Video Projection of Brownian Motion in Milk to Determine the Avogadro Constant: An Activity for the High School and Undergraduate Laboratory

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Summary

In 1905 Albert Einstein presented a theory of Brownian motion that suggested the experimental means to determine Avogadro's number N. Jean Perrin reported an experimental value for N three years later. We recreate Perrin's experiment to estimate Avogadro's constant using dilutions of dairy milk and video projection. The method is readily adaptable to the high school or undergraduate chemistry and physics laboratory. The merits of the procedure are the unsurpassed ease of set up and data collection. Preliminary experimental work by the authors and high school students using this approach yield estimates of Avogadro's constant in good agreement with the accepted value.

Einstein's Brownian motion equation: $$\lambda_x^2 = \frac{t}{N}\left(\frac{RT}{3\pi kP}\right)$$

where λ_x^2 = the mean square displacement of a suspended particle in the x direction
t = the duration of the displacement
R = the gas constant
T = the absolute temperature
k = the viscosity of the suspension
P = The radius of the suspended particle
N = Avogadro's constant

Procedure

1. Adjust a compound light microscope so that a 10× wide field ocular and the 40× objective lens are in place. Indirect illumination is provided by a cool fluorescent light and a mirror to avoid warming of the microscope slide.
2. Mount the video camera on the microscope eyepiece and connect the camera to the video projector.
3. Determine the magnification of the image with a stage micrometer graduated in 0.01 mm. Magnification in the order of 10 000 to 13000 times renders milk fat droplets of sufficient size for tracking and measurement. Record the magnification in Table 3 on page 2.
4. Dilute 1% or 2% milk to 0.1% with water. Add a drop of the dilute milk suspension to the well of a depression slide and close with a cover slip. View with the microscope and focus the image on screen. Large sheets of white banquet table paper make for excellent screens.
5. Select a fat droplet for tracking near the centre of the screen. Trace several outlines of the chosen droplet to estimate the diameter of its image. Record the diameter of the image in Table 3.
6. Mark and number the consecutive position of the fat droplet every 30 s for a period of 10 minutes. This will generate 21 successive positions. It is convenient for one individual to continually adjust the focus of the microscope to keep the droplet in view and to call out time while a second person is at the screen to mark positions and number positions.
7. At the end of the run, measure the distance between successive positions. These distances represent 20 successive displacements of the image on the x-y plane. Enter these values into column 2 of Table 1 on page 2.
8. Complete the remainder of Tables 2 and 3. Use the headings as guides to perform the required calculations. For example, the actual displacement of the fat droplet is found by dividing the displacement of the image on-screen by the magnification.
9. Record the ambient temperature of the room in the vicinity of the microscope in Table 3.
10. Substitute the measured values into the rearranged form of the equation above and solve for N.

$$N = \frac{t}{\lambda_x^2}\left(\frac{RT}{3\pi kP}\right)$$

Data Collection Sheet for an Estimate of Avogadro's Number

Table 1: Obtaining the Mean Square Displacement.

Interval (n)	Displacement of the Image of the Fat Droplet on Screen (cm)	Actual Displacement of the Fat Droplet (cm)	Square of Actual Displacement of the Fat Droplet λ^2_{xy} (cm^2)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
		Average λ^2_{xy}	
		Average $\lambda^2_x = \frac{1}{2}\lambda^2_{xy}$	

Table 2 The Viscosity of Water as a Function of Temperature.

Temperature (^oC)	Viscosity (k) ($g\ cm^{-1}s^{-1}$)
20	0.01002
21	0.009779
22	0.009548
23	0.009325
24	0.009111
25	0.008904
26	0.008705
27	0.008513
28	0.008327
29	0.008148
30	0.007975

Table 3: Other Relevant Data.

Time Interval t (s)	30
Gas Constant R ($\times 10^7 g\ cm^2\ s^{-2} mol^{-1} K^{-1}$)	8.314
Ambient Temperature T (K)	
Fat Droplet Diameter on Screen (cm)	
Magnification	
Actual Fat Droplet Radius P (cm)	

$$N = \frac{t}{\lambda_x^2}\left(\frac{RT}{3\pi kP}\right) = \_\_\_\_\_\_\_\_\_\_$$

"… one never ceases to experience surprise at this result, which seems, as it were, to come out of nowhere: prepare a set of small spheres which are nevertheless huge compared with simple molecules, use a stopwatch and a microscope, and find Avogadro's number."

Abraham Pais
Subtle is the Lord
The Science and the Life of Albert Einstein
1982
p. 97

Reference:
Deruyter, H. H. and Brubacher, L. J. 2008. Chem 13 News, January 2008.