Integer Programming

CO 452/652

Winter 2013

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1. Basic information:

Lecture times and locations: 12:30-1:20 MWF, MC 4064.

Website: https://learn.uwaterloo.ca/

We will be using UW-learn this term as our course website. Log on using your username and password (same as your UW email account). The course notes, assignments and solutions will be posted there. Various announcements such as the date of exams will be posted there. It is the responsibility of the students to check the web page regularly. We always encourage students to give feedback on the lectures or the assignments. To facilitate the process we have setup on UW-learn, an online, anonymous survey. Note, you may fill in the survey as many times as you want. You are also welcome to talk to the instructor directly.

Contact information:

	Name	Office & Contact	Office hours
Instructor	*Bertrand Guenin*	*MC6056, x33641*	*TBA*

In case of emergency you may leave a message on my voice mail at x33641 or stop by my office. For non-emergencies, please either send email or visit me during office hours. When contacting me via email, please include "CO452" or "CO652" in the subject line (failure to abide by this rule may result in your email being delayed, or ignored). For questions pertaining to the current assignment please use the bulletin board which is setup on the course web page.

Course notes:

There are no course notes available. I will use a number of references for the material, including some recently published papers. Here are some books which I use as reference,

Integer and Combinatorial Optimization, George Nemhauser, Laurence Wolsey.
Theory of Linear and Integer Programming, Alexander Schrijver.

However, I do not suggest that you buy any of the above books as the material in the course will differ substantially from either of these.

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2. Course description:

Overview:

An integer program (IP) is obtained by adding to a linear program (LP) the additional condition that some of the variables have to take integer values. We will show how to formulate a number of problems as IP, including the satisfiability problem, the traveling salesperson tour (TSP) problem, and the Steiner tree problem. Despite obvious similarities between LP's and IP's, solving the former one can be done efficiently (i.e. in time polynomial in the size of the input), while no efficient algorithm is believed to exist to solve IP. We will introduce elementary concepts from complexity theory to formalize these statements.

Farkas' lemma gives sufficient and necessary conditions for the existence of a solution to a linear program. We give a self-contained geometric proof of this result and we will prove an analogous result for integer programs. The basic idea for solving an integer program is to construct a relaxation. If the optimal solution to that relaxation is integer, then it is also an optimal solution to the original integer program. The key for building efficient algorithm is to be able to construct "good" relaxations. We will see that in a sense integer programming can be reduced to linear programming, though that reduction does not necessarily lead to efficient algorithms.

We will define the notions of cones and polytope, and prove that every polyhedron can be decomposed in polytopes and cones. We will also define the notion of faces for convex sets and give a characterization of the maximal and minimal faces of polyhedra. Using these tools we will then be able to present several algorithmic strategies for solving integer programs including branch and cut, and basis reduction. In the remainder the the course we focus on the former and derive technique for generating cutting planes. We first consider specific optimization problems, such as the knapsack problem, and the TSP, and then consider general instances of integer programs. For the latter case we will study split and intersection cuts.

Tighter relaxations can be obtained by adding extra variables and constraints to the formulation. This is known as lifting. The new relaxation can be either solved in the lifted space or in the projection of that lifted space onto the space of the original variables. We review lift and project relaxations, including ones using semi-definite constraints. We present a result that shows that limits of the lifting approach.

Schedule:

This is a tentative schedule only.

Week	Topic
1	Formulations

2	Hardness
3	Good certificates
4	Relaxations
5	Decomposition of polyhedra
6	Faces
7-8	Algorithms
8	Algorithms
9	Cutting planes (specific instances)
10-11	Cutting planes (general)
12	Extended formulations

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3. Grades and expectations:

Objectives:

Students will be expected to master the following tasks and concepts:

- ability to formulate problems as IP
- complexity
- Farkas' lemma and it's integer variant
- The Weyl-Minkowsky and Meyer's theorems
- The face lattice, the characterization of minimal and maximal faces
- Branch and cut and basis reduction algorithms
- Cuts for structured integer program
- Intersection and split cuts
- Extended formulations

Class attendance:

Attendance is mandatory. If for any reason you are not able to attend class, you should let me know as soon as possible. Students who repeatedly fail to come to class will not be able to take the midterm or the final.

Assignments:

There will be five assignments to be handed in. These are to be returned *at the beginning of class* on the due date (which may vary) in class (do not use the drop off boxes). Late assignments will not be graded. You will find a discussion board on the UW-learn site. I will visit the board on a regular basis and will try to answer questions in a timely manner. Note, that it is not acceptable to discuss the assignments on any other discussion board.

While it is acceptable for students to discuss the course material and the assignments, you are expected to do the assignments on your own. Copying or paraphrasing a solution from some fellow student or old solutions qualifies as cheating. As assignments count for a significant percentage of your grade I will be particularly vigilant to ensure that students only get credit for their own work. All students found to be cheating will automatically be given a mark of 0 on the assignment (where that grade will not be ignored as one of the two lowest assignment) and further action may be taken as described in the section on Academic Integrity.

In case you have any question about the way your assignment was graded, please contact me.

Midterm:

It will cover material from weeks 1-6.
The date for the midterm has been tentatively set for *TBA*.
In case you have a conflict with that date, please contact me as soon as possible.

Grading:

Assignments:	50%
Midterm	20%
Final	30%

INC grade:

A grade of INC (incomplete) will be *only* awarded to students who cannot write the final exam for reasons acceptable to the instructor, such as a medical certificate by a recognized medical professional. In addition such students need to be in *good standing* prior to the final exam. To be in good standing a student must

- submit and pass 4 out of 5 assignments,
- write and pass the midterm exam, and
- attend classes.

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4. Academic Integrity/Students with Disabilities:

See www.uwaterloo.ca/accountability/documents/courseoutlinestmts.pdf

We reproduce the text on this site below:

Academic Integrity: In order to maintain a culture of academic integrity, members of the University of Waterloo community are expected to promote honesty, trust, fairness, respect and responsibility. [Check www.uwaterloo.ca/academicintegrity/ for more information.]

Grievance: A student who believes that a decision affecting some aspect of his/her university life has been unfair or unreasonable may have grounds for initiating a grievance. Read Policy 70, Student Petitions and Grievances, Section 4, http://www.adm.uwaterloo.ca/infosec/Policies/policy70.htm. When in doubt please be certain to contact the department's administrative assistant who will provide further assistance.

Discipline: A student is expected to know what constitutes academic integrity to avoid committing academic offenses and to take responsibility for his/her actions. A student who is unsure whether an action constitutes an offense, or who needs help in learning how to avoid offenses (e.g., plagiarism, cheating) or about "rules" for group work/collaboration should seek guidance from the course professor, academic advisor, or the undergraduate associate dean. For information on categories of offenses and types of penalties, students should refer to Policy 71, Student Discipline, http://www.adm.uwaterloo.ca/infosec/Policies/policy71.htm. For typical penalties check Guidelines for the Assessment of Penalties, http://www.adm.uwaterloo.ca/infosec/guidelines/penaltyguidelines.htm.

Appeals: A decision made or penalty imposed under Policy 70, Student Petitions and Grievances (other than a petition) or Policy 71, Student Discipline may be appealed if there is a ground. A student who believes he/she has a ground for an appeal should refer to Policy 72, Student Appeals, http://www.adm.uwaterloo.ca/infosec/Policies/policy72.htm.

Note for students with disabilities: The Office for Persons with Disabilities (OPD), located in Needles Hall, Room 1132, collaborates with all academic departments to arrange appropriate accommodations for students with disabilities without compromising the academic integrity of the curriculum. If you require academic accommodations to lessen the impact of your disability, please register with the OPD at the beginning of each academic term.