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The Complexity of a Non-Interior Path Following Method for the Linear Complementarity Problem

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Abstract The complexity of a non-interior path following method for the linear complementarity problem is studied. The method is based on Chen-Harker-Kanzow-Smale smoothing function. Its complexity is analyzed under the condition that the underlying matrix M is either a P-matrix or a positive definite matrix. When M is a P-matrix, it is shown that the algorithm terminates in at most $\mathcal{O}\left((1 + \frac{1}{\ell(M)}^2 |\log \frac{epsilon}{(1+2\beta)\mu_0}|\right)$ Newton iterations. Here β and μ_0 depend on the initial point, and $\ell(M)$ is a fundamental quantity associated with the P-matrix M. When the matrix M is symmetric positive definite, we obtain the complexity bound $\mathcal{O}\left(C^2 |\log \frac{epsilon}{(1+2\beta)\mu_0}|\right)$, where $C = 1 + \frac{\sqrt{n}}{\min\{\lambda_{\min}(M), 1/\lambda_{\max}(M)\}}$, and $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ are the smallest and the largest eigenvalues of M, respectively.

Keywords Linear Complementarity, Non-interior path-following methods, Complexity, P-matrix, Positive definite matrix.