



Elastic–plastic fatigue crack growth analysis under variable amplitude loading spectra

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ABSTRACT

Most fatigue loaded components or structures experience a variety of stress histories under typical operating loading conditions. In the case of constant amplitude loading the fatigue crack growth depends only on the component geometry, applied loading and material properties. In the case of variable amplitude loading the fatigue crack growth depends also on the preceding cyclic loading history. Various load sequences may induce different load–interaction effects which can cause either acceleration or deceleration of fatigue crack growth. The recently modified two-parameter fatigue crack growth model based on the local stress–strain material behaviour at the crack tip [1,2] was used to account for the variable amplitude loading effects. The experimental verification of the proposed model was performed using 7075-T6 aluminum alloy, Ti-17 titanium alloy, and 350WT steel. The good agreement between theoretical and experimental data shows the ability of the model to predict the fatigue life under different types of variable amplitude loading spectra.

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1. Introduction

The fatigue strength of a component or structure can be significantly reduced by the presence of a crack or any other sharp discontinuities. However, in most engineering cases, the initial size of crack or discontinuity is not large enough to cause catastrophic failure. More commonly fatigue cracks propagate from the initial to the critical crack size before the final failure occurs.

The most common type of sub-critical crack growth is due to fatigue in the presence of an existing crack. In materials science, fatigue is the progressive, localised, and permanent structural damage that occurs when a material is subjected to a cyclic load. Many fatigue crack growth studies available in literature have been carried out under constant amplitude loading. As a result, constant amplitude fatigue crack growth data is in general repeatable and well understood.

The problem of predicting fatigue crack growth life becomes increasingly more complex when the loading spectrum is not constant amplitude. This is commonly referred to as *variable amplitude* or *spectrum loading* and produces what is known as *memory effects* or *load-history effects*.

One of the first fatigue crack growth models capable of predicting fatigue crack growth under spectrum loading has been pro-

posed by Wheeler [3]. The model is based on the analysis of the plastic zone size ahead of the crack tip. The model was shown to be successful for estimating the fatigue crack growth life under constant amplitude loading spectra interrupted by single or repeated overloads. However, it has some difficulties when dealing with under-load cycles occurring periodically within a load spectrum.

A very popular approach to account for the stress-ratio dependence and load–interaction effect is the use of the closure-corrected stress intensity range, ΔK_{eff} . The closure model was initially proposed by Elber [4] and it is based on the plastic deformations and crack face interaction in the wake of the crack and it was later modified to model the fatigue crack growth under variable amplitude loading [5]. Numerous studies have been attempted to explain the fatigue crack growth behaviour using the crack tip closure model.

However, there is no general agreement concerning the importance of the closure effect for the crack growth analysis. Based on observations of stress-ratio dependence of threshold values in vacuum for both steel and aluminium alloys, Louat et al. concluded [6], "... closure cannot be expected to provide a rationale for many fatigue crack growth phenomena, such as load-ratio effects on threshold". Vasudevan et al. in a separate article [7] challenged the validity of plasticity-induced closure in general. Another difficulty with applying the closure methodology is that it requires either measurement or prediction of the opening load.

The UniGrow fatigue crack growth model initially proposed by Noroozi et al. [1] is based on the elastic–plastic stress–strain

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Nomenclature

a	crack length	m	fatigue crack growth exponent
C	fatigue crack growth constant	N_f	number of cycle to fail the first element
da/dN	crack growth rate	p	driving force constant
ΔK_{appl}	applied stress intensity range	ρ^*	elementary material block size
ΔK_{tot}	total stress intensity range	σ_r	residual stress distribution
$\Delta \kappa$	two-parameter driving force	σ_{min}	minimum stress distribution
$K_{\text{max,appl}}$	maximum applied stress intensity factor	σ_{max}	maximum stress distribution
$K_{\text{max,tot}}$	total maximum stress intensity factor	$\sigma_{\text{min,res}}$	minimum resultant stress field
$K_{\text{min,appl}}$	minimum applied stress intensity factor	SIF	Stress Intensity Factor
$K_{\text{min,tot}}$	total minimum stress intensity factor		
K_r	residual stress intensity factor		

response of the material at the crack tip region. This model can be also related to the group of so-called ‘residual stress based models’, according to the Skorupa’s classification [8]. Residual or compressive stresses ahead of the crack tip can either delay or accelerate the subsequent fatigue crack growth depending on the current crack length and previous loading cycles.

The present work is an addition of further modification to the two-parameter fatigue crack growth model proposed by Glinka and Noroozi in order to make it to be applicable for all kinds of variable amplitude loading spectra. The UniGrow fatigue crack growth model denotes the modified two-parameter fatigue crack growth model. The modifications include among others a very important feature called here as the ‘memory effect’.

2. Basics of the two-parameter fatigue crack growth model

As it could be found in the original work by Noroozi et al. [1,2] the two-parameter fatigue crack growth model is based on the following assumptions:

- the material consists of the elementary particles or blocks of size, ρ^* ,
- the fatigue crack is understood as a notch with a notch tip radius, ρ^* ,
- the material stress–strain behaviour can be described by the Ramberg–Osgood stress–strain curve [9],
- the modified for stress multiaxiality Neuber rule [10,11] can be used to determine the actual elastic–plastic stresses and strains in the crack tip region,
- the number of cycles required to break one elementary material block, N_f , can be determined using Smith–Watson–Topper damage parameter [12] and the Manson–Coffin strain-life material curve,
- the instantaneous fatigue crack growth rate can be determined as a ratio of the elementary material block size and the number of cycles needed to break the material block $\frac{da}{dN} = \frac{\rho^*}{N_f}$.

Based on these assumptions a fatigue crack growth expression was derived in the form of

$$\frac{da}{dN} = C(K_{\text{max,tot}}^p \Delta K_{\text{tot}}^{1-p})^m \quad (1)$$

It should be clearly stated that Manson–Coffin strain-life approach and Smith–Watson–Topper fatigue model were used only to derive the form of the UniGrow fatigue crack growth model (Eq. (1)). Fatigue crack growth constants ‘ C ’ and ‘ m ’ in Eq. (1) can be expressed in terms of the Manson–Coffin and Ramberg–Osgood material constants. However, if constant amplitude fatigue crack growth data is available, it is preferable to present this data in

terms of total two-parameter driving force, $K_{\text{max,tot}}^p \Delta K_{\text{tot}}^{1-p}$, and fit the required fatigue crack growth constants using the linear regression method. Thus, the accuracy of the proposed model won’t depend on the Manson–Coffin strain-life approach.

A similar expression was also proposed by Walker [13], with the only difference being that the applied values of maximum stress intensity factor and the stress intensity range were used instead of total ones. Total and applied stress intensity parameters differ only by the amount of the residual stress intensity factor, K_r , corresponding to the compressive residual stresses in the crack tip region, σ_r .

$$\begin{aligned} K_{\text{max,tot}} &= K_{\text{max,appl}} + K_r \\ \Delta K_{\text{tot}} &= \Delta K_{\text{appl}} + K_r \end{aligned} \quad (2)$$

The instantaneous fatigue crack growth should be estimated using Eq. (1) on cycle by cycle base. The variable amplitude loading effects are accounted for by including the residual stresses due to reverse cyclic plasticity into the fatigue crack growth analysis. Therefore, the correct estimation of residual stresses produced by all previous loading cycles and corresponding residual stress intensity factor becomes one of the most important (and complicated) part of the UniGrow fatigue crack growth model.

The following list shows step-by-step procedure for the determination of the fatigue crack increment due to one loading cycle based on the UniGrow approach:

1. Estimate the residual stresses induced by the current loading cycle based on the multiaxial Neuber rule (or non-linear FE analysis).
2. Combine residual stresses induced by the current loading cycle with residual stresses induced by the preceding loading cycles based on five rules described in the next chapter.
3. Calculate the residual stress intensity factor, K_r , base on the weight function technique [14].
4. Determine the total maximum stress intensity factor, $K_{\text{max,tot}}$, and the total stress intensity range, ΔK_{tot} , based on Eq. (2).
5. Determine the instantaneous fatigue crack growth based on Eq. (1).

3. Modifications of the UniGrow model for a variable amplitude loading

3.1. Definition of the resultant minimum stress field

It can be noted that each cycle of the loading spectrum produces a qualitatively similar type of near the crack tip stress field. As it was mentioned before, the UniGrow fatigue crack growth model

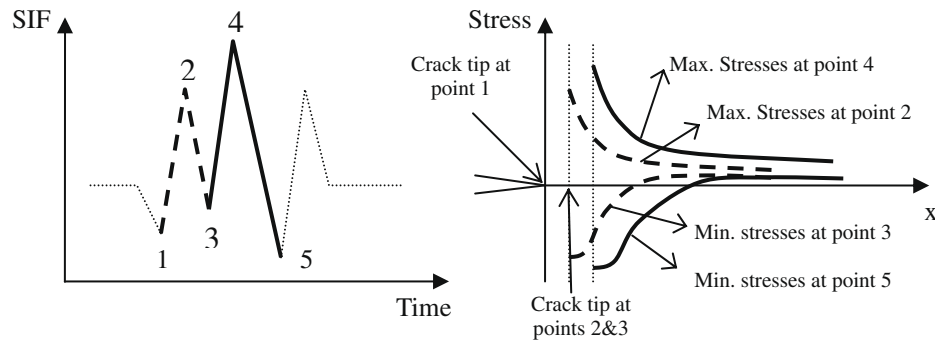


Fig. 1. Schematic stress field in the crack tip region.

suggest using the Neuber rule to determine the actual crack tip stresses induced by a loading cycle. Let us consider several consecutive reversals of an arbitrary variable amplitude loading history shown in Fig. 1.

Application of the tensile load reversal from point 1 to the point 2 may extend the fatigue crack by certain increment, Δa , and therefore, the maximum stresses corresponded to the maximum load level (2) have to be associated with the new crack tip position. On the other side, the fatigue crack does not grow during the unloading reversal from load level (2) to load level (3), thus the static notch analysis can be used to determine the minimum stresses corresponding to the minimum load level (3). Next load reversal (3–4) may again propagate the fatigue crack and a new compressive minimum stress field can be created at the load level (5). However, it is important to understand that the actual fatigue crack growth increment due to load reversal (3–4) is less than it could be under the applied range of stress intensity factor due to the presence of the compressive (residual) stresses left behind the crack tip.

Based on the experimental observations of fatigue crack behaviour under variable amplitude loading it was concluded that the residual stress intensity factor for the current cycle is not only a function of the residual stress field ahead of the crack tip induced by the last cycle, but can also depend on the residual stress fields produced by the previous cycles of the loading history. Therefore this effect has to be taken into account while calculating fatigue crack growth increment induced by the current load cycle. It is also necessary to define when the effect of the previous cycle (or cycles) can be neglected due to the fact that the crack tip has propagated out of its zone of influence.

Based on the available experimental data [15] a new methodology for obtaining the residual stress intensity factor representing the current load cycle has been proposed. Five rules have been formulated necessary for the determination of the residual stress intensity factor required for subsequent estimation of the instantaneous fatigue crack growth rate and crack increments. According to the proposed methodology all crack tip stress distributions induced by previous cycles have to be combined into one resultant minimum stress field influencing current fatigue crack growth rate.

Following is the description of four rules concerning the material memory and the overload retardation effect. The last rule concerning the under-load acceleration effect is described in the next section.

- First, only the compressive part of the crack tip stress field corresponding to the minimum load affects the fatigue crack growth rate. In the case of the stress history shown in Fig. 2 the compressive part of the minimum stress distribution induced by the first loading cycle constitutes the initial resultant minimum stress field used for the determination of the residual

stress intensity factor. The rule is schematically explained in Fig. 2a.

- Secondly, if the compressive part of the minimum stress distribution of the current loading cycle is fully or partly outside of the previous resultant minimum stress field they should be combined (Fig. 2b).
- Thirdly, if the compressive part of the minimum stress distribution induced by the current loading cycle is completely inside of the previous resultant minimum stress field, the material does not “feel” it and the current minimum stress distribution should be neglected (Fig. 2c)
- The fourth rule states that, each minimum stress distribution should be included into the resultant one only when the crack tip is inside of its compressive stress zone. In other words, when the crack tip has propagated across the entire compressive stress zone of given minimum stress field it should be neglected (Fig. 2d).

3.2. Theoretical analysis of simple loading spectra

The four rules established in the previous section enable to determine the resultant minimum stress field corresponding to the current loading cycle. However, before trying to predict the fatigue life under completely variable amplitude loading one has to be sure that the proposed model gives correct qualitative trend in the case of selected simple loading histories.

3.2.1. Constant amplitude loading interrupted by a single overload

A tensile constant amplitude loading history with relatively high overload, minimum stress distributions, and the corresponding residual stress intensity factor are shown in Fig. 3.

It should be noted, that the compressive minimum stress distribution generated by the overload is higher in the magnitude than the compressive minimum stress distributions generated by the constant amplitude loading cycles. Therefore, the application of the high tensile overload results in the decrease of the fatigue crack growth called often as the retardation effect. This fatigue crack growth phenomenon has been confirmed by a wide variety of experimental data [15].

The effect of the single overload resulting from its residual stress distribution extends much longer than that one induced by the constant amplitude loading cycles. This is due to the fact that the range of influence of the overload effective zone is larger than the one induced by smaller constant amplitude loading cycles. However, as soon as the crack propagates through the highest part of the residual stress field induced by the overload the residual stress intensity factor starts decaying and should eventually reach the same level as that for the constant amplitude loading. This effect comes from the fact that the part of the stress field close to the crack tip is much more important than the remaining one.

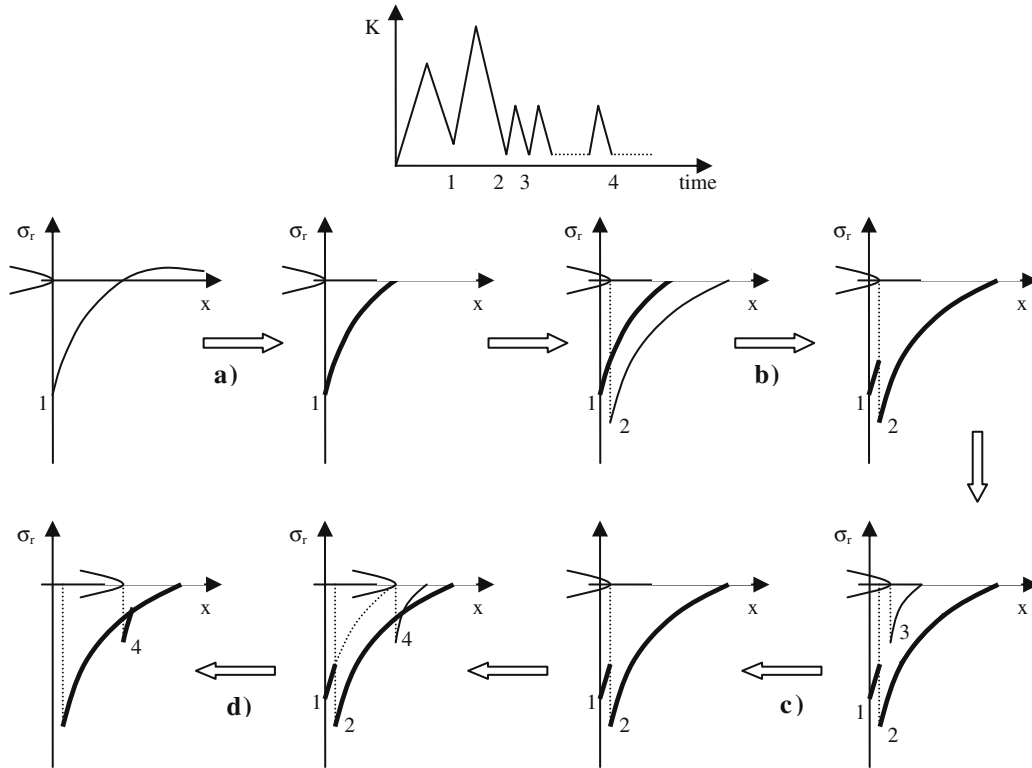


Fig. 2. Four memory rules.

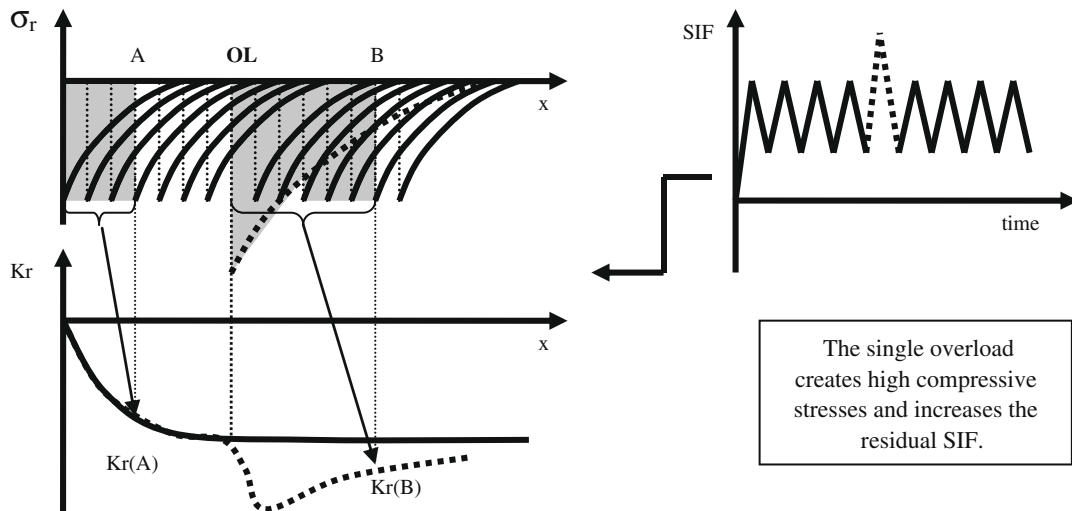


Fig. 3. Minimum compressive stress distributions produced by constant amplitude loading history interrupted by a single overload.

3.2.2. Constant amplitude loading interrupted by a single under-load

Let consider now the same tension to tension constant amplitude loading interrupted by an under-load (Fig. 4), and analyze the compressive minimum stress distribution corresponding to the under-load minimum load level (2)

It should be noted again that the minimum stress distribution corresponding to the absolute minimum load level (2) in Fig. 4 is greater in magnitude than that one generated under constant amplitude loading. However, experiments show that under-loads do not create retardation effect analogously to overloads and they can even cause a minor acceleration of the fatigue crack growth. Additionally, it coincides with the theory of static notches where the application of a high tensile or compressive under-load does

not reduce a lot the mean value of the following constant amplitude loading cycles.

Therefore, it was concluded that the compressive minimum stress field associated with the load level (2) affects the fatigue crack growth only for the immediate reversal just after the under-load (2–3). However its magnitude is different (less) at the next minimum load level (4). Therefore the following rule has been developed for the calculation of the compressive minimum stresses corresponding to the minimum load level (4) following the under-load.

- The minimum stresses distribution at the load level (4) in Fig. 4 has to be determined by taking into account minimum stresses

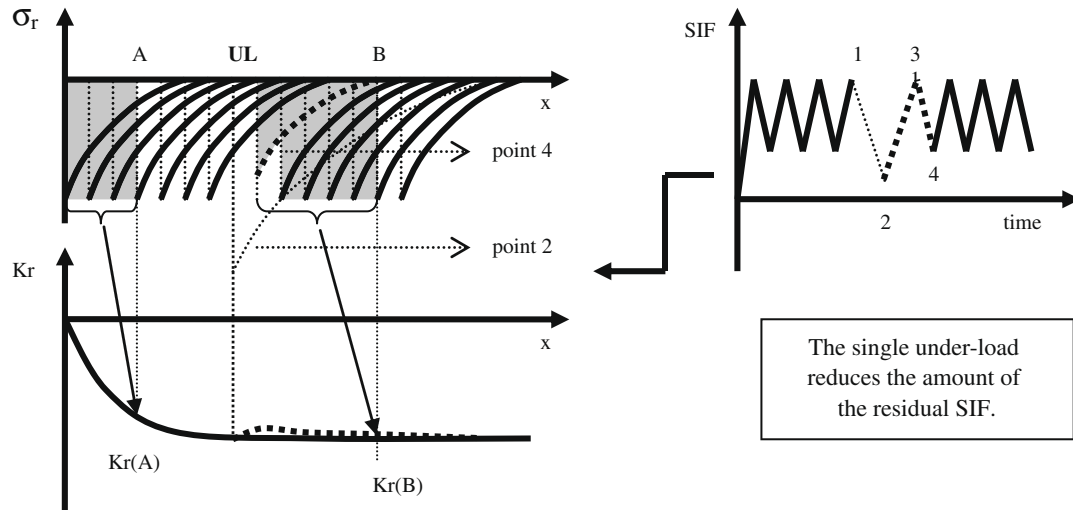


Fig. 4. Minimum compressive stresses under constant amplitude loading history interrupted by a single under-load.

generated by the previous under-load (2) but following the cyclic stress–strain curve from load level (2)–(4). The compressive minimum stress distribution which corresponds to the current loading cycle (load level (4)) and influenced by the previous under-load is that one to be used for the determination of subsequent crack increments caused by the following load cycles. This is general rule which has to be applied to all tensile and compressive under-loads.

3.2.3. Mixed loading spectra

The same rule applies in the case of more complicated stress histories containing both the over- and under-loads (Fig. 5).

The compressive minimum stress distribution induced by the overload (1) effects the fatigue crack growth up to the application of the under-load reversal with the minimum load level at point (2). During the application of the under-load reversal (2) another significant compressive stress field was generated which may overlap the compressive stress field created by previous overload (third rule). However, according to the rule proposed in the previous section the stress field corresponding to the load level (2) af-

fects the fatigue crack growth generated only by the immediate next reversal. The fatigue crack growth caused by subsequent loading cycles is affected by the minimum stress distribution corresponding to the load level (3) which is relatively small comparing with the minimum stress distributions induced by the over- or under-load.

Therefore, the retardation effect created by the overload (overloads) can be fully or partially eliminated by the following under-load reversal depending on relative magnitude of applied loads and the stress–strain material behaviour in the crack tip region. For example, in the case of almost elastic perfectly plastic material (like Ti-17) even small under-loads can eliminate the effect of high tensile overloads.

4. Accuracy of the proposed fatigue crack growth model

The model is based on a series of phenomenological assumptions: Ramberg–Osgood rule, Neuber plasticity rule, Manson–Coffin strain-life approach and Smith–Watson–Topper fatigue model. Therefore, it is important to understand how the assumptions

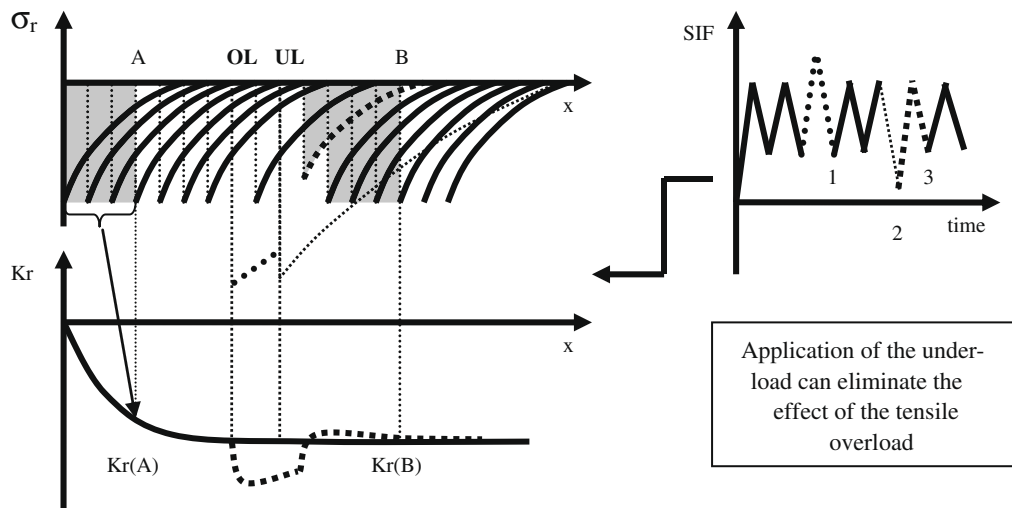


Fig. 5. Minimum compressive stresses for the combined load.

apply to the particular material and how does the error on one of the assumptions apply an overall error on the final result.

It should be clearly stated that all the phenomenological assumptions described above were used in order to derive the form of the Eq. (1). However, the UniGrow fatigue crack growth model does not require using them in the subsequent stress–strain analysis in the crack tip region. Any other approach can be used to calculate the residual stresses induced due to reverse cyclic plastic deformations.

As it was mentioned earlier, the accuracy of the model does not depend on the accuracy of the Manson–Coffin fatigue approach. Instead, it depends on the accuracy of the experimental constant amplitude fatigue crack growth data similarly to most of the existing fatigue crack growth models.

It has to be verified based on the experimental measurements that the strain–stress material behaviour can be modeled by the Ramberg–Osgood expression (for each particular material used in the analysis). If not, another and more suitable approach has to be chosen and used instead of the Ramberg–Osgood expression.

According to the Ref. [15], stresses and strains estimated based on the Neuber rule generally tend to be reasonably accurate or somewhat larger than those from more accurate non-linear numerical analysis, or from careful measurements. Therefore, one has to keep in mind that the strain–stress analysis based on the Neuber rule leads to conservative estimation of the fatigue life of a component. In order to reduce the error any other approach including the non-linear finite element analysis can be used.

It has been checked that the Ramberg–Osgood stress–strain rule and the Neuber plasticity rule work well for all three materials discussed in the next section.

It is apparent that the size of the elementary material block ρ^* has an effect on the calculated crack tip residual stresses, σ_r , and resulting residual stress intensity factor K_r . Subsequently, the residual stress intensity factor, when included into the driving force, influences predicted fatigue crack growth rate. Therefore it is important to estimate the effect of the error or variation of the ρ^* parameter on the predicted fatigue crack growth rate. Several

methods for the elementary material block size estimation have been proposed. The description of these methods is a subject of a separate discussion and not presented in this paper.

Therefore, several elementary material block sizes, ρ^* , were analysed in order to evaluate differences between corresponding residual stress distributions and resulting residual stress intensity factors. The analysis was performed for the Al 7075-T6 alloy, $K_{max,appl} = 10 \text{ MPa}\sqrt{\text{m}}$, $K_{min,appl} = 2 \text{ MPa}\sqrt{\text{m}}$.

The reference residual stress distribution was obtained for the parameter of $\rho^* = 4\text{e-}6 \text{ m}$. The lower residual stress distribution was determined for $\rho^* = 8\text{e-}6 \text{ m}$ and the upper one for $\rho^* = 2\text{e-}6 \text{ m}$. All three residual stress distributions have different magnitudes but character of the distribution remains the same. The difference between the lowest and highest residual stress magnitudes was approximately 9%. After including the difference into the total stress intensity factors the difference in the predicted fatigue crack growth was calculated from Eq. (1).

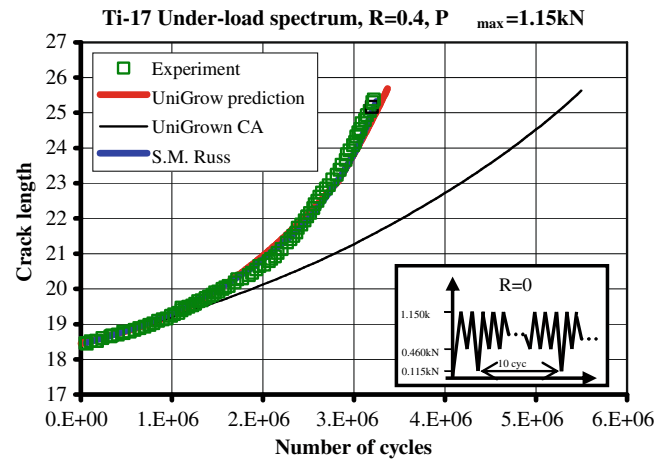


Fig. 7. Fatigue life prediction; Ti-17 alloy, under-loads, $R = 0.4$, $P_{max} = 1.15 \text{ kN}$.

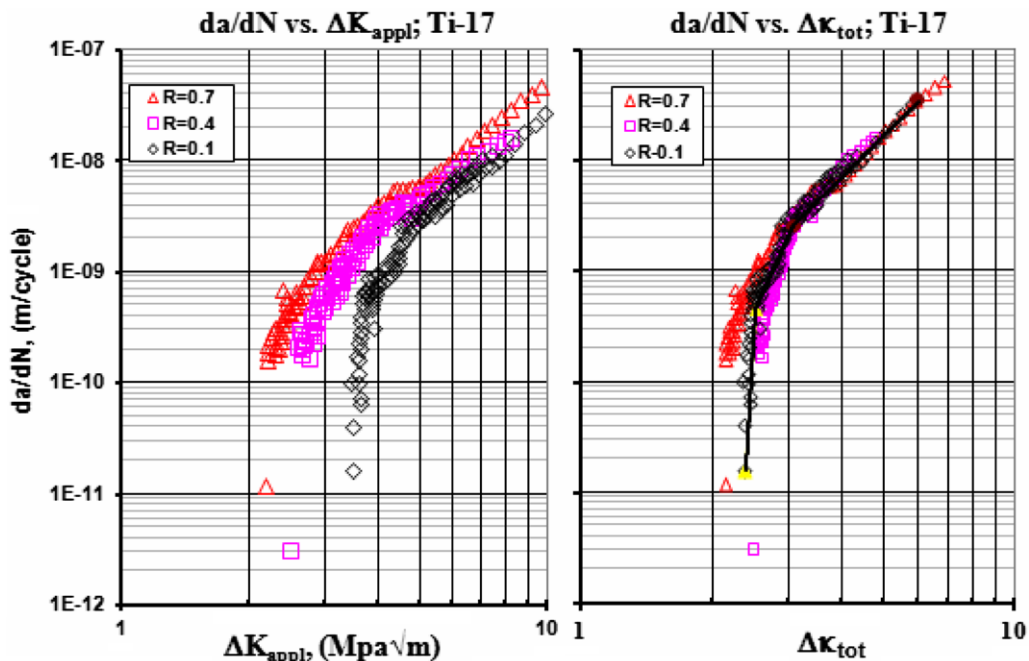


Fig. 6. Constant amplitude fatigue crack growth data in terms of applied stress intensity range (left) and total driving force (right); Ti-17 alloy (Ref. [16]).

$$\frac{(da/dN)_{\rho^*=4e-6}_1}{(da/dN)_{\rho^*=2e-6}_2} \approx \frac{C \left[(K_{\text{appl,max}} - K_{r,1})^{0.1} (\Delta K_{\text{appl}} - K_{r,1})^{0.9} \right]^{3.5}}{C \left[(K_{\text{appl,max}} - K_{r,2})^{0.1} (\Delta K_{\text{appl}} - K_{r,2})^{0.9} \right]^{3.5}}$$

$$\approx \frac{\left[(10 - 2.22)^{0.1} (8 - 2.22)^{0.9} \right]^{3.5}}{\left[(10 - 2.42)^{0.1} (8 - 2.42)^{0.9} \right]^{3.5}} = 1.127$$

It appears that the dependence of the fatigue crack growth rate on the accuracy of the ρ^* parameter is not very strong because twofold (200%) change of the ρ^* parameter resulted in less than 13% difference in predicted fatigue crack growth rates.

5. Fatigue crack growth under spectrum loading – predictions vs. experiments

In order to verify whether the model is capable of predicting fatigue crack growth under variable amplitude loading spectra the calculated fatigue crack growth results were compared with experimental data. The comparison between them shows the level of

accuracy of the proposed UniGrow fatigue crack growth model and its ability to predict the fatigue crack life for a variety of materials and loading spectra.

5.1. Loading spectra with repeatable under-load cycles (Ti-17 alloy)

Compact tension specimens made of titanium alloy Ti-17 were used in order to generate fatigue crack growth data. Ramberg-Osgood stress-strain material constants and the experimental constant amplitude fatigue crack growth data were taken from Ref. [16]. The estimated value of the elementary material block size parameter is $r^* = 7e-6$ m. All experimental constant amplitude fatigue crack growth data points were presented in terms of the total driving force, $K_{\text{max,tot}}^p \Delta K_{\text{tot}}^{1-p}$, resulting in one ‘master’ curve which was subsequently divided into three regions approximated by three linear pieces in log-log scale by using the linear regression method (Fig. 6). Numerical values of parameters of the three curves are $C_1 = 6e-34, m_1 = 59, C_2 = 1e-13, m_2 = 8.6, C_3 = 3e-11, m_3 = 3.89$.

The predicted and experimental crack length vs. number of cycles ($a-N$) data sets are shown in Fig. 7 for $P_{\text{max}} = 1.15$ kN, $P_{\text{min,BL}} = 0.45$ kN, and $P_{\text{min,UL}} = 0.115$ kN.

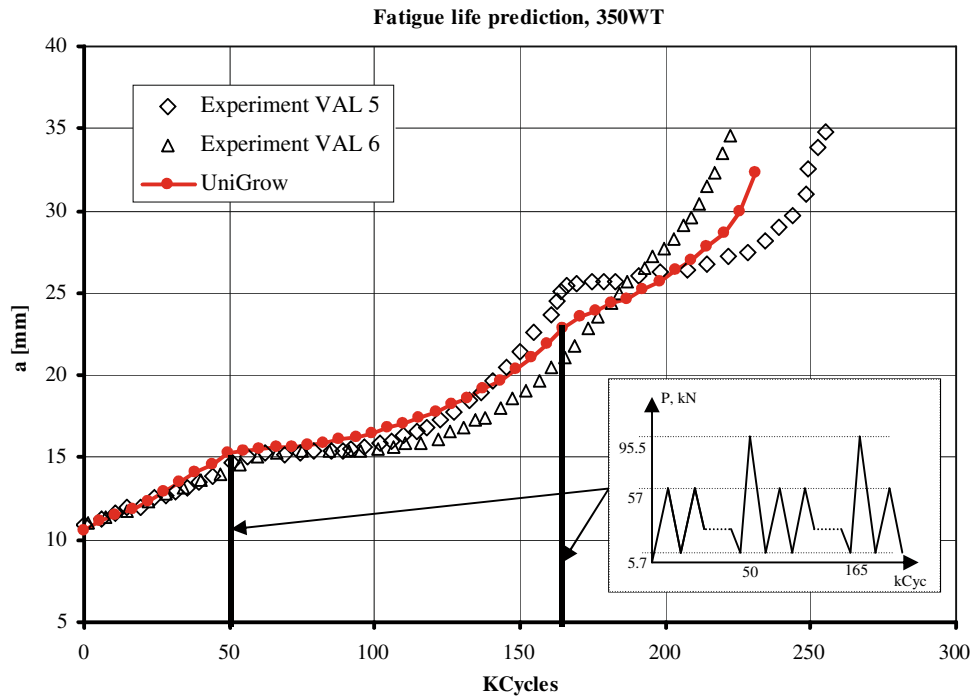


Fig. 8. Fatigue life prediction for the 350WT steel material specimen with through central crack and two overloads.

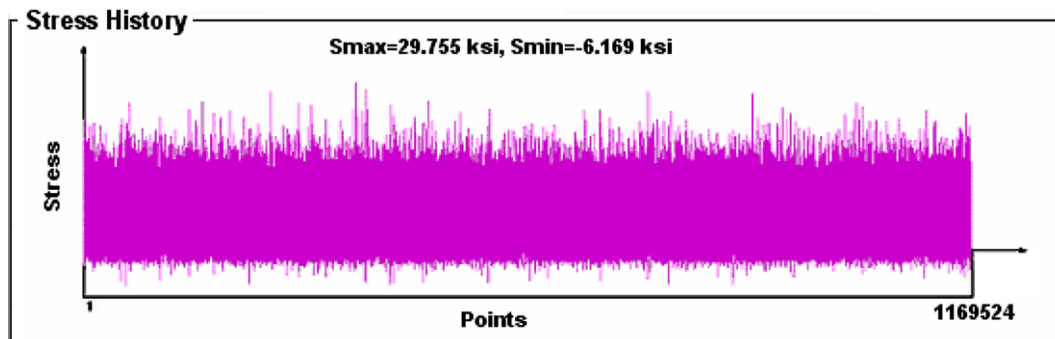


Fig. 9. Variable amplitude P3 loading spectra, Al 7075 – T6 material, specimens with a central through crack.

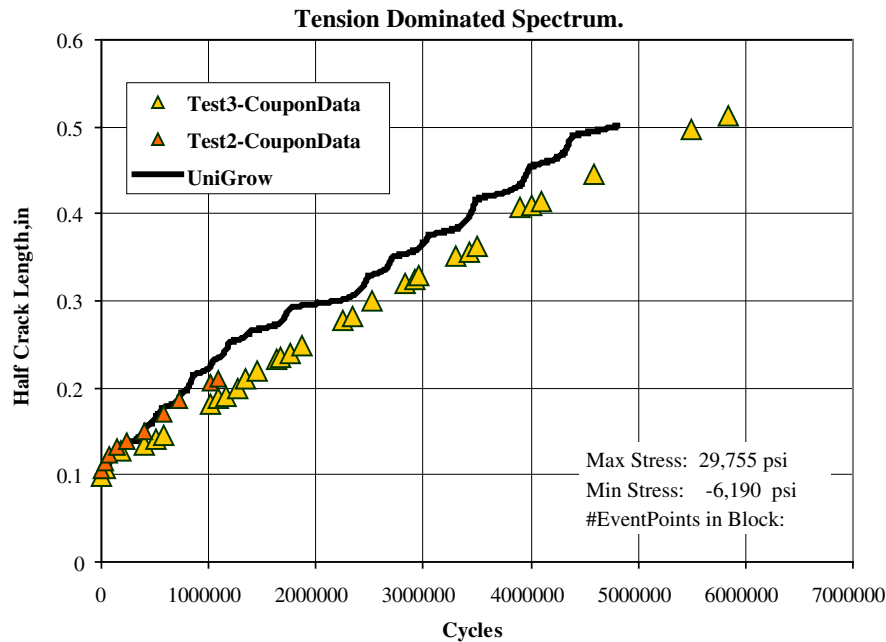


Fig. 10. Fatigue life prediction for the tension dominated P3 load spectrum, Al 7075 – T6 alloy material, specimens with a central through crack.

For comparison the UniGrow fatigue life prediction for a constant amplitude spectrum (under-loads removed) is also presented in Fig. 7. As it can be noted the proposed fatigue crack growth model gives the fatigue life similar to the experimental, and the under-loads effect is clearly shown by the difference in the fatigue life between the baseline and the modified spectra. For comparison the UniGrow fatigue life prediction for a constant amplitude spectrum (under-loads removed) is also presented in Fig. 7. As it can be noted the proposed fatigue crack growth model gives the fatigue life similar to the experimental, and the under-loads effect is clearly shown by the difference in the fatigue life between the baseline and the modified spectra.

5.2. Loading spectra with periodic overload cycles (350WT steel)

Numerous studies have demonstrated the occurrence of the fatigue crack growth retardation following a single overload [17–19]. In order to verify the UniGrow fatigue crack growth model in the presence of overloads the fatigue crack growth has been analysed and compared with the experimental results obtained from Ref. [20].

The fatigue crack growth has been studied in the central through crack specimen made of 350WT steel. Monotonic material properties as well as experimental cyclic stress–strain data were taken from a report by Chen [21]. The estimated value of the elementary material block size parameter is $r^* = 2.6\text{e-}6$ m. Numerical values of the fatigue crack growth parameters are $C_1 = 6\text{e-}18$, $m_1 = 10.62$, $C_2 = 3.1\text{e-}11$, $m_2 = 3.43$.

Two different experimental data sets are presented in Fig. 8 together with the UniGrow fatigue crack growth life prediction. The retardation effect due to each overload is clearly visible in the ‘a vs. N’ diagram in Fig. 8.

5.3. Variable amplitude loading spectra (Al 7075 – T6 material)

In order to verify the capability of the UniGrow fatigue crack growth model to predict fatigue crack propagation life under realistic loading histories, the fatigue crack growth analysis was car-

ried out for P3 aircraft loading spectra shown in Fig. 9 provided by the TDA, Inc. USA [22].

The loading spectrum was predominantly tensile dominated with a number of high tensile overloads (Fig. 9).

Specimens with central through cracks made of Al 7075-T6 alloy were used in order to generate the fatigue crack growth data. The estimated value of the elementary material block size parameter is $\rho^* = 4.4\text{e-}6$ m.

The fatigue crack life predicted based on the UniGrow fatigue crack growth model and the experimental measurements, supplied by the TDA, are shown in Fig. 10. Both the final fatigue life and the fatigue crack growth ‘a vs. N’ profiles are quantitatively and qualitatively well simulated by the proposed model. In spite of the fact that the fatigue crack growth law proposed by Eq. (1) is a power law, both a–N curves have the same shape close to a straight line. This effect comes from the fact that the general minimum stress field was created mostly by stresses induced by high periodic overloads. Therefore the instantaneous residual stress intensity factor was mostly dependent on the overloads.

6. Conclusions

The UniGrow fatigue crack growth model based on the analysis of the elastic–plastic stress/strain behaviour in the crack tip region has been modified in order to make it applicable to a wide variety of loading spectra. The load-interaction effect occurring in the case of variable amplitude loading was simulated by accounting for residual compressive stresses produced by the reverse plastic deformation in the crack tip region.

It was concluded that the instantaneous fatigue crack growth rate depends not only on the residual stresses produced by the previous loading cycle, but depend on all stress fields generated by the previous loading history. Based on this observation, several rules have been established in order to combine residual stresses fields generated by all preceding loading cycles into one resultant minimum stress field which affects the current fatigue crack growth rate.

Experimental verification of the proposed methodology was carried out using three different materials (Ti-17 alloy, St 350WT steel, and Al 7075 – T6 alloy) and three different types of loading spectra (under-loads, overloads, and general variable amplitude). The comparison between experimental and predicted data sets clearly shows the ability of the UniGrow fatigue crack growth model to simulate the load – interaction effect for a variety of variable amplitude loading spectra.

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