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A modification of Morrow and Smith–Watson–Topper mean stress correction models

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ABSTRACT A modification of the Morrow and the Smith, Watson and Topper (SWT) mean stress correction models is proposed to account for the mean stress effect on fatigue life. The capability and accuracy of the proposed model are compared to those of the original Morrow and the SWT model using published mean stress fatigue test data. The proposed mean stress correction model was found to be superior to both the SWT and the Morrow model in the case of the Incoloy 901 superalloy and the ASTM A723 steel. On the other hand both the proposed and the original SWT model provided equally good correlation with experimental data in the case of 7075-T561 aluminium alloy and 1045 HRC 55 steel. The Morrow model was found to give the least accurate predictions for all four materials analysed.

Keywords fatigue; life predictions; mean strain; mean stress; mean stress correction.

INTRODUCTION

Engineering components and structures are often subjected to cyclic loads with the presence of mean stress and mean strain. Various methods have been developed to model mean stress effects on the fatigue behaviour of metals. It is known that the mean stress has a significant influence on the fatigue life.[1] Tensile mean stress is known to be detrimental to the fatigue life, while compressive mean stress is beneficial. Earlier approaches accounting for the mean stress effect were used in the conventional stress-life method for correcting the fatigue limit or fatigue strength in the high cycle fatigue regime of 10^6–10^8 cycles. These contemporary mean stress correction models are given by

Gerber[2]

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1, \tag{1}$$

Goodman[3]

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1, \tag{2}$$

Soderberg[4]

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_y} = 1, \tag{3}$$

Morrow[5]

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f} = 1, \tag{4}$$

Walker[16]

$$\sigma_{ar} = \sigma_{\max}^{1-\gamma}\sigma_a^{\gamma} = \sigma_{\max}\left(\frac{1-R}{2}\right)^{\gamma} = \sigma_a\left(\frac{2}{1-R}\right)^{\gamma}, \tag{5}$$

SWT[8]

$$\sigma_{ar} = \sqrt{\sigma_{\max}\sigma_a} = \sigma_{\max}\sqrt{\frac{1-R}{2}} = \sigma_a\sqrt{\frac{2}{1-R}} \tag{6}$$

where σ_a is the stress amplitude, σ_m is the mean stress, σ_{ar} is the equivalent fully reversed stress amplitude resulting in the same fatigue life as the σ_a–σ_m combination, σ_u is the material ultimate strength, σ_f is the true fracture strength, $\sigma_{\max}$ is the maximum stress, R is the stress ratio and γ is a material constant. The fatigue life and the equivalent fully reversed stress amplitude–life relation based on Basquin's

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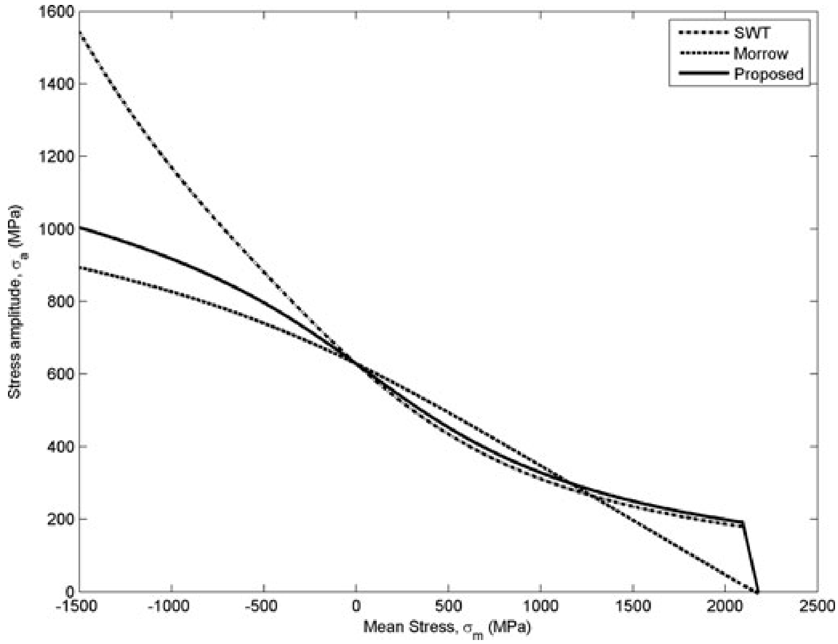


Fig. 1 Stress amplitude versus mean stress diagram for constant fatigue life of $N_f =$ 1000 000– SAE 8620 alloy steel.

Table 1 Cyclic and fatigue properties of the SAE 8620 alloy steel

Cyclic and fatigue properties of the SAE 8629 steel alloy	
Fatigue strength coefficient, σ'_f	2 090 MPa
Fatigue strength exponent, b	−0.087
Fatigue ductility coefficient, ε'_f	0.29 754
Fatigue ductility exponent, c	−0.58
Cyclic strength coefficient, K'	2 506 MPa
Cyclic strain hardening exponent, n'	0.15
Modulus of elasticity, E	200 GPa

equation[17] is given as

$$\sigma_{ar} = \sigma'_f \left(2N_f\right)^b \tag{7}$$

where σ'_f is the fatigue strength coefficient, N_f is the fatigue life and b is the fatigue strength exponent.

The methods represented by Eqs (1)–(6) are generally applicable to cases of long fatigue lives where applied stresses generate predominantly elastic strain amplitudes.

In several past decades, research studies accounting for the mean stress effect on fatigue life were based on the strain-life approach, where the cyclic response of the material is within the elastic–plastic stress–strain range. The strain-life approach is represented by the total strain versus life Eq. (8).

$$\frac{\Delta\varepsilon}{2} = \varepsilon_a = \frac{\Delta\varepsilon^e}{2} + \frac{\Delta\varepsilon^p}{2} = \frac{\sigma'_f}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \tag{8}$$

where $\frac{\Delta\varepsilon}{2}$ is the strain amplitude, ε'_f is the fatigue ductility coefficient, c is the fatigue ductility exponent and E is the elastic modulus.

Since Eq. (8) can only be used to predict the fatigue life at zero mean stress, it needs to be modified to include the mean stress effect.

The Morrow mean stress correction in the form of Eq. (4) was included by Morrow[14] into the elastic term of the strain-life equation given as

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \tag{9}$$

The Morrow model predicts that the mean stress has a significant effect on longer lives, where the elastic strain amplitudes dominate. The model also predicts that the mean stress has little effect on shorter lives, where the plastic strain is large. The prediction trend of the Morrow mean stress correction model is consistent with observations that the mean stress has greater impact at longer lives.

Manson and Halford[6] suggested that both the elastic and plastic terms of the strain-life equation should be modified to account for the mean stress effects and maintain the independence of the elastic to plastic strain ratio from the mean stress.

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}\left(2N_f\right)^b + \varepsilon'_f\left(\frac{\sigma'_f - \sigma_m}{\sigma'_f}\right)^{c/b}\left(2N_f\right)^c \tag{10}$$

The Manson and Halford mean stress model tends to overestimate the mean stress effect on short lives, where the plastic strain dominates.

Table 2 Monotonic and cyclic properties of Incoloy 901, 7075-T651, ASTM A723 and SAE 1045 HRC55 materials

Monotonic properties	Incoloy 901 [10]	7075-T651 [11]	ASTM A723 [9]	SAE 1045HRC [12]
Yield strength, σ_y	958 MPa	501 MPa	1170 MPa	1713 MPa
Ultimate strength, σ_u	1200 MPa	561 MPa	1262 MPa	2165 MPa
Modulus of elasticity, E	202 GPa	71.7 GPa	200 GPa	205 GPa
Strength coefficient, K	1615 MPa	–	1483 MPa	3088 MPa
Strain hardening exponent, n	0.101	–	0.037	0.092
Reduction in area, RA	15%	29.1%	50%	38%
Cyclic and fatigue properties				
Fatigue strength coefficient, σ'_f	1977 MPa	1576 MPa	2123 MPa	3372 MPa
Fatigue strength exponent, b	−0.1228	−0.1609	−0.110	−0.103
Fatigue ductility coefficient, ε'_f	0.125	0.1575	0.49	0.038
Fatigue ductility exponent, c	−0.6478	−0.6842	−0.783	−0.47
Cyclic strength coefficient, K'	1566 MPa	747 MPa	1581 MPa	3082 MPa
Cyclic strain hardening exponent, n'	0.09	0.0597	0.071	0.075

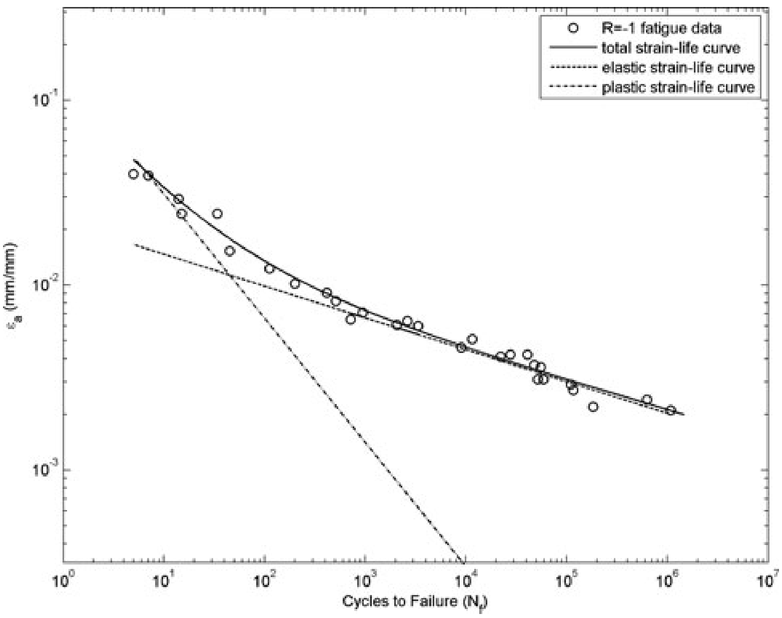


Fig. 2 Experimental fatigue data and the fitted strain-life curve for the 7075-T651 aluminium alloy.

An extension of the SWT[8] parameter can be applied to the strain-life equation by replacing σ_a in Eq. (6) with the strain amplitude, ε_a. The product of the maximum tensile stress, $\sigma_{\max}$, and the strain amplitude, ε_a, in the strain-life model controls the influence of both the mean stress and the strain amplitude:

$$\sigma_{\max}\varepsilon_a = \sigma_{\max}\frac{\Delta\varepsilon}{2} = \frac{(\sigma'_f)^2}{E}\left(2N_f\right)^{2b} + \varepsilon'_f\sigma'_f\left(2N_f\right)^{b+c} \quad (11)$$

where $\sigma_{\max} = \sigma_m + \sigma_a$ is the maximum stress of given cycle. The SWT model assumes that the $\sigma_{\max}\varepsilon_a$ parameter, at a given life, remains constant for different combinations of the strain amplitude and the maximum stress. The model Eq. (11) gives good estimation of mean stress effect in the long life regime, but it is conservative in the low cycle fatigue region.[9] The SWT parameter $\sigma_{\max}\varepsilon_a$ can be also interpreted in terms of the strain energy density.

While studying the partial unloading on the cyclic creep of copper, Lorenzo and Laird[15] reported that a semilog plot of the plastic strain range versus mean stress for constant values of the maximum stress resulted in a family of straight lines having constant slope. When this family of lines is extrapolated to zero mean stress, the extrapolated values of the plastic strain ranges coincide with the values obtained from the same maximum stresses, but under zero mean stress conditions. Based on this finding, Lorenzo and Laird suggested that the same fatigue life, N_f, is observed for a metal when the product of the stress amplitude, σ_a, and the plastic strain amplitude, $\frac{\Delta\varepsilon_p}{2}$, for a fully reversed test is equal to the product of the maximum stress, $\sigma_{\max}$, and the associated plastic strain range, $\frac{\Delta\varepsilon_{pm}}{2}$,

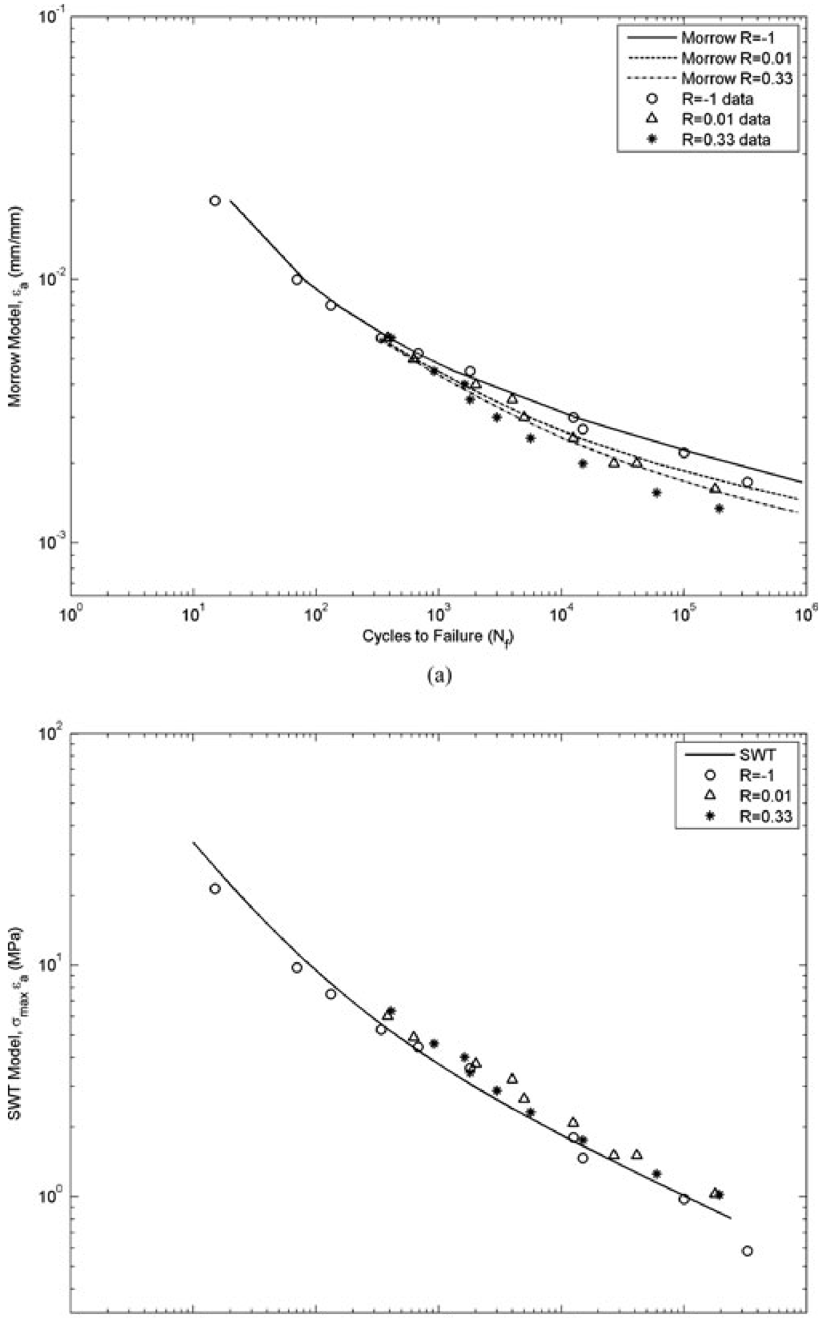


Fig. 3 (a) Comparison of the Morrow model for various strain ratios with experimental fatigue data of Incoloy 901. (b) Comparison of the SWT model for various strain ratios with experimental fatigue data of Incoloy 901. (c) Comparison of the proposed model for various strain ratios with experimental fatigue data of Incoloy 901.

for a test having non-zero mean stress. Therefore, the cyclic plastic strain amplitude and maximum stress can be combined in order to predict fatigue lives under various men stress levels similar to the SWT parameter:

$$\sigma_a \frac{\Delta \varepsilon_p}{2} = \sigma_{\max} \frac{\Delta \varepsilon_{pm}}{2} = \text{constant}. \quad (12)$$

However, this model can only be used in the presence of significant cyclic plastic deformation.

Dowling[13] has recently analysed the mean stress effect in several steel materials and aluminium alloys using the Morrow[5] model, the SWT[8] and the Walker[16] parameter for the mean stress correction in the stress-life and the strain-life fatigue analysis method.

It was subsequently shown by Dowling that the Walker and the SWT parameters Eq. (5) and Eq. (6) can be transformed into the strain-life equation given by expressions Eq. (13) and Eq. (14) respectively:

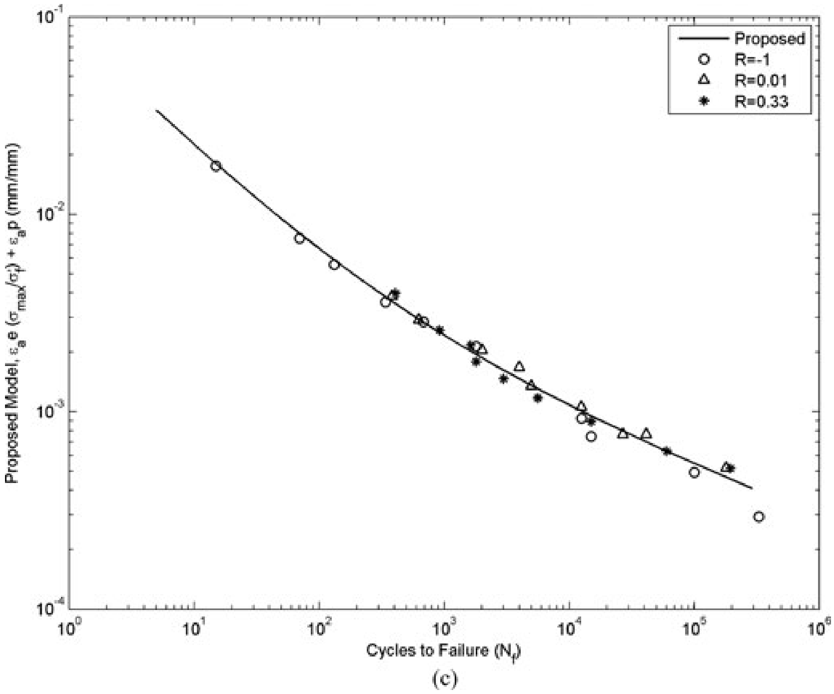


Fig. 3 Continued

$$\varepsilon_a = \frac{\sigma'_f}{E}\left[2N_f\left(\frac{1-R}{2}\right)^{1/2b}\right]^b + \varepsilon'_f\left[2N_f\left(\frac{1-R}{2}\right)^{1/2b}\right]^c \tag{13}$$

$$\varepsilon_a = \frac{\sigma'_f}{E}\left[2N_f\left(\frac{1-R}{2}\right)^{1-\gamma/b}\right]^b + \varepsilon'_f\left[2N_f\left(\frac{1-R}{2}\right)^{1-\gamma/b}\right]^c. \tag{14}$$

Dowling[13] has also reported that the Morrow mean stress correction of Eq. (4) in the stress life is suitable for steel materials, but it provides non-conservative prediction for aluminium alloys.

The SWT mean correction parameter gives good results for both steel and the aluminium materials. The Walker equation with its adjustable fitting parameter γ provides on the other hand equally good mean stress correction for any material. Unfortunately, additional effort and experiments are necessary for the determination of parameter γ. Dowling *et al*.[19] have shown later that in the case of steel materials this parameter can be correlated linearly with the ultimate tensile strength, and estimated for cases where the non-zero mean stress data are not available.

Some high-strength aluminium alloys have parameter, $\gamma \approx 0.5$, which coincides with the SWT method, but higher values of γ apply to low-strength aluminium alloys. For both steels and aluminium alloys, a decrease of the γ parameter with increasing material strength has been observed, indicating an increasing sensitivity to the mean stress level.

The Walker mean stress Eq. (5) is of the same nature as the SWT parameter Eq. (6) except that the maximum stress and the stress or strain amplitude are differently weighted with respect to each other and the Walker relation requires additional adjustable parameter, γ, in order to fit the stress/strain-life equation into experimental data obtained under various means levels. Therefore, the Walker relation is not discussed any further in this present paper.

Although many more mean stress correction models and their extensions have been developed, the strain-life-based approach with the Morrow model Eq. (9) and the SWT model Eq. (11) are the most popular in engineering applications. However, it is known that the Morrow model indirectly suggests that the mean stress changes the relation between the plastic and elastic strain amplitude while the SWT parameter is non-conservative in the presence of compressive mean stresses. Therefore a new modification to those models is proposed in order to avoid those inconsistencies mentioned above. The proposed model is found to provide noticeable improvements to both the Morrow and the SWT models in predicting the fatigue life for the published mean stress fatigue data.

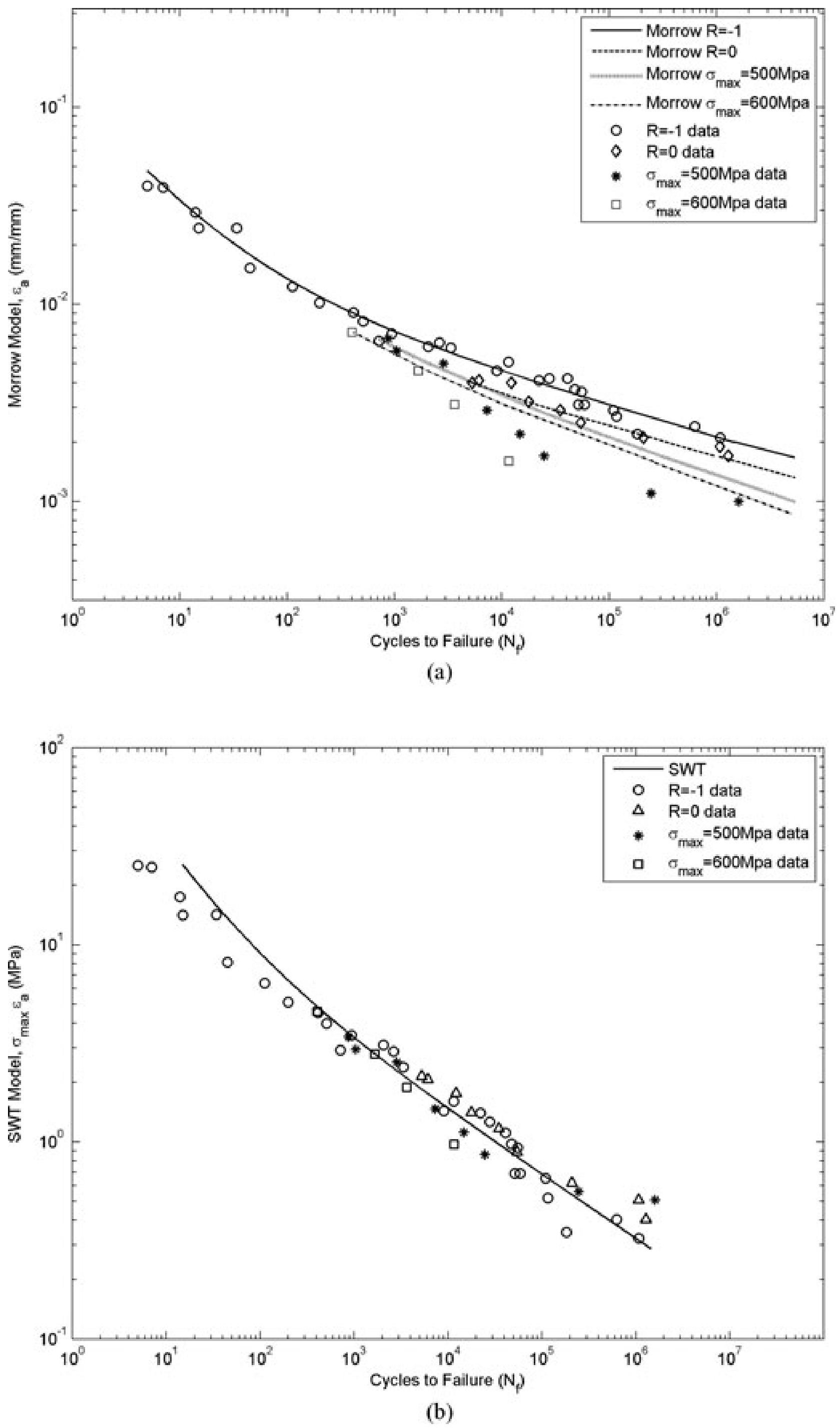


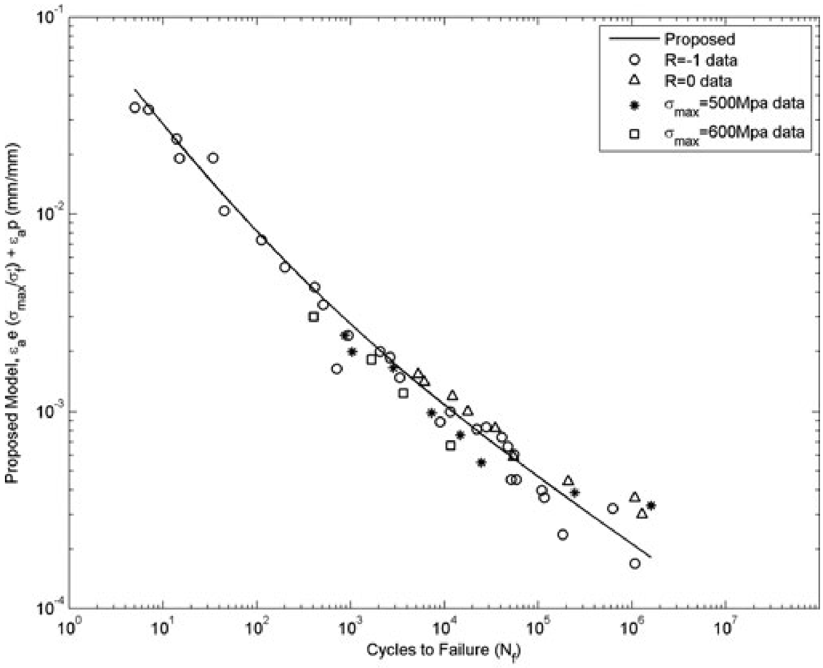
Fig. 4 (a) Comparison of the Morrow model with experimental mean stress fatigue data of 7075-T651. (b) Comparison of the SWT model with experimental mean stress fatigue data of 7075-T651. (c) Comparison of the proposed model with experimental mean stress fatigue data of 7075-T651.

THE PROPOSED MEAN STRESS CORRECTION MODEL

It is well known that both the Morrow and the SWT models have their shortcomings for predicting fatigue life in low and high cycle fatigue regimes.[7,9,12,13] Therefore it seems to be natural to combine the good features of the Morrow and SWT mean stress correction models. For this reason certain modifications of the Morrow and SWT mean stress correction models have been proposed, i.e. by applying the SWT correction only to the elastic part of the strain cycle.

If only the elastic strain amplitude is to be corrected, the resultant equivalent (corrected) strain amplitude can be written as

$$\varepsilon_{eq,a} = \varepsilon^{e}_{a,eq} + \varepsilon^{p}_{a} = f(N_f) \tag{15}$$



(c)

Fig. 4 Continued

where $\varepsilon_{eq,a}$ is the total equivalent strain amplitude, $\varepsilon^{e}_{a,eq}$ is an equivalent elastic strain amplitude corrected for the mean stress effect and ε^{p}_{a} is the plastic strain amplitude.

The elastic strain term based on Basquin's equation[17] can be given as

$$\varepsilon^{e}_{a} = \frac{\sigma'_f}{E}\left(2N_f\right)^{b} \tag{16}$$

In the case of fully reversed loading ($R = -1$), the maximum stress, $\sigma_{\max}$, can be expressed as

$$\sigma_{\max} = \varepsilon^{e}_{\max}E = \varepsilon^{e}_{a}E = \frac{\sigma'_f}{E}(2N_f)^{b}E = \sigma'_f(2N_f)^{b} \tag{17}$$

After multiplying Eq. (16) by the maximum stress $\sigma_{\max}$ it is transformed to the form of Eq. (18):

$$\sigma_{\max}\varepsilon^{e}_{a} = \frac{\sigma'^{2}_{f}}{E}\left(2N_f\right)^{2b} \tag{18}$$

The $\sigma_{\max}\varepsilon^{e}_{a}$ term in Eq. (18) is scaled with σ'_f in order to determine the equivalent elastic strain term.

$$\frac{\sigma_{\max}}{\sigma'_f}\varepsilon^{e}_{a} = \varepsilon^{e}_{a,eq} = \frac{\sigma'_f}{E}\left(2N_f\right)^{2b} \tag{19}$$

The plastic strain term in Eq. (15) based on the Manson–Coffin strain-life equation can be expressed as

$$\varepsilon^{p}_{a} = \varepsilon'_f\left(2N_f\right)^{c} \tag{20}$$

The equivalent elastic term accounting mean stress effects, Eq. (19), and the plastic strain term, Eq. (20), are both strains and can be summed up resulting in the total equivalent strain amplitude as shown below in Eq. (21):

$$\begin{aligned}\varepsilon_{eq,a} &= \varepsilon^{e}_{a,eq} + \varepsilon^{p}_{a} = \frac{\sigma_{\max}}{\sigma'_f}\frac{\Delta\varepsilon^{e}}{2} + \frac{\Delta\varepsilon^{p}}{2}\\ &= \frac{\sigma'_f}{E}(2N_f)^{2b} + \varepsilon'_f(2N_f)^{c}\end{aligned} \tag{21}$$

Similar to the SWT model, the proposed mean stress correction model is applicable providing that the following condition is not violated $\frac{\sigma_{\max}}{\sigma'_f}\frac{\Delta\varepsilon^{e}}{2} + \frac{\Delta\varepsilon^{p}}{2} > 0$.

The Morrow model Eq. (9), the SWT model Eq. (11) and the proposed model Eq. (21) for the SAE 8620 alloy steel are shown in Fig. 1 as $\sigma_a - \sigma_m$ curves corresponding to the constant fatigue life of 10^6 cycles. Obviously, those curves vary depending on the fatigue life, but the general trend remains same.

Cyclic and fatigue properties of the SAE 8620 alloy steel are listed in Table 1. Prediction trends for the proposed mean stress correction model for fatigue lives in the range of 10^6 cycles are clearly seen in Fig. 1. The proposed model provides close predictions to the SWT model for tensile mean stresses and approaches the Morrow model for compressive mean stress values.

COMPARISON OF MEAN STRESS FATIGUE MODELS WITH EXPERIMENTAL DATA

Only four sets of experimental fatigue data from the literature[9–12] have been chosen for the purpose of preliminary assessment of the proposed model. These fatigue data sets are given at various means stresses for Incoloy 901

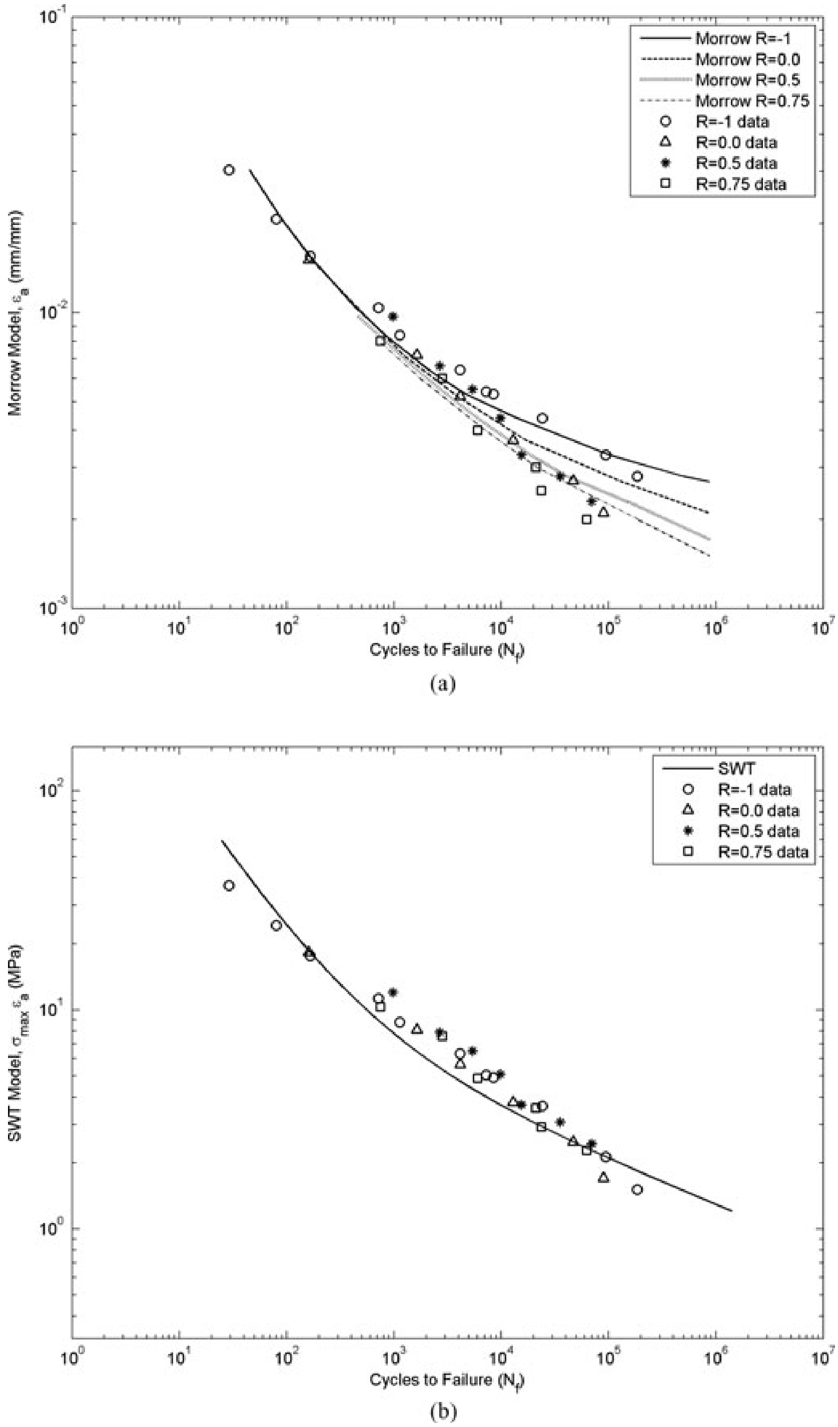
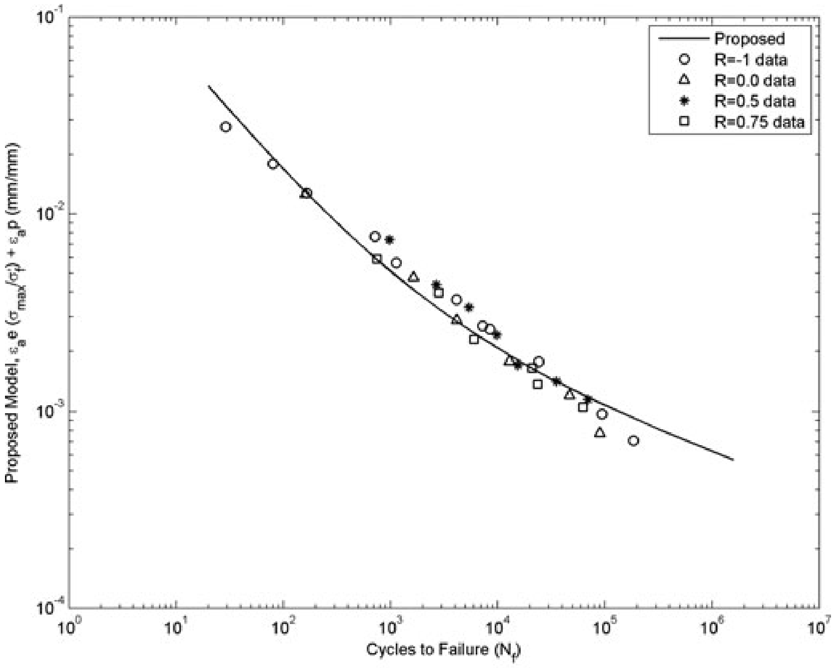


Fig. 5 (a) Comparison of the Morrow model for various strain ratios with experimental fatigue data of ASTM A723. (b) Comparison of the SWT model for various strain ratios with experimental fatigue data of ASTM A723. (c) Comparison of the proposed model for various strain ratios with experimental fatigue data of ASTM A723.

superalloy, ASTM A723 steel, 7075-T561 aluminium alloy and 1045 HRC 55 steel. Monotonic and fatigue properties of these materials at zero mean stress are also given Table 2. The cyclic stress–strain and fatigue properties given in Table 2 are based on the original regression analyses presented in the quoted literature sources.

The tensile mean stress effect is predominantly studied in this paper, because this is the most important range of practical applications. Strain ranges as well as the mean stress and the maximum stress were obtained from experimental half-life stress–strain hysteresis loops.

In order to assess the correctness of analysed fatigue models, it was necessary to establish valid criteria for the comparison. Thus the accuracy of the assessment of the mean stress effect resulted from given model was analysed based on how close the predicted and the experimental



(c)

Fig. 5 Continued

data collapsed when compared on a diagram 'fatigue damage parameter versus fatigue life'. The data points which fall on the right-hand side of the fatigue damage parameter versus fatigue life line were considered as conservative. Conversely, the data points falling on the left-hand side of this line resulting in lives shorter than expected were considered as non-conservative.

Figure 3a shows the Morrow model gives reasonable fatigue life predictions for the fatigue data at $R = -1$ and $R = 0.01$ strain ratios for Incoloy 901 superalloy in Fig. 3a. A relatively poor correlation of the Morrow model at $R = 0.33$ strain ratio can be seen in Fig. 3a and the Morrow model overestimates fatigue lives for $R = 0.33$ strain ratio particularly in the fatigue life region of 10^4–10^6 cycles.

The SWT model provides good correlation for all given strain ratios ($R = -1$, $R = 0.01$, $R = -1$ and $R = 0.33$) in Fig. 3b even though the fatigue life predicted by the SWT model is slightly conservative. The proposed model shows an excellent correlation with the fatigue data at all three strain ratios in Fig. 3c as the experimental data collapse very close the fatigue damage parameter versus fatigue life line.

The results shown in Fig. 4a present reasonable correlations resulted from the Morrow model for fatigue data at $R = -1$ (as also shown in Fig. 2) and $R = 0.0$ strain ratios for 7075-T561 aluminium alloy. However, the non-conservative life predictions by the Morrow model are clearly seen for $\sigma_{max} = 500$ MPa and $\sigma_{max} = 600$ MPa mean stress fatigue data in Fig. 4a. The experimental fatigue data of 7075-T561 aluminium alloy[11] were generated under stress control load conditions at $\sigma_{max} = 500$ and $\sigma_{max} = 600$ mean stress.

It has been widely accepted that the SWT model is particularly good for aluminium alloys.[11,18] This study also indicates that a fairly good correlation between the SWT model and the mean stress fatigue data for 7075-T561 aluminium can be obtained as seen in Fig. 4b. However, the SWT model appears to overestimate fatigue lives in the range less than 10^2 cycles.

Similar to the SWT model, the proposed model provides good correlations for all mean stress fatigue data in Fig. 4c. However, it is not clear whether the proposed model is more accurate in the case of 7075-T651 aluminium alloy than the SWT model.

It is obvious that all three methods are not very accurate in the case of stress-controlled tests at high mean stresses for 7075-T651 aluminium alloy. This might be due to ratcheting effects. There may be some amount of the ratcheting hidden in the mean stress fatigue data particularly for $\sigma_{max} = 600$ MPa conditions even though no ratcheting was reported under these test conditions.[11]

Prediction of fatigue lives using the Morrow, the SWT and the proposed model for the ASTM A723 steel are shown in Fig. 5a–c respectively. The results shown in Fig. 5a indicate that Morrow model seems to overestimate the fatigue lives greater than 10^4 cycles for fatigue data at R = 0.0, $R = 0.50$ and $R = 0.75$ strain ratios.

As seen from Fig. 5b, The SWT model indicates slightly conservative predictions by underestimating fatigue lives in the range of 10^3–10^5 cycles.

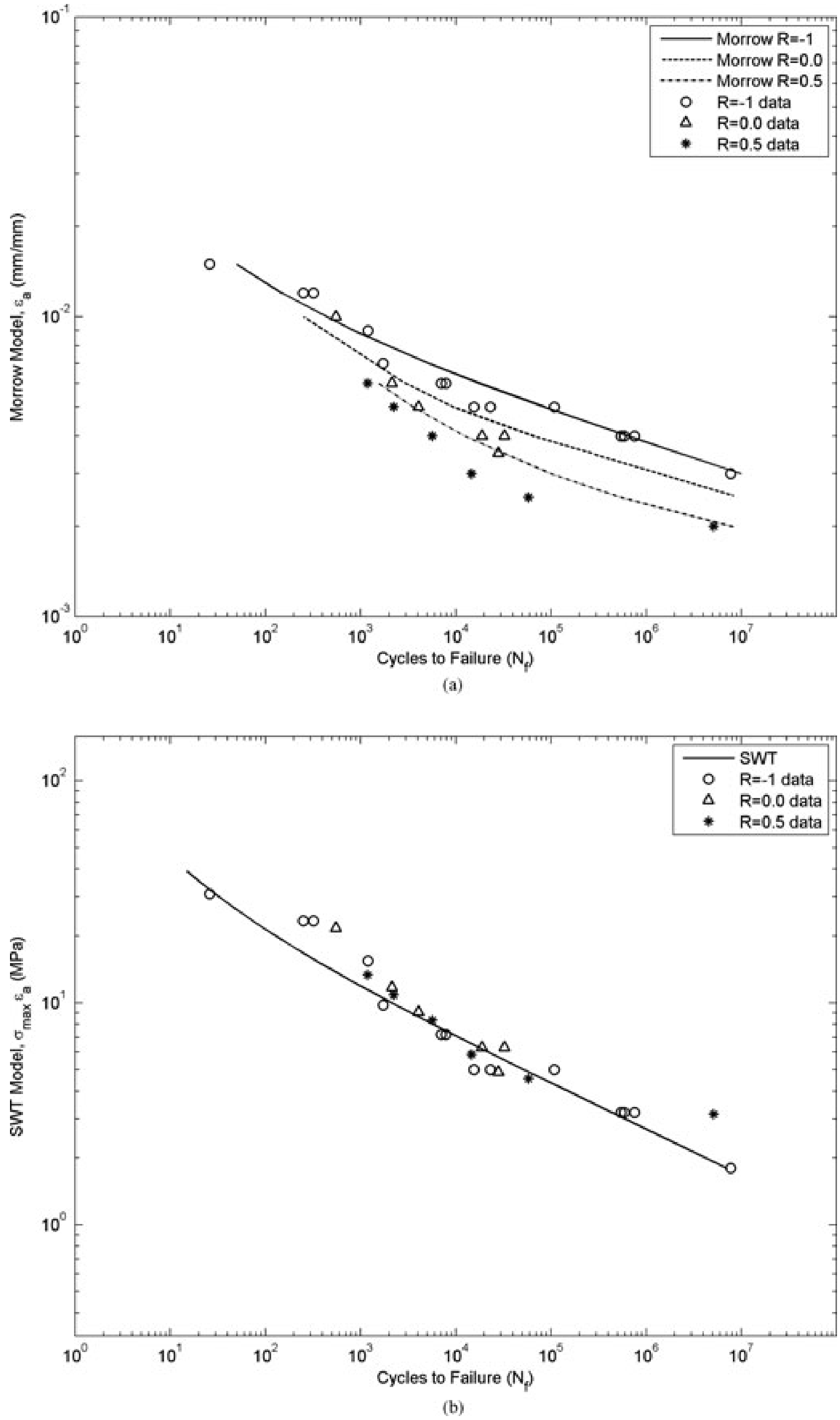


Fig. 6 (a) Comparison of the Morrow model for various strain ratios with experimental fatigue data of 1045 HRC55. (b) Comparison of the SWT model for various strain ratios with experimental fatigue data of 1045 HRC55. (c) Comparison of the SWT model for various strain ratios with experimental fatigue data of 1045 HRC55.

The excellent fatigue life predictions by the proposed model for all three strain ratios ($R = 0.0$, $R = 0.50$ and $R = 0.75$ strain ratios) can be clearly seen in Fig. 5c.

As reported by Wehner and Fatemi,[12] predictions made by the Morrow model are non-conservative for tensile mean stress data at $R = 0.0$, $R = 0.50$ and $R = 0.75$ strain ratios for 1045 HRC 55 steel as shown in Fig. 6a.

All tensile mean stress fatigue data points collapse near the SWT model damage parameter line as illustrated in Fig. 6b. Similar to the conclusion reached by Wehner and Fatemi,[12] the SWT model correlated well with the mean stress fatigue data as shown in Fig. 6b.

The proposed model shows similarly good predictions as the SWT model for 1045 HRC 55 steel in Fig. 6c.

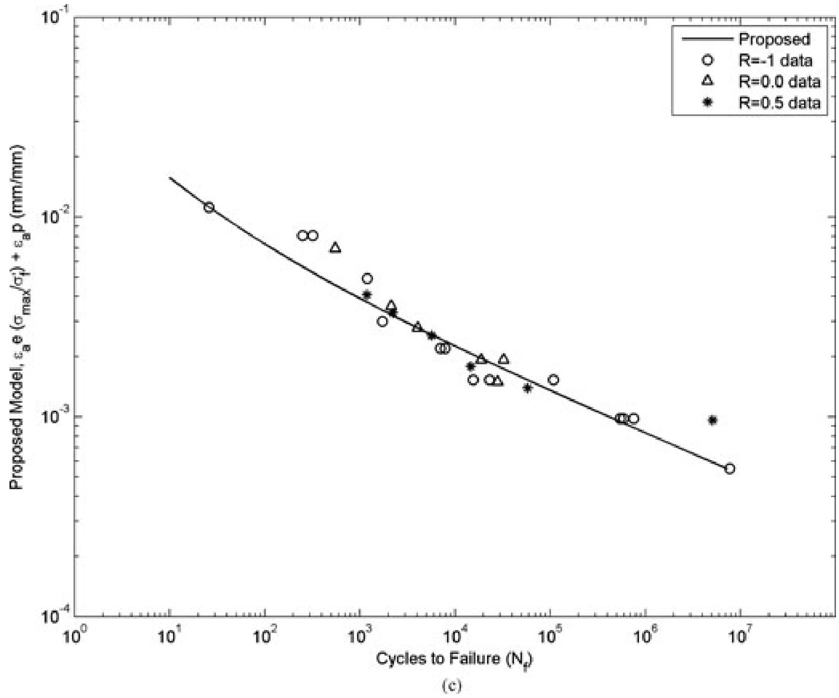


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Table 3 Statistical analysis for the comparison of the models prediction errors

Incoloy 901	Morrow	SWT	Proposed
Mean value	0.1191	−0.1018	0.0231
Standard deviation	0.1943	0.1926	0.1520
7075-T651			
Mean value	−0.0206	−0.2506	−0.2335
Standard deviation	0.4158	0.3245	0.2787
ASTM A723			
Mean value	0.0307	−0.1592	−0.01 955
Standard deviation	0.2968	0.2476	0.2133
SAE 1045HRC			
Mean value	0.2667	0.0551	0.1810
Standard deviation	0.3668	0.3481	0.3578

Similarity of the prediction trends makes it difficult to distinguish the prediction capabilities between SWT and proposed model for 1045 HRC 55 steel.

STATISTICAL ANALYSIS OF MODEL CORRELATIONS

The fatigue life predictions by the proposed model for tensile mean stress fatigue data for Incoloy 901 superalloy and ASTM A723 steel were found to correlate very well with the experimental lives. For the case of 7075-T651 aluminium alloy and SAE 1045 steel, both the SWT and the proposed models gave reasonably good predictions.

In order to further study the prediction accuracy of the proposed model against Morrow and SWT models for Incoloy 901 superalloy, 7075-T561 aluminium alloy, ASTM A723 steel and 1045 HRC 55 steel, a probability analysis is applied to the fatigue life prediction errors for the given tensile mean stress fatigue data. The prediction error which is the difference between the predicted logarithmic life and experimental logarithmic life for a given mean stress data point is defined as

$$\text{error} = \log_{10}\left(N_f^p\right) - \log_{10}\left(N_f^e\right) \quad (22)$$

where N_f^p is the predicted fatigue life and N_f^e is the experimental fatigue life for a given mean stress data point. If the error is positive, the prediction is non-conservative and if the error is negative the prediction is conservative.

The mean and standard deviations are considered to be good statistical quantities to summarize central tendency and spread of the prediction errors for the models. Therefore, the mean and standard deviation values for the models prediction errors are given in Table 3. As clearly seen in Table 3, the Morrow model tends to overestimate fatigue lives with the largest mean and standard deviation values. The SWT model provides conservative predictions with negative mean values and lower standard deviations. The proposed model gives very good predictions with very low mean and standard deviation values. Fitted probability density functions (PDF) for the prediction errors for the Morrow, SWT and proposed models are shown in Fig. 7–Fig. 10 for Incoloy 901 superalloy, 7075-T561

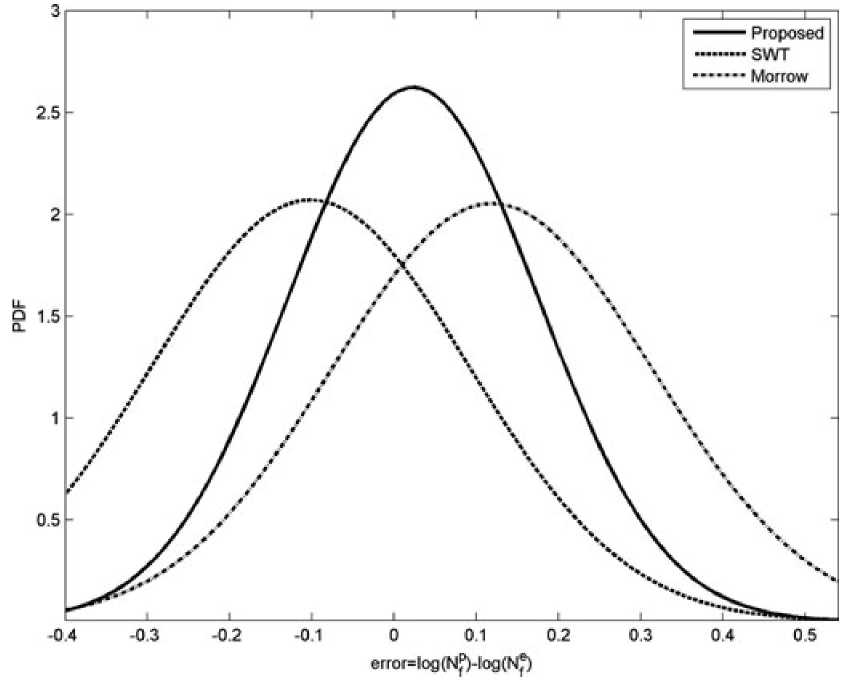

Fig. 7 Probability density function of prediction errors for the Incoloy 901 superalloy.

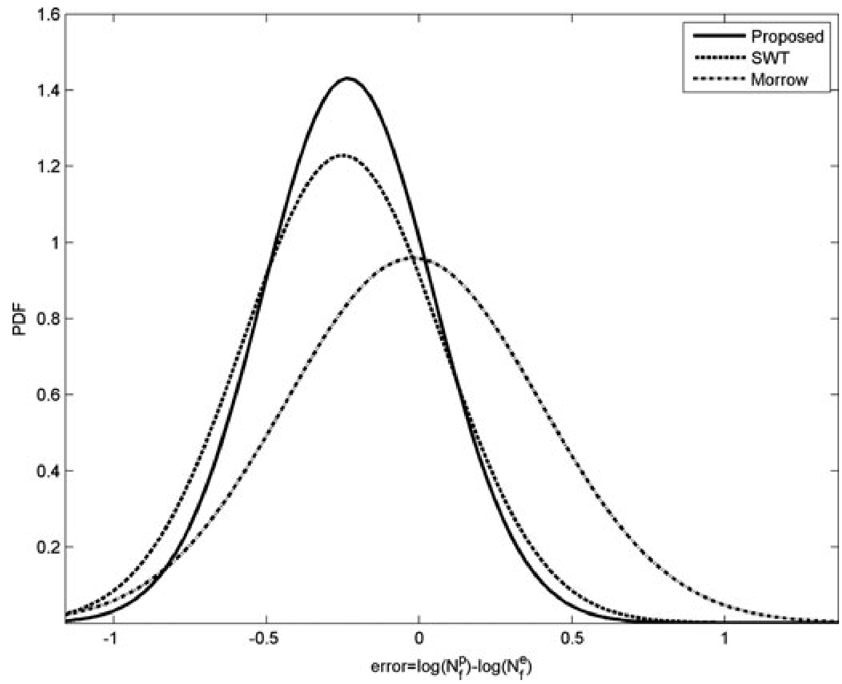

Fig. 8 Probability density function of prediction errors for 7075-T651 aluminium.

aluminium alloy, ASTM A723 steel and 1045 HRC 55 steel respectively. The PDF functions can reveal the prediction accuracy trend of the models. Figures 7 and 9 indicate that the proposed model predictions are superior to both the SWT and the Morrow models for Incoloy 901 superalloy and ASTM A723 steel with the lowest mean value and scatter of the prediction errors. The PDF of the prediction errors for 7075-T561 aluminium alloy and 1045 HRC 55 steel are shown in Figs 8 and 10 respectively. Both the SWT and the proposed model appear to provide equally good correlation with experimental data even though the proposed model predicts a slightly more non-conservative life than SWT model for 1045 HRC 55 steel. As clearly seen in Fig. 7–Fig. 10, the Morrow model tends to overestimate the fatigue lives as the PDF of the prediction errors has positive mean values and the scatter of the prediction errors is more inclined to the positive side.

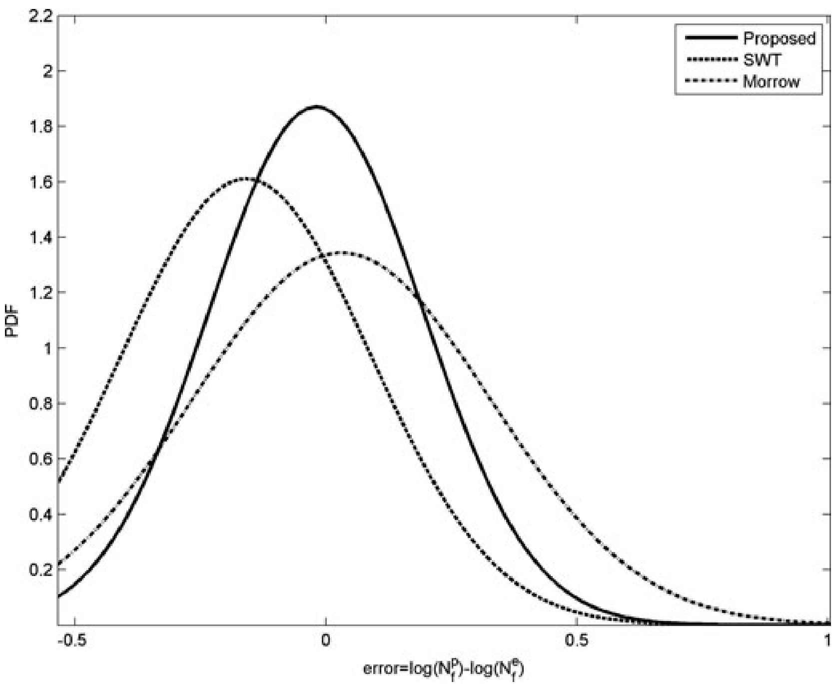

Fig. 9 Probability density function of prediction errors for ASTM A723 steel.

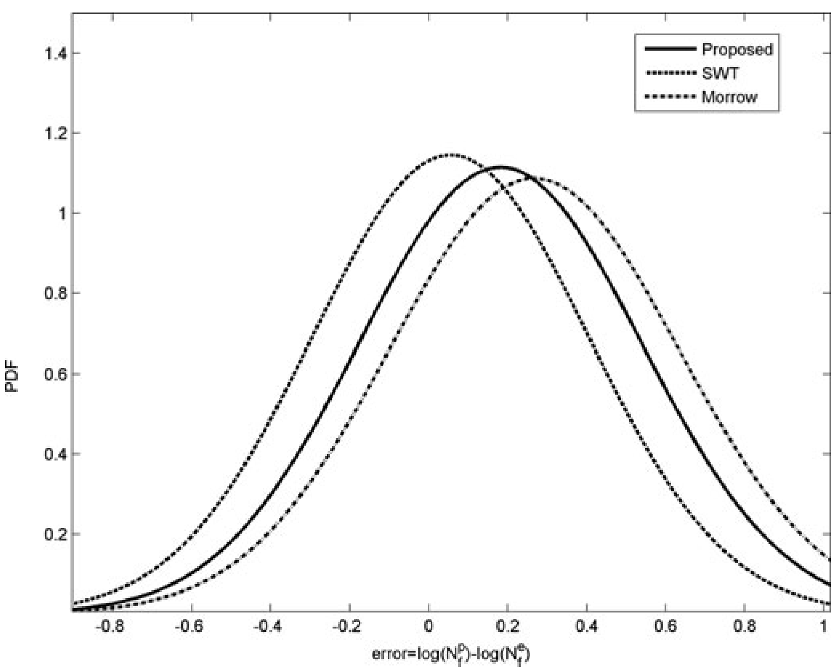

Fig. 10 Probability density function of prediction errors 1045 HRC 55 steel.

CONCLUSIONS

A modification of the Morrow and the SWT mean stress correction models has been proposed to account for the mean stress effect on fatigue life. Only four sets of experimental fatigue data for Incoloy 901 superalloy, ASTM A723 steel, 7075-T561 aluminium alloy and 1045 HRC 55 steel have been selected to investigate the prediction capabilities of the proposed model against the Morrow and the SWT model. The Morrow model provided a reasonably good correlation for Incoloy 901, ASTM A723 steel and 1045 HRC 55 steel even though the predictions were generally non-conservative in the longer life regime. However, the Morrow model gave poor correlation for 7075-T561 aluminium alloy. The SWT model correlated well with the experiments of all four materials; however, the prediction trends were slightly conservative. The proposed mean stress correction model was found to be superior to both the SWT and the Morrow model for

Incoloy 901 superalloy and ASTM A723 steel. Both the proposed and SWT models provided equally good correlation with experimental data for 7075-T561 aluminium alloy and 1045 HRC 55 steel.

While the proposed model seems very promising, based on limited data used in this study, it can be considered as moderate improvement to the Morrow and SWT models. Therefore, the proposed model should be re-examined using more experimental mean stress data sets in the future, especially under the compressive mean stresses. Any limitation of the proposed model then can be explored particularly under compressive mean stress conditions.

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