Statistical Assessments of Systemic Risk Measures

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Abstract

In this chapter, we review existing statistical measures for systemic risk and discuss their strengths and weaknesses. Among them we discuss the Conditional Value-at-Risk (CoVaR) introduced by Adrian and Brunnermeier (2010) and the Systemic Expected Shortfall (SES) of Acharya, Pedersen, Philippon and Richardson (2011). Systemic risk is also highly related to financial contagion and we will explain drawbacks and advantages of looking at “coexceedances” (simultaneous extreme events) or at the local changes in “correlation” that have been proposed in the literature on financial contagion (Bae, Karolyi and Stulz (2003), Baig and Goldfajn (1999) and Forbes and Rigobon (2002)).

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1 Introduction and Background on Systemic Risk

During the financial crisis of 2007-2009, worldwide taxpayers had to bailout many financial institutions. Governments are now trying to understand why the regulation failed, why capital requirements were not enough and how a guaranty fund should be built to face the next crisis. To implement such a fund, one needs to understand the risk that each institution represents to the financial system and why regulatory capital requirements were not enough. In the financial and insurance industry, capital requirements have the following common properties. First, they depend solely on the distribution of the institution’s risk and not on the outcomes in the different states of the world. Second, capital requirements and marginal calculations treat each institution in isolation. An important element is missing in the above assessment of risk: it is the dependency between the individual institution and the economy or the financial system. The regulation should “be regulating each bank as a function of both its joint (correlated) risk with other banks as well as its individual (bank-specific) risk” (Acharya (2009)).

There is already an important literature to assess “systemic risk”. One can distinguish two major approaches. One approach consists of using network analysis and works directly on the structure and the nature of relationships between financial institutions in the market. Another approach is to investigate the impact of one institution on the market and its contribution to the global system risk. In this chapter we focus on the second approach to quantify systemic risk.

First Adrian and Brunnermeier (2010) introduced the CoVaR measure. The idea is to compare the Value-at-Risk (VaR) of the system under “normal conditions” and the VaR of the system conditional on the fact that a given institution is under stress. Acharya, Pedersen, Philippon and Richardson (2011) define “the systemic expected shortfall (SES)”, it is linked to the marginal expected shortfall, that is “the average return of each firm during the 5% worst days for the market”. An empirical extension is given by Brownlees and Engle (2011). Systemic risk is linked to financial contagion. Financial contagion refers to the extra dependence in the financial market during times of crisis (often measured as extra correlation). Bae, Karolyi and Stulz (2003) measured financial contagion by studying “coexceedances” or simultaneous occurrences of extreme events (defined as “one that lies either below (above) the 5th (95th) quantile of the marginal return distribution”). Other studies have used changes in correlation to show evidence of contagion but there are considerable statistical difficulties involved in testing hypotheses of changes in correlations across quiet and turbulent periods (see Baig and Goldfajn (1999) and Forbes and Rigobon (2002)). Billio et al. (2010) explain that a single risk measure for systemic risk is not enough. Using monthly equity returns, they discuss how to use correlations,
return illiquidity, principal components analysis, regime switching (fitting a 2-state Markov process) and Granger causality tests (using networks).

In this literature, systemic risk appears to be determined by the dependency between the individual institution and the economy or “financial system” in a stressed economy. It is indeed well documented that companies tend to be strongly dependent in a crisis whereas they may only be weakly dependent in good times. Using Sklar (1959)’s theorem, it is possible to separate the marginal distributions of the financial institution and of the financial system and their dependence structure (copula). We show how this separation can be useful to better understand the proposed measures for systemic risk. First we review the CoVaR in Section 2 then the SES in Section 3. In these sections we show how the CoVaR and the SES depend on the marginal distributions for the market’s returns and for the individual financial institution’s returns as well as the copula between the financial institution and the system. We then briefly review existing tail dependency measures in Section 4 and discuss coexceedances and exceedance correlation as they may be useful to measure contagion and systemic risk.

2 CoVaR

2.1 Original Definition

Let $M$ represent the aggregate value of the financial system, and $X$ represent the assets of an individual financial institution. The Value-at-Risk (VaR) at the level $q$ of the system is denoted by $VaR^M_q$ and computed as the quantile

$$P(M \leq VaR^M_q) = q.$$  \hspace{1cm} (1)

Similarly one defines $VaR^X_q$ as the Value-at-Risk of the financial institution.

The original definition of CoVaR (Adrian and Brunnermeier (2010)) is the VaR when the institution is under stress denoted by $CoVaR^X_{q}=VaR^X_q$,

$$P \left( M \leq CoVaR^X_{q} \mid X = VaR^X_q \right) = q.$$  \hspace{1cm} (2)

The systemic risk is then measured by the difference with the unconditional VaR as $\Delta CoVaR^X_{q} = CoVaR^X_{q} = VaR^X_q - VaR^M_q$ or in their more recent working paper, they investigate the difference with the “median” situation.

Note that the Value-at-Risk may take negative values (depending on the support of the distribution). The smaller it is, the riskier the company and the higher the corresponding capital requirements are.
We denote it by $\Delta CoVaR_q^=\text{ }q$ and define it as
\[ \Delta CoVaR_q^= = CoVaR_q^{X=VaR_q^X} - CoVaR_q^{X=VaR_{50\%}^X}. \] (3)

2.2 Alternative Definition

The fact that the institution is under stress when $X$ is at its Value-at-Risk level is arguable. It would make more sense to say that the institution is under stress when $X$ is below its Value-at-Risk level. In their paper, Adrian and Brunnermeier (2010) make use of quantile regression and need the equality to apply this technique. A more appropriate definition of $CoVaR$ would be
\[ P(M \leq CoVaR_q^{X \leq VaR_q^X} | X \leq VaR_q^X) = q, \] (4)
where the financial institution is under stress when $X \leq VaR_q^X$. The corresponding systemic risk is denoted by $\Delta CoVaR_q^{\leq}q$ and defined as
\[ \Delta CoVaR_q^{\leq} = CoVaR_q^{X \leq VaR_q^X} - CoVaR_q^{X=VaR_{50\%}^X}. \] (5)

Note that we are using the equality with the median for the “normal” conditions case which seems to make more sense than the inequality.

In the case when $(M, X)$ is a bivariate normal distribution, then $M\{|X = x\}$ is normally distributed and it is straightforward to derive closed-form expressions for $\Delta CoVaR_q^=$. However the conditional distribution of $M\{|X \leq x\}$ is more complicated. It is not normal anymore but is a skewed distribution.

2.3 Closed-form Expressions

From Sklar (1959)’s theorem, the joint distribution of $(X, M)$ is characterized by a copula $C$ and the respective margins $F_X$ and $F_M$. In other words,
\[ P(X \leq x, M \leq y) = C(F_X(x), F_M(y)). \]

In particular when $X$ and $M$ have uniform margins then $P(X \leq x, M \leq y) = C(x, y)$. For a couple $(U, V)$ with uniform margins and copula $C$, the conditional distribution (also called $h$-function by Aas et al. (2009)) can be calculated as follows
\[ h_v(u) := P(U \leq u | V = v) = \frac{\partial C(u, v)}{\partial v}. \] (6)
Then Equation (2) can be written as

\[ h_{FX(VaR_X^q)} \left( F_M \left( CoVaR_{q}^{X=VaR_X^q} \right) \right) = q, \]

where \( U \equiv F_M(M) \) and \( V \equiv F_X(X) \). The copula \( C \) describes the dependence between \( M \) and \( X \). Then \( CoVaR_{q}^{X=VaR_X^q} \) is given by \( F_M^{-1}(h_{FX(VaR_X^q)}^{-1}(q)) \), that is

\[ CoVaR_{q}^{X=VaR_X^q} = F_M^{-1}(h_{q}^{-1}(q)), \quad (7) \]

since \( VaR_X^q = F_X^{-1}(q) \). Similarly, we can derive a closed-form expression for Equation (4). It is

\[
q = P \left( \frac{M \leq CoVaR_{q}^{X=VaR_X^q} \mid X \leq VaR_X^q}{P(X \leq VaR_X^q)} \right) \\
= \frac{P \left( M \leq CoVaR_{q}^{X=VaR_X^q}, X \leq VaR_X^q \right)}{F_X(VaR_X^q)} \\
= C \left( F_M \left( CoVaR_{q}^{X=VaR_X^q} \right), F_X(VaR_X^q) \right).
\]

Let \( C_q^{-1}(\cdot) \) denote the inverse of \( C_q : x \mapsto C(\cdot, q) \), then \( CoVaR_{q}^{X=VaR_X^q} = F_M^{-1}(C_{FX(VaR_X^q)}^{-1}(qF_X(VaR_X^q))) \) and therefore

\[ CoVaR_{q}^{X=VaR_X^q} = F_M^{-1} \left( C_q^{-1}(q^2) \right). \quad (8) \]

Note that in case of Archimedean copulas with generator \( \varphi \), \( C_q^{-1} \) can easily be derived in closed-form as \( C_q^{-1} : x \mapsto \varphi^{-1}(\varphi(x) - \varphi(q)), x \in (0, q) \). For other copulas such as the Gaussian, numerical inversion is needed.

These analytical derivations ((7) and (8)) show that the \( \Delta CoVaR \) as measure of systemic risk is independent of the marginal distribution of \( X \). In particular, it is independent of characteristic properties such as the volatility of \( X \). If one however defines the \( CoVaR \) by

\[ P \left( X \leq CoVaR_{q}^{M=VaR_M^q} \mid M = VaR_M^q \right) = q, \]

and similarly for \( CoVaR_{q}^{M=VaR_M^q} \), then the corresponding \( \Delta CoVaR \) depends on the marginal distribution of \( X \) but no longer on that of \( M \). Adrian and Brunnermeier (2010) call this \( \Delta CoVaR \) the “exposure CoVaR”, since it measures how strongly an institution is affected in case of a crisis.
2.4 Numerical Example

In this section we discuss with examples how the definitions (3) and (5) are different and how the dependence structure affects $\Delta CoVaR$.

2.4.1 Difference between the definitions (3) and (5)

Figure 1 shows $\Delta CoVaR_{0.05}^=$ defined by (3) and $\Delta CoVaR^\leq_{0.05}$ defined by (5) for Student-t margins with two degrees of freedom and different copulas. Evidently $\Delta CoVaR_{0.05}^=$ reaches its minimum for moderate levels of dependence when a Gaussian or a Clayton copula are chosen. For high levels of dependence, which indicate a high systemic risk, $\Delta CoVaR_{0.05}^=$ however increases. $\Delta CoVaR^\leq_{0.05}$ does not show such odd behaviour.

Figure 1: $\Delta CoVaR_{0.05}^=$ (left panel) and $\Delta CoVaR_{0.05}^\leq$ (right panel) for $M$ having a Student-t distribution with two degrees of freedom and for different copulas with parameters chosen according to Kendall’s tau, which specify the dependence between $X$ and $M$.

This shows that we should prefer the formulation (3) to the formulation (5) with the stressed state of a company being modelled as the company’s assets being lower than its Value-at-Risk level.

Figure 1 also shows the importance of the copula, and its impact on systemic risk. It significantly increases when the financial institution has lower tail dependence with the financial market (such as with the Clayton copula). Evidence of lower tail dependence between asset returns is often found in the literature as, for example, in Longin and Solnik (2001).
2.4.2 Effects of the marginal distribution

We now show that the marginal distribution of the market may have an important effect on $\Delta CoVaR$. Systemic risk increases when the marginal distribution is negatively skewed or heavy-tailed, both being stylized facts of asset returns. This is illustrated in Figure 2 where we evaluate $\Delta CoVaR_{0.05}^{\leq}$ for different marginals for the market.

Figure 2: $\Delta CoVaR_{0.05}^{\leq}$ for $M$ with different zero mean and unit variance distributions. A Gaussian copula with parameters chosen according to Kendall’s tau specifies the dependence between $X$ and $M$. The skewness parameters are $\gamma = 2/3$ meaning a negative skew (using the parametrization of Fernandez and Steel (1998)). The Student-t distributions have five degrees of freedom.

3 Marginal Expected Shortfall

In this section we review the $SES$, systemic expected shortfall, introduced by Acharya et al. (2011). The $SES$ measures the propensity of a company to be undercapitalized when the system as a whole is undercapitalized. Acharya et al. (2011) explain that in the current regulation context, financial institutions do maximize their risk-adjusted returns without taking into account “the loss they impose in default on creditors and the externality they impose on the society at large in a systemic crisis”.

3.1 Systemic Expected Shortfall

The Systemic Expected Shortfall ($SES$) is closely related to the Marginal Expected Shortfall ($MES$). For example the $MES$ of a stock can be calcu-
lated as the average of the returns of this stock in the worst 5% days of
the value weighted market return. Let $M$ be the return of the aggregate
banking sector. It can be seen as a weighted sum of the returns $X_i$ of each
bank (out of the $N$ banks)

$$
M = \sum_{i=1}^{N} y_i X_i.
$$

In this sum, $y_i$ represents the weight of bank $i$ in the total aggregated value
of the market (it could be seen as the bank $i$’s assets divided by the aggregate
value of the market). The expected shortfall for the market can be evaluated
as

$$
ES_q = -E \left[ M | M \leq VaR^M_q \right].
$$

It is straightforward to decompose this expected shortfall as

$$
ES_q = \sum_{i=1}^{N} y_i MES^i_q,
$$

where

$$
MES^i_q = -E \left[ X_i | M \leq VaR^M_q \right]
$$
is the “marginal expected shortfall” of bank $i$ when the market is under
stress. This can be seen as the contribution of bank $i$ to the overall expected
shortfall of the system.

Using a similar idea as above, Acharya et al. (2011) define the systemic
expected shortfall of a bank $i$ denoted by $SES^i_i$. It is its “propensity to
be undercapitalized when the system as a whole is undercapitalized”. It
roughly corresponds to the expected loss of the bank $i$ when the market
is in a crisis. They further define the $DES^i_i$ which is the default expected
shortfall of bank $i$, in other words the expected loss of bank $i$ in case it goes
bankrupt. Finally they decompose optimal taxation for systemic risk into
two components: the first one is based on the $DES$ (which is based on the
company’s individual risk) and the second one is based on the $SES$ (which
is the bank’s contribution to systemic risk).

### 3.2 Closed-form Expressions

We can give a formula for $MES^i_q$ given in (10) as a function of the copula
between $X_i$ and $M$ and the marginal distribution of $X_i$. Denote by $F_M$
the cdf of $M$, $F_{X_i}$ the cdf of $X_i$ and $f_{X_i}$ its corresponding density. Using the
notation of the $h$-function \([9]\), we obtain

$$MES_q^i = -E[X_i | M \leq VaR^M_q] = -\int_{-\infty}^{\infty} x P\left(X_i = x | M \leq VaR^M_q\right) dx$$

$$= -\int_{-\infty}^{\infty} \frac{P\left(M \leq VaR^M_q | X_i = x\right)}{F_M(VaR^M_q)} f_{X_i}(x) dx$$

$$= -\frac{1}{F_M(VaR^M_q)} \int_{-\infty}^{\infty} x h_{F_{X_i}(x)}(F_M(VaR^M_q)) f_{X_i}(x) dx$$

$$= -\frac{1}{F_M(VaR^M_q)} \int_{0}^{1} F_{X_i}^{-1}(u) h_u(F_M(VaR^M_q)) du \quad \text{[substitute } u := F_{X_i}(x)]$$

$$= -\frac{1}{q} \int_{0}^{1} F_{X_i}^{-1}(u) h_u(q) du,$$

where the last equality follows from $VaR^M_q = F_M^{-1}(q)$. That is, $MES_q^i$ \([10]\) is independent of the distribution of $M$ and only depends on the market return through the copula $C$, which specifies the dependence between $M$ and $X_i$.

### 3.3 Numerical Example

We perform a similar numerical study as for the CoVaR, and study the impact of the choice of the copula and of the margin of $X_i$ with respect to skewness and heavy-tailedness (see Figure 3).

Similarly to $\Delta CoVaR^\leq$, the marginal expected shortfall indicates a higher systemic risk when the Gaussian or the lower tail dependent Clayton copulas are used. Negative skewness and heavy-tailedness also increase the systemic risk. In contrast to $\Delta CoVaR^\leq$, the marginal expected shortfall is more sensitive to skewness. Here the different $y$-scales of Figure [1] and [3] have to be taken into account.

In this chapter, we restrict ourselves to the analysis of statistical properties of the systemic risk measures proposed in the literature. Recently, Brownless and Engle (2011) and Hautsch et al. (2011) extend the work of Acharya et al. (2011) on the SES for empirical use. They explain how to estimate this quantity from financial time series.

### 4 Other Tail Dependence Measures

There is an important literature on tail dependence, see Juri and Wüthrich (2003), Embrechts et al. (2003), McNeil et al. (2005), Nelsen (2006) and Joe (1997). The estimation of the standard tail dependence coefficient is rather difficult and prone to bias due to the small proportion of observations which
Figure 3: Left panel: $MES_{0.05}^i$ as a function of Kendall’s tau for $X_i$ having a Student-t distribution with two degrees of freedom and for different copulas with parameters chosen according to Kendall’s tau. Right panel: $MES_{0.05}^i$ as a function of Kendall’s tau for several marginal distribution for $X_i$ with different zero mean and unit variance. A Gaussian copula with parameters chosen according to Kendall’s tau specifies the dependence between $X_i$ and $M$. The skewness parameters are $\gamma = 2/3$ meaning a negative skew (using notation of Fernandez and Steel 1998). The Student-t distributions have five degrees of freedom. To facilitate comparison to $\Delta CoVaR$ (Figures 1 and 2), $-MES_{0.05}^i$ is shown here.

can be used for estimation. Alternative methods are needed in practice to determine the tail behavior of pairs of random variables. Some research has been done on coexceedances and exceedance correlation and we now review these two approaches.

4.1 Exceedances

Bae et al. (2003) define contagion as a significant increase in market co-movement after a shock in the market. As Adrian and Brunnermeier (2010) note, “increases of comovement give rise to systemic risk”. Measures for contagion are therefore potential measures for systemic risk. Bae et al. (2003) focus on occurrences of extreme returns. Extreme returns (or exceedances) are defined as returns that lie below the 5th percentile or above the 95th percentile of the marginal distribution. They treat separately the “bottom tail” that consists of negative extreme returns and “top tail” that consists of positive extreme returns. The study of Bae et al. (2003) is based on daily index returns. A coexceedance of $i$ on a given day means that there has been $i$ extreme returns (that is $i$ exceedances) observed among the indices
4.2 Exceedance Correlation

Other references in the literature, such as Baig and Goldfajn (1999) and Forbes and Rigobon (2002) have used exceedance correlations. However, it is shown by Beine et al. (2010), that the use of “correlations leads to underestimate the impact of trade and financial integration on stock market comovement”. As Forbes and Rigobon (2002) point out, “correlation coefficients are conditional on market volatility.” Precisely consider $A$ a positive probability event, there are some issues with the conditional correlation. First Bradley and Taqqu (2004) illustrate that the choice of the conditioning event is critical. For example there is a strong difference between $\rho(X,Y \mid |X| > VaR_X(q))$ and $\rho(X,Y \mid X > VaR_X(q))$ which is illustrated in Figure 1 of their paper. Furthermore Bradley and Taqqu (2004) show that if the conditional variance $\text{var}(X|X \in A)$ is bigger than the unconditional variance $\text{var}(X)$ then the conditional correlation $\rho_A := \rho(X,Y|X \in A)$ also satisfies $|\rho_A| > |\rho(X,Y)|$ (when $(X,Y)$ is bivariate Gaussian). Therefore if the conditioning sample is more variable than the original sample, the correlation may increase whereas there has been no change in structure (this is referred as “heteroscedasticity bias”). This issue typically occurs when one compares data during a financial crisis to data in normal conditions. Under some assumptions, Forbes and Rigobon (2002) show how to adjust for this bias by

$$\rho_{\text{adjusted}} := \frac{\rho_A}{\sqrt{1 + \delta(1 - \rho_A^2)}}$$

where $\delta = \text{var}(X|A)/\text{var}(X) - 1$ represents the relative increase in market volatility during the crisis period relative to normal conditions. Their empirical study then contradicts previous literature by showing that there was virtually no increase in unconditional correlation during crisis between 1980 and 2000.

Campbell et al. (2008) study “truncated correlation” and “exceedance correlation”. They are defined respectively as correlations between two indices when one of them is beyond some level for the truncated estimator or when both of them are beyond some levels for the exceedance estimator. Campbell et al. (2008) compute these indicators for the bivariate normal distribution and for the bivariate Student-t distribution and show significant difference. Their empirical study further suggests that the excess in conditional correlation can be overestimated by assuming bivariate normality.
4.3 Coexceedances and Exceedance Correlation for Systemic Risk Measurement

In our context it is natural to investigate coexceedances and exceedance correlation between the financial system and an individual financial institution as possible systemic risk measures, since they measure the joint tail behavior of two random variables and may be used, for example, to rank different companies according to their risk. Formally, the probability that the return $M$ of the financial market and the return $X$ of a financial institution jointly fall below their quantile at level $q$ is

$$P(X \leq F_X^{-1}(q), M \leq F_M^{-1}(q)) = C(q,q),$$

where $C$ is the copula of $M$ and $X$. It is illustrated in Figure 4 for independent and lower tail dependent data.

Ang and Chen (2002) define exceedance correlation with certain thresholds $\delta_1$ and $\delta_2$ as

$$\text{corr}(X, M|X \leq \delta_1, M \leq \delta_2).$$

This measure is however not independent of the margins of $X$ and $M$, since it is based on Pearson’s product-moment correlation coefficient. A simple modification of this definition can cope with this issue by using the common dependence measures Kendall’s $\tau$ instead of the Pearson correlation:

$$\tau(X, M|X \leq \delta_1, M \leq \delta_2).$$
Theoretical expressions of lower exceedance Kendall’s $\tau$ for continuous random variables with copula $C$ can be obtained by

$$
\tau(X,M|X \leq \delta_1, M \leq \delta_2) = \frac{4}{C(F_X(\delta_1), F_M(\delta_2))^2} \int_0^{F_M(\delta_2)} \int_0^{F_X(\delta_1)} C(u_1, u_2) dC(u_1, u_2) - 1. \tag{13}
$$

See Theorems 5.1.1 and 5.1.3 in Nelsen (2006). In most cases, explicit solutions of the integrals in (13) are hard to obtain. To evaluate lower exceedance Kendall’s $\tau$ for the quantile at level $q$, we set $\delta_1 = F_X^{-1}(q)$ and $\delta_2 = F_M^{-1}(q)$. Then it is independent of the margins of $X$ and $M$.

In an extensive Monte Carlo study, Brechmann (2010) compares both measures (11) and (13) across different copulas (see Chapter 3 of Brechmann (2010)). It is shown that the lower exceedance Kendall’s $\tau$ is empirically able to discriminate between pairs of random variables that exhibit strong joint tail behavior and those that do not. The measure of coexceedances, which Brechmann (2010) refers to as “tail cumulation”, is however not able to clearly distinguish between pairs with or without strong joint tail behaviour, except for the asymmetric tail dependence induced by the observations from the Clayton copula. For both measures but especially for coexceedances, there are some problems in discriminating Gaussian and Student-$t$ copulas, although their dependence structure is strongly different in terms of tail behaviour. As a result, coexceedances should be used carefully, while exceedance Kendall’s $\tau$ is quite useful in assessing tail dependence.

5 Conclusions & Alternative Systemic Measure

It is clear from the literature that systemic risk is linked to the left tail dependency. We have shown that the $CoVaR$ proposed by Adrian and Brunnermeier (2010) depends on the marginal distribution of the financial market as well as the dependence between the financial institution and the economy in the left tail of the market’s marginal distribution. The marginal distribution of the company (including its characteristics, such as volatility or returns) does not influence its contribution to systemic risk. On the contrary, the marginal expected shortfall used by Acharya et al. (2011) to determine the $SES$ depends on the marginal distribution of the financial institution and its dependency with the economy but not on the marginal distribution of the financial system.

We adjusted the definition of $CoVaR$ by defining the stress of a company as being below its Value-at-Risk level and not at its Value-at-Risk level. After this adjustment the Marginal Expected Shortfall and the $CoVaR$ have similar sensitivities with respect to the skewness and heavy-tailedness of the
marginal distributions and with respect to the copula between the bank and the economy. Coexceedances and exceedance correlations are alternative measures to CoVaR and SES. However they may not be as useful to assess the left tail dependency between the bank and the financial system.

Most measures for systemic risk (except CoVaR) depend not only on the dependency between the company and the economy but also on the marginal distribution of the company. In some sense, standard capital requirements already incorporate the risk represented by the marginal distribution, therefore it might be more appropriate to have a measure that depends solely on the interaction between the company and the financial system. A risky company will have high capital requirements when considered in isolation. It might however not represent a high systemic risk, therefore it is important that the systemic risk measure does not penalize companies with risky marginals. It is indeed also not fair for a small company to have a big premium because it will need the fund if one of the big bank, or if a company such as Ambac (US company providing financial guarantees) is going bankrupted. The cost of that should be paid by the big banks, the ones that take the risks and make the system at risk and get the return associated by this additional risk. However in such a situation a small company will have little effect on the VaR of the system and therefore small CoVaR. However its marginal expected shortfall can be large. The CoVaR measure seems therefore more reasonable.

Existing systemic risk measures investigate what happens when the system is under stress or when an institution is under stress but not on why it is under stress and what the causes are. Companies responsible for the stress may not be the ones that suffer most from the system being under stress. A company may indeed be responsible for systemic risk without being under stress when the system is under stress. Systemic risk has been so far identified in the left tail dependency between the institution and the financial market. It might be more appropriate to extend these measures in order to capture abnormal profits when the market is under stress, but also abnormal profits due to excessive risk taken with the financial system. Such abnormal profits are not necessarily reflected on the individual risk and are not necessarily linked to huge losses when the system is under stress.

For example Ambac is a company that provides guarantees. If Ambac is under stress and fails, many guarantees (from the counterparties of Ambac) will become uncovered and risky. Therefore many other institutions will observe a significant increase in risk. As a consequence the global risk of the system may increase significantly. But should Ambac be penalized as much as a company that benefits from a crisis? It is clear that Ambac is trying to help the stability of the system. On the opposite, Fabrice Tourre was a VP at Goldman Sachs. From the press, one could read: “his job was fairly straightforward. He helped large institutions take positions in the housing
sector by creating customized collateralized debt obligations, essentially collections of residential mortgages.” But in some emails he explained to some of his friends that these products were so complex that nobody could understand them and that he knew that the business was over but he kept selling and buying them for the profit of Goldman Sachs. By these activities Goldman Sachs was contributing to systemic risk and this may not be reflected by extreme losses when the market is under stress but by extreme gains!

A good systemic risk measure should be such that companies that take an excessive amount of risk should not be rewarded for luck but should pay back part of these benefits to the system to the guarantee fund. They are indeed taking risk with the system and earn benefits thanks to that activity. A systemic risk measure should encourage companies to hedge and reward companies that do not play with the system to increase their benefits. Abnormal profits made by a company even when the system goes well should also be taken into account. To do so it is important to look closely at the dependence between the financial institution and the financial system and not only in the left bottom corner of the picture in Figure 4 but in the four corners.
References


