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3-connected planar spaces uniquely embed in the sphere

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Abstract  We characterize those locally connected subsets of the sphere that have a unique embedding in the sphere – i.e., those for which every homeomorphism of the subset to itself extends to a homomorphism of the sphere. This implies that if $\overline{G}$ is the closure of an embedding of a 3-connected graph in the sphere such that every 1-way infinite path in $G$ has a unique accumulation point in $\overline{G}$, then $\overline{G}$ has a unique embedding in the sphere. In particular, the standard (or Freudenthal) compactification of a 3-connected planar graph embeds uniquely in the sphere.