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Stress intensity factor for an embedded elliptical crack under arbitrary normal loading

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ABSTRACT

In this paper we introduce the boundary value problem of three-dimensional classical elasticity for an infinite body containing an elliptical crack. Using the method of simultaneous dual integral equations, the problem is transformed to the system of linear algebraic equations. Stress intensity factor is obtained in the form of the Fourier series expansion. Several solutions for specific cases of applied polynomial stress fields are derived and compared with existing results. Eligibility of the method for more complicated stress fields is demonstrated on the example of partially loaded elliptical crack.

1. Introduction

Fatigue durability, damage tolerance and strength evaluations of cracked structural components require calculation of *stress intensity factors* (SIF). Most practical crack configurations are embedded planar cracks subjected to complex two-dimensional stress fields.

The only crack geometry which has been studied for all types of applied loadings is a penny shaped crack. Sneddon [1] first introduced the system of dual integral equations with Bessel kernels for the case when applied stress depends on the polar angle only. Later his solution was extended by Kassir and Sih [2] for all types of applied loading.

Elliptical crack as an example of more complicated crack geometry became a real challenge for researchers in Fracture Mechanics. So far only limited number of analytical solutions for applied stress in the form of specific polynomials has been obtained (see, for example, [2–4]).

In the present paper we use the method of simultaneous dual integral equations for a problem of elliptical crack subjected to the arbitrary normal loading, compare our results with available solutions for polynomial applied loadings and introduce the solution for more complicated applied stress.

2. Method of simultaneous dual integral equations for an elliptical crack

Consider the following boundary value problem for a crack. Suppose that a planar elliptical crack (with semiaxes a and b, $a \geqslant b$) enclosed in a three-dimensional infinite elastic body (with material constants E and ν), occupies the open domain $S: \frac{x^2}{a^2} + \frac{y^2}{b^2} < 1$ in the plane $z = 0$ (in Cartesian coordinates (x, y, z)). The crack is opened up by an applied normal stress $p(x, y)$, symmetric with respect to the crack plane. Due to the symmetry of the applied load, it is sufficient to analyze the problem only in the upper half-space $z > 0$. Together with the standard equations of elastic equilibrium [5] the following boundary conditions are imposed

$$\begin{aligned} &\tau_{xz} = \tau_{yz} = 0, \quad \text{for } z = 0; \\ &\sigma_z^I(x, y, 0) = -p(x, y), \quad \text{for } (x, y) \in S \\ &w(x, y, 0) = 0, \quad \text{for } (x, y) \in S_1, \end{aligned} \tag{1}$$

where $S_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} > 1$ is an open domain in the plane $z = 0$ outside of the crack, $\tau_{xz}\tau_{yz}$, $\sigma_z^I(x, y, 0)$ are components of the elastic stress tensor and $w(x, y, 0)$ is the displacement in z-direction in the plane $z = 0$. We seek for the crack opening displacement $w(x, y, 0)$ at any arbitrary point $Q(x, y) \in S$. After the crack opening displacement has been found, stress intensity factor (SIF) at the corresponding point of the crack contour $Q'(x' = a\cos\varphi, y' = b\sin\varphi)$ can be obtained as [6]

$$K(\varphi) = \frac{E}{4(1-\nu^2)}\sqrt{\frac{\pi}{2}}\lim_{s\to 0}\frac{w(x, y, 0)}{\sqrt{s}}, \tag{2}$$

where s is the distance from the point $Q(x, y)$ to the crack front, i.e. the point $Q'(x', y')$.

A common approach to solving such a problem is based on the representation of elastic stresses and displacements by means of one function, harmonic in the half-space $z > 0$ and satisfying

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mixed boundary conditions in the plane $z = 0$ [7]. By using the representation of a harmonic function in the form of the single-layer potential, one can transform the first boundary condition in (1) into the integral equation

$$\Delta_{xy} \int\int_S \frac{w(\xi, \eta, 0)\mathrm{d}\xi\mathrm{d}\eta}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} = -\sigma(x, y), \tag{3}$$

where $\sigma(x,y) = \frac{4\pi(1-\nu^2)}{E} p(x,y)$ and Δ_{xy} is the two-dimensional Laplace operator

$$\Delta_{xy} = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{4}$$

We apply the method of simultaneous dual integral equations, used earlier by Sneddon [8] and Kassir and Sih [2] for the problem of a circular crack to solve the Eq. (3). For convenience Cartesian coordinates (x, y, z) are changed to the elliptic system $(x = ar\cos\theta, y = br\sin\theta)$. Two domains S and S_1 can be defined as

$$S : r < 1, \quad 0 \leqslant \theta \leqslant 2\pi \qquad S_1 : r > 1, \quad 0 \leqslant \theta \leqslant 2\pi. \tag{5}$$

The next step is to expand the load into Fourier series

$$\sigma(x,y) = \sigma(r,\theta) = \frac{\sigma_0^c(r)}{2} + \sum_{n=1}^{\infty} \left(\sigma_n^c(r)\cos n\theta + \sigma_n^s(r)\sin n\theta\right). \tag{6}$$

Further, we make use of the inverse Fourier transform

$$\frac{1}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} = \int_{\mathbb{R}^2} \frac{1}{\sqrt{p_1^2 + p_2^2}} \mathrm{e}^{-i(xp_1 + yp_2)} \mathrm{e}^{i(\xi p_1 + \eta p_2)} \mathrm{d}p_1 \mathrm{d}p_2 \tag{7}$$

and introduce new unknown function

$$F(p_1, p_2) = \int\int_S \mathrm{e}^{i(\xi p_1 + \eta p_2)} w(\xi, \eta, 0)\mathrm{d}\xi\mathrm{d}\eta \tag{8}$$

expanded in polar coordinates $(ap_1 = R\cos\psi, bp_2 = R\sin\psi)$ into Fourier series

$$F(R,\psi) = \sum_{m=0}^{\infty} (F_m^c(R)\cos m\psi + F_m^s(R)\sin m\psi). \tag{9}$$

Using the expansion

$$\mathrm{e}^{-iRr\cos(\theta-\psi)} = J_0(Rr) + \sum_{n=1}^{\infty} 2(-i)^n J_n(Rr)\cos n(\theta - \psi), \tag{10}$$

we can transform (3) and the last displacement boundary condition in (1) into the system of dual integral equations for every component $F_n^c(R)$

$$\begin{aligned} &\int_0^{\infty} \sum_{m=0}^{\infty} \alpha_{nm}^c F_m^c(R) J_n(Rr) R^2 \mathrm{d}R = \frac{ab^2 \sigma_n^c(r)}{2(-i)^n}, \quad 0 \leqslant r < 1 \\ &\int_0^{\infty} F_n^c(R) J_n(rR) R\mathrm{d}R = 0, \quad r > 1, \end{aligned} \tag{11}$$

where

$$\alpha_{nm}^c = \int_0^{2\pi} \sqrt{\frac{b^2}{a^2}\cos^2\psi + \sin^2\psi}\cos n\psi \cos m\psi\,\mathrm{d}\psi \tag{12}$$

and $J_n(rR)$ is the Bessel function of the first kind of order n. A similiar system can be constructed for components $F_n^c(R)$ with parameters

$$\alpha_{nm}^s = \int_0^{2\pi} \sqrt{\frac{b^2}{a^2}\cos^2\psi + \sin^2\psi}\sin n\psi \sin m\psi\,\mathrm{d}\psi. \tag{13}$$

Following Sneddon [8], we seek the solution of (11) in the form

$$F_n^c(R)R = R^{-1/2} \sum_{k=0}^{\infty} A_{kn}^c J_{n+2k+3/2}(R). \tag{14}$$

Substituting (14) into the second Eq. (11) and making use of Weber-Schnafheitlin integral [9] we note that the second equation is satisfied automatically and the first equation can be transformed into the system of linear equations with unknown coefficients A_{km}^c

$$\sum_{m=0}^{\infty} \sum_{k=0}^{\infty} C_{nmki}^c A_{km}^c = B_{ni}^c \tag{15}$$

$$C_{nmki}^c = \begin{cases} \alpha_{nm}^c, & m + 2k = n + 2i \\ 0, & \text{otherwise} \end{cases} \tag{16}$$

$$B_{ni}^c = \frac{2\left(n + 2i + \frac{3}{2}\right)\Gamma(n+i+1)}{(-i)^n \Gamma(1+n)\Gamma(i+3/2)} \int_0^1 \sigma_n^c(r) r^{n+1}\sqrt{1-r^2} \times \mathfrak{F}_i(n + \frac{3}{2}, n+1, r^2)\mathrm{d}r, \tag{17}$$

where $\mathfrak{F}_i(n + \frac{3}{2}, n+1, r^2)$ is Jacobi Polynomial of degree k in r^2, defined by the hypergeometric function

$$\mathfrak{F}_i(n + \frac{3}{2}, n+1, r^2) = {}_2F_1(-k, k+n+3/2, n+1, r^2). \tag{18}$$

Using the same approach we can obtain a similiar system for unknown A_{km}^s

$$\sum_{m=1}^{\infty} \sum_{k=1}^{\infty} C_{nmki}^s A_{km}^s = B_{ni}^s \tag{19}$$

After solving (15) and (19) and using the inverse of transformation (8) the crack opening displacement can be obtained as

$$w(r,\theta) = \frac{\pi b}{2}\sqrt{1-r^2} \sum_{n=0}^{\infty} \frac{(-i)^n r^n}{\Gamma(n+1)} \sum_{k=0}^{\infty} \frac{\Gamma(n+k+1)}{\Gamma(k+\frac{3}{2})} \times \mathfrak{F}_k\left(n + \frac{3}{2}, n+1, r^2\right)[A_{kn}^c \cos n\theta + A_{kn}^s \sin n\theta] \tag{20}$$

In order to obtain the limiting values of the crack opening displacement, according to (2), we use the following asymptotic expansion in the vicinity of $r = 1$

$$1 - r^2 = 2s\frac{1}{b}\sqrt{\frac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi} + O(s^2). \tag{21}$$

The SIF can be now obtained as

$$K(\varphi) = \pi\sqrt{b}\left(\frac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi\right)^{1/4} \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} (-i)^n(-1)^k [A_{kn}^c \cos n\varphi + A_{kn}^s \sin n\varphi]. \tag{22}$$

3. Comparison with existing solutions

To validate our solution we compare our results with analytical expressions for SIF in five different cases of polynomial applied stress obtained in [10,2–4].

1. Uniform applied pressure

$$\sigma(x,y) = \sigma_0 \quad \text{or} \ \sigma_0^c(r) = 2\sigma_0 \tag{23}$$

There is only one non-zero coefficient in (22)

$$A_{00}^c = \frac{3}{\alpha_{00}^c} B_{00}^c = \frac{\sigma_0}{\sqrt{\pi}E(k)}, \tag{24}$$

where $E(k)$ denotes elliptic integral of the second kind with $k = \frac{\sqrt{a^2-b^2}}{a}$.
The stress intensity factor in such a case is

$$K(\varphi) = \frac{\sqrt{\pi}b\sigma_0}{E(k)}\left(\frac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi\right)^{1/4}. \tag{25}$$

This solution coincides with the well known solution, first obtained by Irwin in [10].

2. Linear stress field dependent on x

$$\sigma(x,y) = \sigma_0\frac{x}{a} \quad \text{or } \sigma(r,\theta) = \sigma_0 r\cos\theta \quad \text{with } \sigma_1^c(r) = \sigma_0 r. \tag{26}$$

The appropriate solution for A_{01}^c in (22) is

$$A_{01}^c = \frac{5}{\alpha_{11}^c}B_{10}^c = \frac{4\sigma_0}{3(-i)\sqrt{\pi}\alpha_{11}^c} \tag{27}$$

and

$$\alpha_{11}^c = -\frac{4}{3}\frac{(1-2k^2)E(k) - k'^2K(k)}{k^2}, \tag{28}$$

where $K(k)$ denotes the complete elliptic integral of the first kind and $k' = \frac{b}{a}$. The SIF in such a case is

$$K(\varphi) = -\frac{\sqrt{\pi}b\sigma_0k^2\cos\varphi}{(1-2k^2)E(k) - k'^2K(k)}\left(\frac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi\right)^{1/4}. \tag{29}$$

This solution exactly coincides with the result of Shah and Kobayashi [3]. Shah and Kobyashi also pointed out, that the solution obtained by Kassir and Sih in [11] for this case of applied load and widely used by other authors, is incorrect.

3. Linear stress field dependent on y

$$\sigma(x,y) = \sigma_0\frac{y}{b} \quad \text{or } \sigma(r,\theta) = \sigma_0 r\sin\theta \quad \text{with } \sigma_1^s(r) = \sigma_0 r \tag{30}$$

Similarly to the previous case we obtain

$$A_{01}^s = \frac{5}{\alpha_{11}^s}B_{10}^s = \frac{4\sigma_0}{3(-i)\sqrt{\pi}\alpha_{11}^s} \tag{31}$$

and

$$\alpha_{11}^s = \frac{4}{3}\frac{(1+k^2)E(k) - k'^2K(k)}{k^2}. \tag{32}$$

The SIF in such a case is

$$K(\varphi) = \frac{\sqrt{\pi}b\sigma_0k^2\sin\varphi}{(1+k^2)E(k) - k'^2K(k)}\left(\frac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi\right)^{1/4}. \tag{33}$$

This solution also coincides with the result of Shah and Kobayashi [3].

4. Quadratic stress field odd in x and y

$$\sigma(x,y) = \sigma_0\left(\frac{xy}{ab}\right)^2 \quad \text{or} \quad \sigma(r,\theta) = \frac{\sigma_0 r^2}{2}\sin 2\theta \quad \text{with} \quad \sigma_2^s(r) = \frac{\sigma_0 r^2}{2} \tag{34}$$

The appropriate coefficient in (22) is

$$A_{02}^s = \frac{7B_{20}^s}{\alpha_{22}^s} = -\frac{16\sigma_0}{15\sqrt{\pi}\alpha_{22}^s}, \tag{35}$$

where

$$\alpha_{22}^s = \frac{16}{15}\frac{E(k)(2k'^4 - 2k'^2 + 2) - K(k)(k'^4 + k'^2)}{k^4}. \tag{36}$$

Finally we arrive at the SIF in the form of

$$K(\varphi) = \frac{\sqrt{\pi}b\sigma_0k^4\sin 2\varphi}{2(E(k)(2k'^4 - 2k'^2 + 2) - K(k)(k'^4 + k'^2))} \times \left(\frac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi\right)^{1/4}, \tag{37}$$

which also coincides with the result of Kassir and Sih [2].

5. Quadratic stress field even in x and y

$$\sigma(x,y) = p_{20}\left(\frac{x}{a}\right)^2 + p_{02}\left(\frac{y}{b}\right)^2 \quad \text{or } \sigma(r,\theta) = r^2(\sigma_0 + \sigma_2\cos 2\theta) \tag{38}$$

with

$$\sigma_0 = \frac{p_{20}+p_{02}}{2}, \quad \sigma_2 = \frac{p_{20}-p_{02}}{2}, \tag{39}$$

$$\sigma_0^c(r) = 2r^2\sigma_0, \quad \sigma_2^c(r) = r^2\sigma_2. \tag{40}$$

Solving the system (15), we obtain the coefficients

$$A_{00}^c = \frac{8\sigma_0}{5\sqrt{\pi}\alpha_{00}^c} \tag{41}$$

$$A_{10}^c = -\frac{32}{15\sqrt{\pi}}\frac{\alpha_{22}^c\sigma_0 - \alpha_{02}^c\sigma_2}{\alpha_{00}^c\alpha_{22}^c - \alpha_{02}^{c2}} \tag{42}$$

$$A_{02}^c = -\frac{32}{15\sqrt{\pi}}\frac{\alpha_{00}^c\sigma_2 - \alpha_{02}^c\sigma_0}{\alpha_{00}^c\alpha_{22}^c - \alpha_{02}^{c2}}. \tag{43}$$

Hence, the SIF is

$$\begin{aligned} K(\varphi) &= \pi\sqrt{b}\left(\tfrac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi\right)^{1/4}[A_{00}^c - A_{10}^c - A_{02}\cos 2\varphi] \\ &= \tfrac{8\sqrt{\pi b}}{15}\left(\tfrac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi\right)^{1/4}[C_0 + C_2\cos 2\varphi], \end{aligned} \tag{44}$$

where

$$C_0 = \sigma_0\left(\frac{3}{\alpha_{00}^c} + \frac{2\alpha_{22}^c}{\alpha_{00}^c\alpha_{22}^c - \alpha_{02}^{c2}}\right) - \sigma_2\frac{2\alpha_{02}^c}{\alpha_{00}^c\alpha_{22}^c - \alpha_{02}^{c2}} \tag{45}$$

$$C_2 = 2\frac{\alpha_{00}^c\sigma_2 - \alpha_{02}^c\sigma_0}{\alpha_{00}^c\alpha_{22}^c - \alpha_{02}^{c2}} \tag{46}$$

This result coincides with Martin's result [4]. Martin also pointed out that the result obtained by Kassir and Sih in [2] for this case of applied load is incorrect.

Note, that in all examples above for the applied loading in the form of a polynomial of arbitrary degree, both series in k and n in Eq. (22) are truncated and the method converge. We do not provide here the proof of the convergence of the method for more complicated applied stresses, but as an example of non truncated series in k in Eq. (22) we consider the applied constant stress distributed over the small elliptical region at the center of the crack

$$\sigma(r,\theta) = \frac{\sigma_0(r)}{2} = \begin{cases} \sigma_0, & r \leqslant r_0 \\ 0, & r_0 < r < 1 \end{cases}. \tag{47}$$

Then

$$B_{0i}^c = \sigma_0\frac{(4i+3)\Gamma(i+1)}{\Gamma(i+3/2)}r_0^2\,{}_2F_1\left(-\frac{1}{2}-i, 1+i, 2, r_0^2\right) \tag{48}$$

and for $n > 0$ $B_{ni}^c = 0, B_{ni}^s = 0$. We also make use of the following equation:

$$\sum_{i=0}^{\infty}(-1)^iB_{0i}^c = \frac{4\sigma_0}{\sqrt{\pi}}\left(1 - \sqrt{1 - r_0^2}\right) \tag{49}$$

For any fixed N ($N = 0, 2, 4\ldots$), the Nth approximation of the SIF is represented in the form

$$K^N(\varphi) = \pi\sqrt{b}\left(\frac{b^2}{a^2}\cos^2\varphi + \sin^2\varphi\right)^{1/4}\sum_{k=0}^{N/2}X_{2k}^N\cos 2k\varphi. \tag{50}$$

In what follows we use the notations:

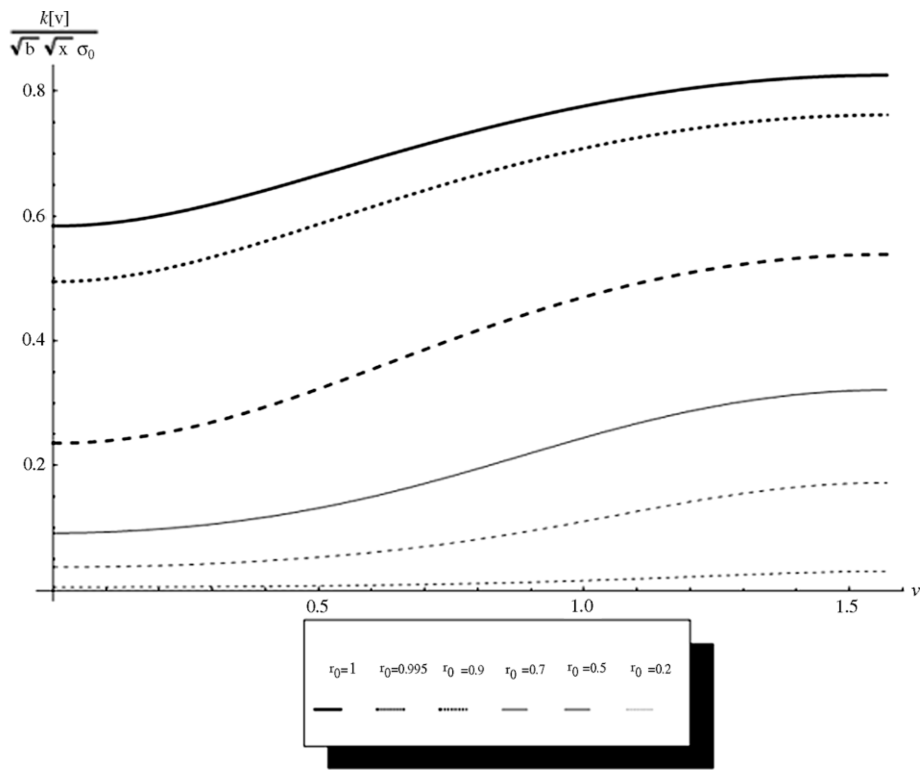

Fig. 1. SIF for an elliptical crack subjected to the constant pressure distributed over the elliptical region.

$$D_{2k} = \begin{pmatrix} \alpha^c_{00} & \alpha^c_{02} & \alpha^c_{04} & \dots & \alpha^c_{0,2k} \\ \alpha^c_{20} & \alpha^c_{22} & \alpha^c_{24} & \dots & \alpha^c_{2,2k} \\ \alpha^c_{40} & \alpha^c_{42} & \alpha^c_{44} & \dots & \alpha^c_{4,2k} \\ \dots & \dots & \dots & \dots & \dots \\ \alpha^c_{2k,0} & \alpha^c_{2k,2} & \alpha^c_{2k,4} & \dots & \alpha^c_{2k,2k} \end{pmatrix}, \quad k = 0, 1, 2, \dots \frac{N}{2}, \tag{51}$$

$\Delta_{2k} = \det D_{2k}, \delta^0_0 = 1$, δ^{2i}_{2k} is the cofactor obtained by removing the first row and $i+1$ column of the matrix D_{2k} for $i \leqslant k$ and $\delta^{2i}_{2k} = 0$ otherwise. Then

$$X^N_{2k} = \frac{\delta^{2k}_N}{\Delta_N}\frac{4\sigma_0}{\sqrt{\pi}}\left(1 - \sqrt{1 - r_0^2}\right) + \sum_{j=0}^{N/2-1}\left(\frac{\delta^{2k}_{2j}}{\Delta_{2j}} - \frac{\delta^{2k}_N}{\Delta_N}\right)(-1)^j B^c_{0j} \tag{52}$$

This solution is a new result and cannot be compared with any other literature data, but we can consider its two limiting cases. First, when $a = b$ matrix D_{2k} has the diagonal form, $\frac{\delta^0_{2j}}{\Delta_{2j}} = \frac{1}{\alpha^c_{00}}$, $\delta^{2k}_{2j} = 0$ for every j and $k \geqslant 1$. Consequently the SIF reduces to the following form:

$$K(\varphi) = 2\sqrt{\frac{a}{\pi}}\left(1 - \sqrt{1 - r_0^2}\right), \tag{53}$$

what coincides with Sneddon's result [1] for a penny-shaped crack. When $r_0 = 1$ the only non-zero coefficient $B^c_{00} = \frac{4\sigma_0}{\sqrt{\pi}}$ yields the only non-zero $X^N_0 = \frac{4\sigma_0}{\sqrt{\pi}\alpha^c_{00}}$ and the SIF reduces to the Eq. (25).

The variation of SIF along the contour for the elliptical crack with $a = 1$, $b = 0.5$ for different values of r_0 is shown in Fig. 1.

4. Conclusions

The method of simultaneous dual integral equations is the most advanced amongst existing methods for the analysis of the elliptical crack under arbitrary applied normal load. The method is much easier for numerical computations in comparison to the method proposed by Shah and Kobayashi [3].

The method of simultaneous dual integral equations is the only method which enables us to consider wide class of applied stress configurations.

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