance of any arbitrary liability $L \neq 1$ is easily obtained by scaling the solutions. We
refer to (4.10) as the normalized empirical optimal reinsurance model, which we will use to analyze the optimal reinsurance contract structure for the Canadian crop insurance program.

5 Reinsurance Premium Principle Calibration

Before applying the normalized reinsurance model (4.10) to the Canadian crop data, we need to make some assumptions on how the reinsurance premium will be determined for each loss that is ceded to the reinsurer. As mentioned earlier, despite utilizing the standard deviation premium principle (4.5), we still must determine the loading factor. An easy solution is to assume a plausible loading factor that is consistent with the market practice. Another solution is to calibrate the standard deviation premium principle to the observed reinsurance data. The latter approach, though more tedious, is adopted here since this is more consistent with our objective of deriving the optimal reinsurance from the empirical data.

Subsection 5.1 describes in detail the procedure we use to calibrate the standard deviation premium principle to the observed reinsurance data. While we will focus our discussion primarily on the standard deviation premium principle, it should be emphasized that the calibration method is quite general and hence other premium principles can be calibrated similarly with only minor modifications. Calibrating the premium principle to the reinsurance data also requires that the distribution of LCR is determined; hence Subsection 5.2 is devoted to fitting an appropriate distribution to the Canadian LCR. Five plausible distributions are fit and compared to the LCR data for Canada. It is of interest to note that while the Vasicek distribution is not a well-known distribution (though it has been used in credit risk modeling, see Vasicek, 2002; Anderson and Sidenius, 2004), it has the best fit compared to other popular distributions including beta, exponential, Gamma, and Weibull.

5.1 Calibration Methodology

To calibrate the loading factor $\theta$, a comprehensive data set obtained from MASC is utilized. The data set is comprised of all private reinsurance experience for the province of Manitoba,
including actual liabilities for reinsurance, the amount sold within the reinsurance layer, the reinsurance layer, the corresponding reinsurance premium, the LCR, and the reinsurance claim, for years 2001 through 2012.

Let \( L_i \) and \( x_i \) denote, respectively, the actuarial liability and the actuarial loss in year \( i \) for \( i = 2001, \ldots, 2011 \). Then \( LCR_i = x_i / L_i \in [0, 1] \) corresponds to the year \( i \) loss cost ratio. The layer reinsurance with attachment points \((d, m)\) corresponding to the \( LCR_i \) is defined as

\[
Y_i = \min[(LCR_i - d)_+, m - d], \quad 0 \leq d, m \leq 1,
\]

(5.11)

where \((a)_+ = \max(a, 0)\). By design, if \( LCR_i \leq d \) then \( Y_i = 0 \); if \( LCR_i \geq m \) then \( Y_i \) is capped at \( m - d \). Hence \( d \) and \( m - d \) can be interpreted as the retention percentage and the maximum indemnity percentage, respectively. Note that the reinsurance coverage \( Y_i \) is expressed in terms of per dollar of liability. In a typical reinsurance transaction, the liability exposure must also be defined so that the actual indemnity is given by \( L_i Y_i \). However, as noted earlier it is more convenient to work with the normalized reinsurance model (4.10) where the liability \( L \) is normalized to $1.

For the Manitoba reinsurance data set, the reinsurance treaty for years 2001-2011 is consistently a layer reinsurance. More specifically, the retention attachment point \( d \) remains constant at 15% while the upper attachment point \( d + m \) is 25% for years 2001-2006 and 27.5% for years 2007-2011. To distinguish the two empirically observed layer reinsurance, for \( i = 2001, \ldots, 2011 \) we set \( Y_i^{(j)} = Y_i \) where \( j = 1 \) if \( i \in \{2001, \ldots, 2006\} \) and \( j = 2 \) if \( i \in \{2007, \ldots, 2011\} \). The observed Manitoba reinsurance data over the years 2001-2011 is reported in Table 1. The columns under “\( \Pi(L_i Y_i) \)”, “\( L_i \)”, and “\( LCR_i \)” are obtained directly from the Manitoba reinsurance data. The column “\( \Pi(Y_i) \)” is easily calculated from the observed “\( \Pi(L_i Y_i) \)” and “\( L_i \)” via \( \Pi(Y_i) = \Pi(L_i Y_i) / L_i \) due to the positive homogeneity property of the standard deviation premium principle.

We conclude this subsection by describing the procedure of calibrating the loading factor of the standard deviation premium principle to the Manitoba data set. The following three steps are proposed:
<table>
<thead>
<tr>
<th>Year</th>
<th>$\Pi(L_i Y_i)(\text{millions})$</th>
<th>$L_i(\text{millions})$</th>
<th>$\Pi(Y_i)$</th>
<th>LCR$_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2001</td>
<td>966.9</td>
<td>241.7</td>
<td>0.00693</td>
<td>10.3%</td>
</tr>
<tr>
<td>2002</td>
<td>1,221.6</td>
<td>305.4</td>
<td>0.01146</td>
<td>6.6%</td>
</tr>
<tr>
<td>2003</td>
<td>1,349.5</td>
<td>337.4</td>
<td>0.01149</td>
<td>4.4%</td>
</tr>
<tr>
<td>2004</td>
<td>1,111.5</td>
<td>277.9</td>
<td>0.01152</td>
<td>18.3%</td>
</tr>
<tr>
<td>2005</td>
<td>970.0</td>
<td>242.5</td>
<td>0.01144</td>
<td>30.8%</td>
</tr>
<tr>
<td>2006</td>
<td>1,046.0</td>
<td>261.5</td>
<td>0.01539</td>
<td>4.3%</td>
</tr>
<tr>
<td>2007</td>
<td>1,338.6</td>
<td>368.1</td>
<td>0.01696</td>
<td>5.7%</td>
</tr>
<tr>
<td>2008</td>
<td>1,794.1</td>
<td>493.4</td>
<td>0.01683</td>
<td>3.6%</td>
</tr>
<tr>
<td>2009</td>
<td>1,804.6</td>
<td>496.3</td>
<td>0.01646</td>
<td>8.5%</td>
</tr>
<tr>
<td>2010</td>
<td>1,856.0</td>
<td>510.4</td>
<td>0.01546</td>
<td>11.1%</td>
</tr>
<tr>
<td>2011</td>
<td>2,245.0</td>
<td>617.4</td>
<td>0.01706</td>
<td>15.0%</td>
</tr>
</tbody>
</table>

Table 1: Manitoba Reinsurance Data

1) First, a distribution is fit to the observed Canadian LCR data set. Some elementary analysis on the histogram (see Figure 1 below) of the LCR data reveals that the LCR distribution has a very light tail. Therefore, distributions such as beta, Vasicek, exponential, Gamma, and Weibull are plausible candidates due to their light-tailed property. The best fit distribution according to the Bayesian Information Criterion (BIC) is then selected. We relegate the details of the fit to Subsection 5.2.

2) The fitted LCR distribution from Step 1 enables us to easily compute the indemnity’s moments corresponding to the observed Manitoba layer reinsurance. In particular, we are primarily interested in the following means and standard deviations of the layer reinsurance:

$$a_0 = E(Y_i^{(1)}), \quad a_1 = \sqrt{\text{Var}(Y_i^{(1)})}, \quad b_0 = E(Y_i^{(2)}), \quad b_1 = \sqrt{\text{Var}(Y_i^{(2)})}.$$  \hspace{1cm} (5.12)

Recall that $Y_i^{(1)}$ represents the layer reinsurance for years 2001-2006 with attachment points (15%, 25%) while $Y_i^{(2)}$ denotes the same layer reinsurance but for years 2007-
2011 and with attachment points (15%, 27.5%).

The above estimated parameters, together with a loading factor \( \theta \) of the standard deviation premium principle, allows us to calculate the reinsurance premium of the corresponding observed layer reinsurance; i.e.,

\[
\tilde{\Pi}(Y^{(1)}_i) = a_0 + a_1 \theta
\]

for years 2001-2006 and

\[
\tilde{\Pi}(Y^{(2)}_i) = b_0 + b_1 \theta
\]

for years 2007-2011.

3) Equations (5.13) and (5.14) yield the reinsurance premium, which is a function of the loading factor and the respective layer reinsurance that we observed in practice. Note that in practice we also have the observed reinsurance premiums \( \Pi(Y_i) \), as reported in the fourth column of Table 1. This suggests that a best estimate of the loading factor can be obtained by minimizing a distance between the observed reinsurance premium and the reinsurance premium computed from (5.13) or (5.14) depending on the type of layer reinsurance. In this study, the corresponding best estimate of the loading factor, \( \hat{\theta} \), is obtained by minimizing the following mean square error:

\[
\hat{\theta} = \min_{\theta \in \mathbb{R}} \left\{ \sum_{i=2001}^{2006} \left[ \Pi(Y_i) - \tilde{\Pi}(Y^{(1)}_i) \right]^2 + \sum_{i=2007}^{2011} \left[ \Pi(Y_i) - \tilde{\Pi}(Y^{(2)}_i) \right]^2 \right\}.
\]

### 5.2 Modelling LCR

The focus of this subsection is on modelling the Canadian LCR data. Recall that this step is needed to complete the calibration of the premium’s loading factor. The histogram of the Canadian LCR data in Figure 1 indicates a very light tail over \([0.5, 1]\). Therefore, light-tailed distributions, such as beta, Vasicek, Gamma, exponential and Weibull, are plausible candidates for fitting to the LCR data. Among these five distributions, only beta and Vasicek have support over \([0, 1]\). The remaining distributions have support over the positive.
half real line beyond the possible range for LCR since LCR ∈ [0, 1]. To alleviate this issue, a transformation is applied so that the resulting distribution has support over [0,1] only. Specifically, let the density function and distribution function be \( f^*(x; \theta) \) and \( F^*(x; \theta) \), respectively, with parameter(s) \( \theta \). The transformed density function is defined as follows

\[
f(x; \theta, u) = \frac{uf^*(ux; \theta)}{F^*(u; \theta)}, \quad 0 < x < 1.
\]

The density functions of the five distributions considered are summarized as follows

1) Beta: \( f(x; \alpha, \beta) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha, \beta)} \) with parameters \( \alpha > 0 \) and \( \beta > 0 \).

2) Vasicek: \( f(x; p, \rho) = \sqrt{1-\frac{\rho}{\rho}} \exp \left\{ \frac{1}{2} \left[ (\Phi^{-1}(x))^2 - \left( \frac{\sqrt{1-\rho}\Phi^{-1}(x) - \Phi^{-1}(p)}{\sqrt{\rho}} \right)^2 \right] \right\} \)
with parameters \( 0 < p, \rho < 1 \).

3) Transformed Exponential: \( f(x; \xi) = \frac{1}{1-e^{-1/\xi}} \left[ \frac{1}{\xi} e^{-x/\xi} \right] \) with parameter \( \xi \geq 0 \).

4) Transformed Gamma: \( f(x; \kappa, \xi) = \frac{1}{\Gamma(k)\Gamma(1/\xi; \kappa)} \frac{1}{\lambda^\kappa} x^{\kappa-1} e^{-x/\xi} \) with parameters \( \kappa > 0 \) and \( \xi > 0 \).

5) Transformed Weibull: \( f(x; \kappa, \xi) = \frac{1}{1-e^{-1/\xi^\kappa}} \left[ \frac{\kappa}{\xi} \left( \frac{x}{\xi} \right)^{\kappa-1} e^{-\left( \frac{x}{\xi} \right)^\kappa} \right] \) with parameters \( \kappa > 0 \) and \( \xi > 0 \).

where \( \Phi^{-1}(\cdot) \) is the inverse of the standard normal distribution function, and the gamma and beta functions are defined as follows:

\[
\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1} e^{-t} dt; \quad \Gamma(x; \alpha) = \frac{1}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} e^{-t} dt; \quad B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1},
\]

for \( \alpha, \beta, x > 0 \). Also note that a random variable \( \Phi \left( \frac{\Phi^{-1}(p) - \sqrt{\rho}Z}{\sqrt{1-\rho}} \right) \) has a Vasicek density function, where \( Z \) is a standard normal random variable.

Each distribution above is fit to the Canadian LCR data using the maximum likelihood estimation. For each calibrated distribution, Step 2 of Subsection 5.1 is conducted to obtain best estimates \( a_0, a_1, b_0 \) and \( b_1 \), as defined in (5.12). Step 3 is then implemented to determine
a best estimate loading factor \( \hat{\theta} \). The results of the estimations are summarized in Table 2. With the exception of the exponential distribution, the means, the standard deviations, and the loading factors are in accordance with the other four distributions. In particular, the calibrated loading factor is around 18%.

Along with the estimates \( a_0, a_1, b_0, b_1 \) and \( \hat{\theta} \), we also tabulate \( \frac{a_0 + a_1 \hat{\theta}}{a_0} \) and \( \frac{b_0 + b_1 \hat{\theta}}{b_0} \). These fractions correspond to the ratio of the calibrated standard deviation premium relative to its pure premium for the layer reinsurance with attachment points (15%, 25%) and (15%, 27.5%), respectively. With the exception of the exponential distribution, the ratio is about 1.5.

Once the five distributions have been fit to the Canadian LCR data, the best goodness of fit must be assessed. With the exception of the exponential distribution, the four remaining distributions are quite similar. The goodness of fit can be compared graphically by plotting the fitted probability density functions in Figure 1, along with the histogram of the observed LCR data. This graph suggests that the Vasicek and the Gamma distributions, relative to the other three distributions, provide a better fit to the LCR data. The inadequacy of the exponential distribution, particularly in fitting the left tail of the distribution, is clearly highlighted. A more scientific approach of model selection is to compare the BIC values. Recall that BIC is defined as \( \text{BIC} = -2l + k \ln(n) \), where \( l \) denotes the log-likelihood function evaluated at the MLE parameters, \( k \) is the number of parameters and \( n \) is the length of sample size (i.e., the number of LCR data points). According to the BIC criterion for model selection, the smallest BIC is preferred. The BIC values in Table 2 confirm that the Vasicek distribution has the best overall fit, with the Gamma distribution a close second best.

6 Optimal Agricultural Reinsurance

Based on the Manitoba reinsurance data, the preceding section demonstrated that the best fit LCR distribution is Vasicek and the best estimated loading factor of the standard deviation premium principle is 0.186105536. In this section, the calibrated results are used together
<table>
<thead>
<tr>
<th>Distribution</th>
<th>Para. Estimates</th>
<th>BIC</th>
<th>Moments (Layer Reinsurance)</th>
<th>Loading factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beta $(\alpha, \beta)$</td>
<td>$\hat{\alpha} = 1.7199$</td>
<td>-1128.25</td>
<td>$a_0 = 0.0089$</td>
<td>$\theta = 0.1873$</td>
</tr>
<tr>
<td></td>
<td>$\hat{\beta} = 16.4053$</td>
<td></td>
<td>$a_1 = 0.0237$</td>
<td>$\frac{a_0 + a_1 \hat{\theta}}{a_0} = 1.4999$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_0 = 0.0096$</td>
<td>$\frac{b_0 + b_1 \hat{\theta}}{b_0} = 1.5172$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_1 = 0.0263$</td>
<td></td>
</tr>
<tr>
<td>Vasicek $(p, \rho)$</td>
<td>$\hat{p} = 0.0935$</td>
<td>-1149.63</td>
<td>$a_0 = 0.0088$</td>
<td>$\theta = 0.1861$</td>
</tr>
<tr>
<td></td>
<td>$\hat{\rho} = 0.1488$</td>
<td></td>
<td>$a_1 = 0.0241$</td>
<td>$\frac{a_0 + a_1 \hat{\theta}}{a_0} = 1.5096$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_0 = 0.0096$</td>
<td>$\frac{b_0 + b_1 \hat{\theta}}{b_0} = 1.5278$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_1 = 0.0270$</td>
<td></td>
</tr>
<tr>
<td>Exponential $(\xi)$</td>
<td>$\hat{\xi} = 0.0939$</td>
<td>-1065.12</td>
<td>$a_0 = 0.0124$</td>
<td>$\theta = 0.0223$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$a_1 = 0.0260$</td>
<td>$\frac{a_0 + a_1 \hat{\theta}}{a_0} = 1.0466$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_0 = 0.0140$</td>
<td>$\frac{b_0 + b_1 \hat{\theta}}{b_0} = 1.0410$</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>$b_1 = 0.0257$</td>
<td></td>
</tr>
<tr>
<td>Gamma $(\kappa, \xi)$</td>
<td>$\hat{\kappa} = 1.9415$</td>
<td>-1143.36</td>
<td>$a_0 = 0.0087$</td>
<td>$\theta = 0.1938$</td>
</tr>
<tr>
<td></td>
<td>$\hat{\xi} = 0.0483$</td>
<td></td>
<td>$a_1 = 0.0237$</td>
<td>$\frac{a_0 + a_1 \hat{\theta}}{a_0} = 1.5281$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_0 = 0.0094$</td>
<td>$\frac{b_0 + b_1 \hat{\theta}}{b_0} = 1.5469$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_1 = 0.0264$</td>
<td></td>
</tr>
<tr>
<td>Weibull $(\kappa, \xi)$</td>
<td>$\hat{\kappa} = 1.3760$</td>
<td>-1121.86</td>
<td>$a_0 = 0.0093$</td>
<td>$\theta = 0.1642$</td>
</tr>
<tr>
<td></td>
<td>$\hat{\xi} = 0.1035$</td>
<td></td>
<td>$a_1 = 0.0244$</td>
<td>$\frac{a_0 + a_1 \hat{\theta}}{a_0} = 1.4299$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_0 = 0.0100$</td>
<td>$\frac{b_0 + b_1 \hat{\theta}}{b_0} = 1.4450$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$b_1 = 0.0271$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: LCR Distribution Calibration Results
Figure 1: Comparison of the Fitted Distributions to the Empirical Histogram.
with the Canada data to analyze the optimal agricultural reinsurance contract structure assuming the insurer adopts the empirical CTE-based normalized reinsurance model (4.10). Note again that we analyze the optimal reinsurance by normalizing the liability to $1 so that the solution for any arbitrary liability can be obtained by mere scaling.

Eight different levels of reinsurance premium budgets are considered:

- \( \tilde{\pi} = 0.01 + 0.002k; k = 0, \ldots, 5. \)
- \( \tilde{\pi} = (a_0 + a_1 \hat{\theta}) = 0.0133 \) where \( a_0 = 0.0088, a_1 = 0.0241 \) and \( \hat{\theta} = 0.1861. \)
- \( \tilde{\pi} = L(b_0 + b_1 \hat{\theta}) = 0.0145L \) where \( b_0 = 0.0095, b_1 = 0.0270 \) and \( \hat{\theta} = 0.1861. \)

The last two cases are of interest as they correspond to the best estimates of the Vasicek distribution when (5.13) and (5.14) are fit to the respective Manitoba reinsurance data (see Table 2). These budgets, therefore, can be interpreted as the premium of a layer reinsurance contract structure of the form (5.11), with respective attachment points (15%, 25%) and (15%, 27.5%).

With the specification of the reinsurance budget and together with the Canadian LCR data, CVX software can be used to obtain solutions to the empirical model (4.10). Recall that the solutions are expressed in terms of \( \{(y_i, g_i^*), i = 1, 2, \ldots, n\} \) and these are plotted in Figure 2. The scatter plots reveal that the optimal reinsurance treaties are special cases of the layer reinsurance, consistent with the shape that we observed in practice. When an insurer has a large reinsurance budget (such as 2% of the liability), the optimal treaty appears to be a stop-loss reinsurance, but, with an upper limit no larger than $1 (since the indemnity cannot be larger than the liability). When the reinsurance budget reduces gradually from 2% of the liability to 1% of the liability, the shape of the optimal reinsurance changes from a stop loss reinsurance to a typical layer reinsurance with a decreasing cap. This is to be expected as an insurer spending less on reinsuring risk, implies that a greater portion of risk must be absorbed, particularly in the higher layer of the risk.

In addition to visually assessing the solutions from the empirical-based reinsurance model as depicted in Figure 2, we can gain more insight of the optimal solutions by fitting a curve
to the scatter plot. In particular, we fit a ceded loss function of the form

\[ f(y) = \min\{c(y - d)_+, m - d\} \]

to the solutions \(\{(x_i, g^*_i), i = 1, 2, \ldots, n\}\) for parameters \(c, d\) and \(m\). The results of the fit are reported in Table 3. The values of parameter \(c\) are not reported since their fitted values are almost one. The fitted values again confirm our expectation that as the insurer is willing to spend more on reinsurance premium, the risk retained is smaller, as signified by the decreasing retention level, the increasing limit on indemnity \((m - d)\), as well as the decreasing CTE of the reinsurance model. Finally, it is of interest to note that under the premium budget of 1.33% and 1.45% of liability, the attachment points of the layer reinsurance implied by the data are \((19.3\%, 47.8\%)\) and \((18.7\%, 54.6\%)\), respectively. While these values do not coincide with the attachment points that we observed in practice, the empirical-based reinsurance model is able to recover the same basic shape of the reinsurance treaty.

<table>
<thead>
<tr>
<th>Reinsurance budget</th>
<th>(d)</th>
<th>(m)</th>
<th>CTE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0% of liability</td>
<td>0.2053</td>
<td>0.3745</td>
<td>0.2153</td>
</tr>
<tr>
<td>1.2% of liability</td>
<td>0.1979</td>
<td>0.4320</td>
<td>0.2099</td>
</tr>
<tr>
<td>1.33% of liability</td>
<td>0.1925</td>
<td>0.4780</td>
<td>0.2058</td>
</tr>
<tr>
<td>1.4% of liability</td>
<td>0.1897</td>
<td>0.5261</td>
<td>0.2037</td>
</tr>
<tr>
<td>1.45% of liability</td>
<td>0.1869</td>
<td>0.5456</td>
<td>0.2014</td>
</tr>
<tr>
<td>1.6% of liability</td>
<td>0.1785</td>
<td>1</td>
<td>0.1945</td>
</tr>
<tr>
<td>1.8% of liability</td>
<td>0.1669</td>
<td>1</td>
<td>0.1849</td>
</tr>
<tr>
<td>2.0% of liability</td>
<td>0.1565</td>
<td>1</td>
<td>0.1765</td>
</tr>
</tbody>
</table>

Table 3: Implied Optimal Layer Reinsurance with Attachment Points \((d, m)\) for the Canada LCR Data and the Optimal CTE Values
7 Conclusion

In this paper, an attempt is made to gain additional insight to the optimal reinsurance treaty that we observed in practice. We begin by first calibrating the Canadian LCR data. Of the five distributions we investigated, the less known Vasicek distribution is the best. From the fitted Vasicek distribution, we then calibrate the loading factor of the standard deviation premium principle to the Manitoba reinsurance data. Combining these results, we investigate the optimal reinsurance based upon the CTE-based empirical reinsurance model of Weng (2009), and under different levels of reinsurance budget. While there are some discrepancies between the implied attachment points and the observed attachment points of the reinsurance, it is illuminating to learn that the empirical reinsurance model at least is able to reproduce the optimal shape of the reinsurance that is consistent with what we observed in practice.

There are a number of factors which affect the optimal design of the reinsurance contract structure. Some of these include the risk measure for quantifying an insurer’s risk exposure, the premium principle for computing the reinsurance premium, and the reinsurance budget. The power of the empirical-based reinsurance model is that it gives us a relatively simple way to analyze the optimal reinsurance treaty under a wide class of reinsurance models. Therefore, future research will continue to explore the reinsurance model that will best recover the observed market practice.

References


1) Budget = 1.0% of liability
2) Budget = 1.2% of liability
3) Budget = 1.33% of liability
4) Budget = 1.4% of liability
5) Budget = 1.45% of liability
6) Budget = 1.6% of liability
7) Budget = 1.8% of liability
8) Budget = 2.0% of liability

Figure 2: Empirical Optimal Reinsurance for Increasing Levels of Reinsurance Budget as a Function of Liability of $1