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94-01 **A nonlinear duality result equivalent to the Clarke-Ledyaev mean value inequality**

A.S. Lewis and D. Ralph*

A recent result of Clarke and Ledyaev extends the classical mean value theorem: its smooth, finite-dimensional case states that if X and Y are compact, convex, nonempty sets in IR^m, and f is a continuously differentiable function on a neighbourhood of the convex hull Z of $X \cup Y$, then for some z in Z,

$$\langle f'(z), y - x \rangle \geq \min_Y f - \max_X f \text{ for all } y \in Y \text{and } x \in X.$$

We observe that this is equivalent to a nonlinear version of the classical Fenchel duality theorem, and prove some variants.

Key Words and phrases: Mean value theorem, Fenchel duality.

94-02 **Compositions with distinct parts**

B. Richmond and A. Knopfmacher*

The number of compositions $C(n)$ of a positive integer n into distinct parts can be considered as a natural analogue of the number $q(n)$ of distinct partitions of n. We obtain an asymptotic estimate for $C(n)$ and in addition show that the sequence $\{C(n,k)\}$ of distinct compositions of n with k distinct parts is unimodal. Our analysis is more complicated than is usual for composition problems. The results imply however that the behaviour of these functions is of comparable complexity to partition problems.

94-03 **An umbral relation between pattern and commutation in strings**

I.P. Goulden, D. M. Jackson and C.M. Springer*

Two settings for string enumeration are considered in which string statistics can be constructed such that the generating series for the set of all strings have the form $(F^{-1} \ o \ a)^{-1}$ in both cases, where F is a formal power series and a is a sequence. The two settings are qualitatively different, one involving pattern, which is locally testable, and the other involving commutation in strings, which is not locally testable. Evidence for a common generalization of these two settings is considered.

94-04 **Derivatives of spectral functions**

A.S. Lewis

A *spectral* function of a Hermitian matrix X is a function which depends only on the eigenvalues of $X, \lambda_1(X) \geq \lambda_2(X) \geq \ldots \geq \lambda_n(X)$, and hence may be written $f(\lambda_1(X), \lambda_2(X), \ldots, \lambda_n(X))$ for some symmetric function f. Such functions appear in a wide variety of matrix optimization problems. We give a simple proof that this spectral function is differentiable at X if and only if the function f is differentiable at the vector $\lambda(X)$, and we give a concise formula for the derivative. We then apply this formula to deduce an analogous expression for the Clarke generalized gradient of the spectral function. A similar result holds for real symmetric matrices.

Keywords and phrases: matrix functions, spectral functions, eigenvalues, perturbation, unitarily invariant, differentiability, nonsmooth analysis, Clarke derivative

94-05 **Hypercubes and Multicommodity Flows**

B.Yu, J. Cheriyan and P. E. Haxell

The average degree of a subgraph H of the r-dimensional hypercube Q_r equals at most the maximum Hamming distance of any two nodes in H.

A corollary is that the minimum number of edges to delete from Q_r such that any two nodes at Hamming distance ℓ are separated is $(r + 1 - \ell)2^{r-1}$.

This corollary has applications to multicommodity flows.

Keywords and phrases: Cube graphs, compression, average degree, multicommodity flows, minimum cuts.

94-06 **The Quadratic Assignment Problem: A Survey and Recent Developments**

Panos M. Pardalos*, Franz Rendl*, and Henry Wolkowicz

Quadratic Assignment Problems model many applications in diverse areas such as operations research, parallel and distributed computing, and combinatorial data analysis. In this paper we survey some of the most important techniques, applications, and methods regarding the quadratic assignment problem. We focus our attention on recent developments.

Keywords and phrases: Combinatorial optimization, quadratic assignment problem, graph partitioning, survey, exact algorithms, heuristics, algorithms, test problems, bibliography.

94-07 **A Recipe for Best Semidefinite Relaxation for $(0,1)$-Quadratic Programming**

Svata Poljak, Franz Rendl,* and Henry Wolkowicz

Current and new quadratic relaxations of $(0,1)$-quadratic programming problems with linear equality constraints are studied. A recipe for constructing the best quadratic relaxation using the Lagrangian dual of an appropriate quadratically constrained program is presented. The dual of this Lagrangian dual then provides a semidefinite relaxation. The main result shows that a quadratic Lagrangian relaxation yields the best possible bound over all possible quadratic relaxations and this bound is equal to many of the bounds from quadratic type relaxations that are currently being studied in the literature. Moreover, this gives rise to a primal-dual semidefinite interior point method.

These techniques are then applied to find a best semidefinite relaxation for three hard combinatorial problems, i.e. for the quadratic assignment, graph partitioning, and max-clique problems.

Keywords and phrases: Quadratic boolean programming, semidefinite programming, bounds, Lagrangian duality, parametric programming, trust region subproblems, minmax eigenvalue problems, quadratic assignment problem, graph partitioning, max-clique, theta function.

94-08 **Factorisations for partition functions of random Hermitian matrix models, and topological series**

D. M. Jackson and T. I. Visentin*

The partition function Z_N, for Hermitian-complex matrix models can be expressed as an explicit integral over R^n, where N is a positive integer. Such an integral also occurs in connexion with random surfaces and models of two dimensional quantum gravity. We show that Z_N can be expressed as the product of two partition functions, evaluated

at translated arguments, for another model, giving an explicit connexion between the two models. We also give an alternative computation of the partition function for the ϕ^4-model. The approach is an algebraic one and holds for the functions regarded as formal power series in the appropriate ring.

94-09 **Planar Decompositions of Tableaux and Schur Function Determinants**

A.M. Hamel* and I.P. Goulden

In this paper we describe planar decompositions of skew shape tableaux into strips and use the shapes of these strips to generate a determinant. We then prove that each of these determinants is equal to the Schur function for the skew shape. The Jacobi-Trudi identity, dual Jacobi-Trudi identity, Giambelli identity and Rim Ribbon identity of Lascoux and Pragacz are all special cases of this theorem. A compact Gessel-Viennot lattice path argument provides the proof.

94-10 **Distance-Regular Graphs with $b_t = 1$ and Antipodal Double-Covers**

Makoto Araya*, Akira Hiraki*, and Aleksandar Jurišić

Let Γ be a distance-regular graph of diameter d and valency $k > 2$. If $b_t = 1$ and $2t \leq d$, then Γ is an antipodal double-cover. Consequently, if $f > 2$ is the multiplicity of an eigenvalue of the adjacency matrix of Γ and if Γ is not an antipodal double-cover then $d \leq 2m - 3$. This result is an improvement of Godsil's bound and it is very important for the classification of distance-regular graphs with an eigenvalue of small multiplicity (as opposed to a dual classification of distance-regular graphs with small valency).

94-11 **Dichromatic Sums Revisited**

W. T. Tutte

In a previous paper the author studied an enumerating power series Φ in six variables. The typical term was the sum of a dichromatic polynomial over all rooted planar maps of given numbers of vertices and faces and given valencies for the root-face and root-vertex. Loops and multiple joins were allowed. The polynomial was the one now commonly called the"Tutte Polynomial" by other writers. An equation for Φ was obtained. It made possible a recursive calculaton of coefficients in the order of increasing edge-number.

The present paper arose out of the observation that the equation for Φ takes a particularly simple form when the variables x and y of the dichromatic polynomial are each given the value 1. The value of the polynomial is then the number of spanning trees of the map concerned.

In that special case a theoretical solution is obtained. It is stated in terms of a remainder obtained when a certain power series in the four remaining variables, slightly transformed, is divided by a certain polynomial.

94-12 **Eigenpolytopes of Distance Regular Graphs**

C. D. Godsil

Let X be a graph with vertex set V and let A be its adjacency matrix. If E is the matrix representing orthogonal projection onto an eigenspace of A with dimension m then E is positive semi-definite. Hence it is the Gram matrix of a set of $|V|$ vectors in $\mathbb{R}^m$. We call the convex hull of such a set of vectors an eigenpolytope of X. The connection between the properties of this polytope and the graph is strongest when X is distance regular and, in this case, it is most natural to consider the eigenpolytope associated to the second largest eigenvalue of

A. The main result of this paper is the characterisation of those distance regular graphs X for which the 1-skeleton of this eigenpolytope is isomorphic to X.

94-13 Convexity of Minimal Total Dominating Functions in Graphs

Bo Yu

A *total dominating function* (TDF) of a graph $G = (V, E)$ is a function $f : V \to [0, 1]$ such that for each $v \in V$, the sum of f values over the open neighbourhood of v is at least one. Zero-one valued TDFs are precisely the characteristic functions of total dominating sets of G. A minimal total dominating function (MTDF) f is called *universal* if convex combinations of f and any other MTDF are minimal. Generalizing previous results by Cockayne, Mynhardt and Yu, we give a sufficient condition for an MTDF to be universal. Moreover, we define a *splitting operation* on graph G, which gives several classes of graphs having a universal MTDF.

94-14 Ordered Orthogonal Group Divisible Designs

Xiaojun Zhu

In [10], a recursive construction which uses a COQ(conjugate orthogonal quasigroup) of order m to inflate the group size of an OGDD(orthogonal group divisible design) by a factor of m is proved. This is a powerful construction in the sense that knowing a single OGDD could give us a class of OGDDs by applying the above construction, but unfortunately, the spectrum of COQ is far from being determined. In [14], Zhang generalized the above construction, basically he relaxed the definition of a COQ to that of an object called a MCOQ(a type of conjugate orthogonal quasigroup) while strengthening the definition of an OGDD to that of a MOGDD (a type of ordered OGDD) to obtain a new construction. His construction has the advantage that MCOQs are known for orders for which COQs are unknown, but also has the disadvantage that MOGDDs are known only for a limited family. In

this paper, we will prove that any OGDD can be given a special ordering such that an analogue of the construction in [10] can be applied by using a weaker version of COQ, namely NCOQ(near conjugate orthogonal quasigroup). We will also show the spectrum of NCOQ contains $N\backslash\{2,3,6,10,12,14,15,18,20,21,22,24,26,30,34,38,39,42,51\}$. In the mean time, some other weakening of COQ and the corresponding strengthening of OGDD is also introduced.

94-15 **Non Smooth Invexity in Multiobjective Fractional Programming**

S. K. Suneja *

Fritz-John and Kuhn-Tucker type necessary conditions are obtained for efficiency in a multiobjective fractional programming problem involving non smooth Lipschitz functions. Sufficient optimality conditions and duality results are proved under the conditions of non smooth invexity, quasiinvexity and pseudoinvexity. Vector valued saddle point results are also given under the assumptions of invexity. The principal analytic tool used for this purpose is the generalized gradient of Clarke (Ref. 1).

Keywords and phrases: Multiobjective fractional programming, efficiency, non smooth invexity, optimality, duality.

94-16 **The convex analysis of unitarily invariant matrix norms**

A. S. Lewis

A fundamental result of von Neumann's identifies unitarily invariant matrix norms as symmetric gauge functions of the singular values. Identifying the subdifferential of such a norm is important in matrix approximation algorithms, and in studying the geometry of the corresponding unit ball. We show how to reduce many convex-analytic questions of this kind to questions about the underlying gauge function, via an elegant Fenchel conjugacy formula.

Keywords and phrases: unitarily invariant norm, Fenchel conjugacy, subdifferential, matrix optimization, extreme point, exposed point.

94-17 **Which posets are valuable?**

David G. Wagner

We consider the problem of recognizing those posets which admit a valuation–this is a linear-algebraic condition which arose naturally in an algebraic/geometric context. We show that a poset has at most one valuation (which is integer-valued) and present various conditions which are either necessary or sufficient for a poset to be valuable. The main result is a reduction theorem which allows us to restrict attention to those posets which do not have a bounded cutset.

94-18 **Efficiency and Proper Efficiency**

M. J. Best and B. Ding

The relationship between solution sets involving efficient solutions, properly efficient solutions and locally proper efficient solutions are investigated. Some sufficient conditions that the set of efficient solutions is equal to the set of properly efficient solutions (or the set of locally proper efficient solutions) are given, and some known results are generalized.

Keywords and phrases: Efficient solution, proper efficient solution, pseudolinear function.

94-19 **Extremal graphs with given stability number and connectivity (I)**

Taojun Lu*

Call a graph G an (n, α, κ)-graph if G is of order n with stability number α and connectivity κ. Chvátal and Erdös show that if $\alpha \leq \kappa$, then G is hamiltonian. Now if $\alpha - 1 \geq \kappa \geq 2$, how big should G be to ensure the existence of a hamiltonian circuitΓ In this paper we answer this question and further characterize the extremal (n, α, κ)-graphs – the ones with the maximum number of edges.

94-20 **Extremal graphs with given stability number and connectivity (II)**

Taojun Lu*

This is a continuation of [3]. In [3], an (n, α, κ)-graph is defined to be a graph of order n with stability number α and connectivity κ. The extremal (n, α, κ)-graphs – those with the maximum number of edges are characterized. In this paper, we discuss some other related problems. Among them are (i) characterize the extremal (nonhamiltonian) graphs with given connectivity; (ii) characterize the extremal (nonhamiltonian) graphs with given stability number; (iii) characterize the extremal nonhamiltonian graphs; and (iv) characterize the extremal *BC-closed* graphs – graphs in which the degree sum of every two nonadjacent vertices is at most the order of the graph.

94-21 **A Relaxation of the Precedence Constrained Knapsack Problem**

Michael J. Best and Nilotpal Chakravarti*

In this paper we consider a linear programming relaxation of the precedence constrained knapsack problem. We derive an efficient algorithm for solving this problem and further provide better algorithms for some

special cases.

Keywords and phrases: Precedence Constraints, Knapsack Problem, Scheduling

94-22 **Counting pairs of lattice paths by intersections**

Ira Gessel *, Wayne Goddard*, Walter Shur*, Herbert S. Wilf*, and Lily Yen

94-23 **Maps in locally orientable surfaces, the double coset algebra, and zonal polynomials**

I.P. Goulden and D.M. Jackson

The *genus* series is the generating series for the number of maps (inequivalent two-cell embeddings of graphs), in locally orientable surfaces, closed and without boundary, with respect to vertex- and face-degrees, number of edges and genus. A *hypermap* is a face two-colourable map. An expression for the genus series for (rooted) hypermaps is derived in terms of zonal polynomials by using a double coset algebra in conjunction with an encoding of a map as a triple of matchings. The expression is analogous to the one obtained for orientable surfaces in terms of Schur functions.

94-24 **An Algorithm for the Solution of the Parametric Quadratic Programming Problem**

Michael J. Best

We present an "active set" algorithm of the solution of the convex (but not necessarily strictly convex) parametric quadratic programming problem. The optimal solution and associated multipliers are obtained as piece-wise linear functions of the parameter. At the end

of each interval, the active set is changed by either adding, deleting, or exchanging a constraint. The method terminates when either the optimal solution has been obtained for all values of the parameter, or, a further increase in the parameter results in either the feasible region being null or the objective function being unbounded from below. The method used to solve the linear equations associated with a particular active set is left unspecified. The parametric algorithm can thus be implemented using the linear equation solving method of any active set quadratic programming algorithm.

Keywords and phrases: Parametric Quadratic Programming Algorithm, Quadratic Programming, Active Set

94-25 **Graphical Codes Revisited**

Dieter Jungnickel* and S. A. Vanstone

It is well-known that the set of all even subgraphs of a graph G on p vertices with q edges forms a binary linear code $\mathbf{C} = \mathbf{C_E(G)}$ with parameters $[q, q-p+1, g]$, where g is the girth of G. Such codes were studied systematically by Bredesen and Hakimi (1967) and Hakimi and Bredesen (1968) who were concerned with the problems of augmenting $\mathbf{C}$ to a larger $[q, k, g]$-code and of efficiently decoding such augmented graphical codes. We will give a new approach to these problems by requiring the augmented codes to be graphical (i.e., they should still consist of spanning subgraphs of G). On one hand, we present two construction methods which turn out to contain the methods proposed by Hakimi and Bredeson (1968) as special cases (even though it is not clear from their paper that their augmented codes are in fact graphical). As we will show, this not only gives a better understanding of their construction, it also results in augmenting codes of larger dimension. We look at the case of 1-error correcting graphical codes in some detail. In particular, we show how to obtain the extended Hamming codes as "purely" graphical codes by our approach. On the other hand, we use techniques from combinatorial optimization to give decoding procedures for graphical codes which turn out to be considerably more efficient than the approach via majority logic decoding proposed by

Hakimi and Bredeson. We also consider the decoding problem for the even graphical code based on the complete graph K_{2n} in more detail: we discuss an efficient hardware implementation of an encoding/decoding scheme for these codes and we also show that things may be arranged in such a way that one can also correct all adjacent double errors. Finally, we briefly discuss non-linear graphical codes.

94-26 **On a family of conics associated with the Configuration of Pappus**

Leroy J. Dickey*

A fixed Pappus configuration, consisting of nine points and nine lines, gives rise to a family of eighteen conics, many of which may be characterized in either of two ways, as the locus of a point or as a non-degenerate conic determined by five points, no three of which are collinear.

94-27 **Submap Density and Asymmetry Results for Two Parameter Map Families**

Edward A. Bender*, E. Rodney Canfield*, Zhicheng Gao*, and L. Bruce Richmond

Let $M_{n,k}(S)$ be the set of n-edge k-vertex rooted maps in some class on the surface S. Let P be a planar map in the class. We develop a method for showing that almost all maps in $M_{n,k}(S)$ contain many copies of P. One consequence of this is that almost all maps in $M_{n,k}(S)$ have no symmetries. The classes considered include c-connected maps ($c \leq 3$) and certain families of degree restricted maps.

94-28 **Truncated Objective Functions in Bivalent Integer Programming with an Application to Selecting Gymnastic Teams**

C. E. Haff

94-29 **Near-Optima in Linear Programming**

C. E. Haff

Relative to linear program (LP) maximize $z = cx$ over polyhedron $\mathbb{P}$ we define a **strict program** (P) maximize $z = cx$ over Q, where Q is the intersection of $\mathbb{P}$ and the interior of a polyhedron containing $\mathbb{P}$. Of course, z may not have a maximum value over Q, but if (P) is feasible, then we can always find an arbitrarily good **near-optimum** solution for (P). Algorithms are given for deciding (P)-feasibility and determining (P)-near optimality. Applications include finding near-optimum solutions to linear programs in which a specified set of variables are required to have positive values.

94-30 **Connection coefficients, matchings and a combinatorial conjecture for Jack symmetric functions**

I.P. Goulden and D.M. Jackson

A series is introduced that is an extension to three sets of variables of the Cauchy sum for Jack symmetric functions in the Jack parameter α. We conjecture that the coefficients of this series with respect to the power sum basis are nonnegative integer polynomials in b, the Jack parameter shifted by 1. More strongly, we make the *Matchings-Jack Conjecture*, that the coefficients are counting series in b for matchings with respect to a parameter of nonbipartiteness. Evidence is presented for these conjectures, and they are proved for two infinite families.

94-31 **First-Order Cumulative $0-1$ Laws**

Stan Burris*, Kevin Compton*, Andrew Odlyzko*, and Bruce Richmond

Using counting arguments and Feferman-Vaught techniques we show that certain assumptions on the distribution of the directly indecom-

posable members of a class of structures lead to a first-order $0-1$ law. We give a natural model of the limit law theory; and show that the limit law theory is decidable if the theory of the directly indecomposables is decidable.

94-32 **A Semidefinite Framework for Trust Region Subproblems with Applications to Large Scale Minimization**

Franz Rendl*, and Henry Wolkowicz

A primal-dual pair of semidefinite programs provides a general framework for the theory and algorithms for the trust region subproblem (TRS). This problem consists in minimizing a general quadratic function subject to a convex quadratic constraint and, therefore, it is a generalization of the minimum eigenvalue problem. The importance of TRS is due to the fact that it provides the step in trust region minimization algorithms. The semidefinite framework is studied as an interesting instance of semidefinite programming as well as a tool for viewing known algorithms and deriving new algorithms for TRS. In particular, a dual simplex type method is studied that solves TRS as a parametric eigenvalue problem. This method uses the Lanczos algorithm for the smallest eigenvalue as a black box. Therefore, the essential cost of the algorithm is the matrix-vector multiplication and, thus, sparsity can be exploited. A primal simplex type method provides steps for the so-called hard case. Extensive numerical tests for large sparse problems are discussed. These tests show that the cost of the algorithm is $1+\alpha(n)$ times the cost of finding a minimum eigenvalue using the Lanczos algorithm, where $0 < \alpha(n) < 1$ is a fraction which decreases as the dimension increases.

Key Words and Phrases: trust region subproblems, parametric programming, semidefinite programming, min-max eigenvalue problems, large scale minimization.

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