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## Self-Concordant Barriers for Hyperbolic Means

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**Abstract** The geometric mean and the function $(\det(\cdot))^{1/m}$ (on the $m$-by-$m$ positive definite matrices) are examples of "hyperbolic means": functions of the form $p^{1/m}$, where $p$ is a hyperbolic polynomial of degree $m$. (A homogeneous polynomial $p$ is "hyperbolic" with respect to a vector $d$ if the polynomial $t \mapsto p(x + td)$ has only real roots for every vector $x$.) Any hyperbolic mean is positively homogeneous and concave (on a suitable domain): we present a self-concordant barrier for its hypograph, with barrier parameter $O(m)$. Our approach shows, for example, that the function $-m \log(\det(\cdot) - 1)$ is an $m$-self-concordant barrier on a natural domain. Such barriers suggest novel interior point approaches to convex programs involving hyperbolic means.