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Optimal 3-terminal cuts and linear programming

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Abstract Given an undirected graph $G = (V, E)$, and three specified terminal nodes t_1, t_2, t_3, a *3-cut* is a subset of A of E such that no two terminals are in the same component of G/A. If a non-negative edge weight c_e is specified for each $e \int E$, the *optimal 3-cut problem* is to find a 3-cut of minimum total weight. This problem is $\mathcal{NP}$-hard, and in fact, is max-$\mathcal{SNP}$-hard. An approximation algorithm having performance guarantee $\frac{7}{6}$ has recently been given by Călinescu, Karloff, and Rabani. It is based on a certain linear programming relaxation, for which it is shown that the optimal 3-cut has weight at most $\frac{7}{6}$ times the optimal LP value. It is proved here that $\frac{7}{6}$ can be improved to $\frac{12}{11}$, and that this is best possible. As a consequence, we obtain an approximation algorithm for the optimal 3-cut problem having performance guarantee $\frac{12}{11}$. In addition, we show that $\frac{12}{11}$ is best possible for this algorithm.