The Complexity of a Non-Interior Path Following Method for the Linear Complementarity Problem

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Abstract

The complexity of a non-interior path following method for the linear complementarity problem is studied. The method is based on Chen-Harker-Kanzow-Smale smoothing function. Its complexity is analyzed under the condition that the underlying matrix $M$ is either a $P$-matrix or a positive definite matrix. When $M$ is a $P$-matrix, it is shown that the algorithm terminates in at most $O\left((1 + \frac{1}{\mu_0})^2 \log \frac{\epsilon}{(1 + 2\beta)^{\mu_0}}\right)$ Newton iterations. Here $\beta$ and $\mu_0$ depend on the initial point, and $\ell(M)$ is a fundamental quantity associated with the $P$-matrix $M$. When the matrix $M$ is symmetric positive definite, we obtain the complexity bound $O\left(C^2 \log \frac{\epsilon}{(1 + 2\beta)^{\mu_0}}\right)$, where $C = 1 + \frac{\sqrt{n}}{\min(\lambda_{\min}(M), \lambda_{\max}(M))}$, and $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ are the smallest and the largest eigenvalues of $M$, respectively.

Keywords

Linear Complementarity, Non-interior path-following methods, Complexity, $P$-matrix, Positive definite matrix.