

CO 631 — Symmetric functions

This course is an introduction to symmetric functions and their applications to enumerative combinatorics, representation theory, and related areas.

Topics:

- **The algebra of symmetric functions:** Partitions, standard bases, Hall inner product and Cauchy Kernel, the fundamental involution, exponential specialization, determinantal formulae.
- **Young tableaux:** Bumping and sliding algorithms, the plactic monoid, crystal structure, the Robinson–Shensted–Knuth correspondence, enumeration of plane partitions, the Littlewood–Richardson rule.
- **Applications to species theory:** Species, orbit counting, cycle index functions.
- **The symmetric group:** Coxeter generators and relations, the group algebra and its centre, representations, Specht modules, characters, enumeration of planar maps, the Murnaghan–Nakayama rule.
- **Schubert calculus:** The Grassmannian, Schubert varieties, Schubert intersections, puzzles.

References:

- *The symmetric group*, Bruce E. Sagan, Springer, 1991.
- *Young Tableaux*, W. Fulton, Cambridge University Press, 1997.
- *Symmetric functions and Hall Polynomials*, I.G. Macdonald, Oxford University Press, 1979.
- *Enumerative Combinatorics, Volume 2*, Richard P. Stanley. Cambridge University Press, 1999.
- *Combinatorial Enumeration*, David M. Jackson and Ian P. Goulden, John Wiley & Sons, 1983. (Republished by: Dover Publications, 2004.)

Prerequisites: Coming into this course, you should be comfortable with undergraduate level abstract algebra, particularly linear algebra and group theory. I will also assume you have some background in enumeration, including elementary counting and basic ordinary generating function techniques. If you have not worked with generating functions before, I recommend you read Ch. 1–6 of the CO 330 course notes to get up to speed. The algebra background is more crucial. If you feel your abstract algebra foundations are weak, please come talk to me before committing to the course.

CO 630 vs. CO 631: CO 630 is not a prerequisite for CO 631 (although there is some related material), and I will not assume that you have taken it. The two courses are intended to be complementary to each other. CO 630 is about developing algebraic tools to solve combinatorial problems. CO 631 is more about some algebraic problems with rich combinatorial structure.