Asymptotics for the Probability of Connectedness and the Distribution of Number of Components

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Abstract Let \( \rho_n \) be the fraction of structures of “size” \( n \) which are “connected”; e.g. (a) the fraction of labeled or unlabeled \( n \)-vertex graphs having one component, (b) the fraction of partitions of \( n \) or of an \( n \)-set having a single part or block, or (c) the fraction of \( n \)-vertex forests that is convenient to distinguish three cases depending on the nature of the power series for the structures: purely formal, convergent on the circle of convergence, and other. We determine all possible values for the pair \( (\lim \inf \rho_n, \lim \sup \rho_n) \) in these cases. Only in the convergent case can one have \( 0 < \lim \rho_n < 1 \). We study the existence of \( \lim \rho_n \) in this case.

AMS-MOS Subject Classification (1990): 05A16; Secondary: 05C30, 05C40