CORR 2001-21

Two Numerical Methods for Optimizing Matrix Stability

James V. Burke*, Adrian S. Lewis, Michael L. Overton*

Abstract Consider the affine matrix family $A(x) = A_0 + \sum_{k=1}^{m} x_k A_k$, mapping a design vector $x \in \mathbf{R}^m$ into the space of $n \times n$ real matrices. We are interested in the question of how to choose x to optimize the stability of the dynamical system $\dot{z} = A(x)z$. A classic example in control is stabilization by output feedback. We take two approaches. The first is to directly minimize $\alpha(A(x))$, the spectral abscissa (the largest real part of the eigenvalues) of $A(x)$, since this quantity bounds the asymptotic decay rate of the trajectories of the dynamical system. The spectral abscissa $\alpha(X)$ is a continuous but nonsmooth, in fact non-Lipschitz, function of the matrix argument X, and finding a global minimizer of $\alpha(A(x))$ is difficult. We introduce a novel random gradient bundle method for approximating *local* minimizers, motivated by recent work on nonsmooth analysis of the function $\alpha(X)$. Our second approach is to minimize a related function $\alpha_\delta(A(x))$, where δ is a *robustness* parameter in $(0,1)$. The motivation for the definition of the "robust spectral abscissa" $\alpha_\delta(X)$ is that it bounds transient peaks as well as asymptotic decay of trajectories of $\dot{z} = Xz$. the function $\alpha_\delta(X)$ is Lipschitz but typically nonconvex for $\delta \in (0,1)$, approaching $\alpha(X)$ as $\delta \to 0$ and the largest eigenvalue of $\frac{1}{2}(X + X^T)$ as $\delta \to 1$. We use a Newton barrier method to approximate local minimizers of $\alpha_\delta(A(x))$. We compare the results of the two approaches on a number of interesting test cases.