Packing Odd-Circuits in Eulerian Graphs

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Abstract Let $\mathcal{C}$ be the clutter of odd circuits of a signed graph $(G, \Sigma)$. For nonnegative integral edge-weights $w$, we are interested in the linear program $\min(w^t x : x(C) \geq 1$ for $C \in \mathcal{C}$, and $x \geq 0)$, which we denote by $(P)$. Solving the related integer program is clearly equivalent to the maximum cut problem, which is NP-hard. Guenin proved that $(P)$ has an optimal solution that is integral so long as $(G, \Sigma)$ does not contain a minor isomorphic to odd-$K_5$. We generalize this by showing that, if $(G, \Sigma)$ does not contain a minor isomorphic to odd-$K_5$ then $(P)$ has an integral optimal solution and its dual has a half-integral optimal solution.