A Strengthened SDP Relaxation via a Second Lifting for the Max-Cut Problem

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Abstract  We present a strengthened semidefinite programming, SDP, relaxation for the Max-cut problem, MC, and for the general quadratic boolean maximization problem. The well-known SDP relaxation can be obtained via Lagrangian relaxation and results in an SDP with variable $X \in \mathcal{S}^n$, the space of $n \times n$ symmetric matrices, and $n$ constraints, $\text{diag}(X) = e$, where $e$ is the vector of ones. The strengthened bound is based on applying a lifting procedure to this well-known semidefinite relaxation after adding the nonlinear constraints $X^2 - nX = 0$ and $X \circ X = E$. The lifting procedure is again done via Lagrangian relaxation and results in an SDP with variable $Y \in \mathcal{S}^{t(r+1)}$, where $t(r) = r(r + 1)/2$, and $2t(n - 1) + 1$ constraints. It is shown that the new bound obtained this way strictly improves the previous SDP bound, both empirically and theoretically.

Keywords  Max-cut problem, semidefinite relaxations.