Abstract

The *perfect matching polytope* of a graph G is the convex hull of the set of incidence vectors of perfect matchings of G. Edmonds (1965) showed that a vector x in $\mathbf{R}^E$ belongs to the perfect matching polytope of G if and only if it satisfies the inequalities: (i) $x \geq 0$ (*non-negativity*, (ii) $x(\partial(\nu)) = 1$, for all $\nu \in V$ (*degree constraints*) and (iii) $x(\partial(S) \geq 1$, for all odd subsets S of V (*odd set constraints*). We are interested in the problem of characterizing graphs whose perfect matching polytopes are determined by non-negativity and the degree constraints. (It is well-known that bipartite graphs have this property.) The appropriate context for studying this problem is the theory of matching covered graphs.

An edge of a graph is *admissible* if there is some perfect matching of the graph containing that edge. A graph is *matching covered* if it is connected, has at least two vertices and each of its edges is admissible. A cut C of a matching covered graph G is *tight* if $|M \cap C| = 1$ for every perfect matching M of G, and is *separating* if each of the two graphs obtained by shrinking a shore of C to a single vertex is also matching covered. Every tight cut is a separating cut, but the converse is not true. A non-bipartite matching covered graph is a *brick* if it has no nontrivial tight cuts and is a *solid brick* if it has no nontrivial separating cuts. We show that the above-mentioned problem may be reduced to one of recognizing solid bricks. (The complexity status of this problem is unknown.) We include a brief account of how we were led to solid bricks, present some examples and a proof of a recent theorem of Reed and Wakabayashi.