26th Ontario Combinatorics Workshop

University of Waterloo, Waterloo, Canada

May 13-14, 2022

http://www.fields.utoronto.ca/activities/21-22/combinatorics

Welcome

Dear workshop participants,
Welcome to the 26th Ontario Combinatorics Workshop. It is our delight to welcome you to Waterloo for this workshop.

All events events will be held in QNC 0101. The easiest way to get to the room is to enter the Quantum-Nano Centre through the ring road entrance and take a left through the first set of doors. Signs will be posted.

University policy is that masks must be worn at all times while indoors, except while eating or drinking. If you are eating or drinking indoors, you should be seated.

If you would like to connect to internet while on campus, you can do that through eduroam or, if you don't have an eduroam account, the UniversityOfWaterloo guest network. When you connect to the network, you can register for an account, which will then create a username and password you can use to sign on.

We would like to thank Fields Institute and the University of Waterloo for their support, both financial and administrative. We would also like to thank all our speakers and the participants for making this workshop possible. We look forward to many interesting talks and discussions during the course of the workshop.

The University of Waterloo acknowledges that much of our work takes place on the traditional territory of the Neutral, Anishinaabeg and Haudenosaunee peoples. Our main campus is situated on the Haldimand Tract, the land granted to the Six Nations that includes six miles on each side of the Grand River. Our active work toward reconciliation takes place across our campuses through research, learning, teaching, and community building, and is centralized within the Office of Indigenous Relations

Yours sincerely,

Soffía Árnadóttir (University of Waterloo)
Ada Chan (York University)
Tina Chen (University of Waterloo)
Sabrina Lato (University of Waterloo)
Maxwell Levit (University of Waterloo)
Mike Newman (University of Ottawa)
Mariia Sobchuk (University of Waterloo)

Schedule

	Friday	Saturday
8:30	Opening	Breakfast
9:00	Rosa Orellana	Gabriel Coutinho
10:00	Nathan Nicholson	Sabrina Lato
10:30	Ian Seong	Maxwell Levit
11:00	Lukas Nabergall	Behnaz Refahi
Lunch		
13:00	Sophie Spirkl	Hermie Monterde
13:30		Weichen Xie
14:00	Aristotelis Chaniotis	Tina Chen
14:30	Manoj Belavadi	Mariia Sobchuk
Break		
15:30	Ronen Wdowinski	Bobby Miraftab
16:00	Mohabat Tarkeshian	Daniel Carranza
16:30	Alice Lacaze-Masmonteil	Closing
17:00	Lord Kavi	

All events take place in the Quantum-Nano Centre room 0101.
There will be refreshments and snacks served in the upper level of the room during the breakfast/opening remarks and the mid-afternoon break.

Invited Talks

Speaker: Rosa Orellana, Dartmouth College

Title: Products of characters of the symmetric group

Abstract: One of the main open problems in combinatorial representation theory of the symmetric group is to obtain a combinatorial interpretation for what are known as the Kronecker coefficients. The Kronecker coefficients are obtained when we decompose the tensor product of two irreducible representations of the symmetric group. This talk will be an introduction to Kronecker coefficients and a survey of recent results. This presentation should be accessible to beginning graduate students and will not assume knowledge of representation theory.

Speaker: Sophie Spirkl, University of Waterloo

Title: Chi-boundedness in graphs and digraphs

Abstract: If a graph has large chromatic number, one of the reasons for this could be a large clique in the graph; but what if there isn't one? What can we say about the graph? This question is the motivation behind studying chi-boundedness. I will tell you about some recent results, open questions, and analogous questions in directed graphs.

Based on joint work with Alvaro Carbonero, Patrick Hompe, and Ben Moore.

Speaker: Gabriel Coutinho, Universidade Federal de Minas Gerais

Title: Pretty Good State Transfer

Abstract: It's been known for a while that one can decide if a given graph admits perfect state transfer or not. In this talk, I will show that the same is true for pretty good state transfer, and discuss other applications of our results.

Contributed Talks

Speaker: Nathan Nicholson

Title: The Generalized Terwilliger Algebra of the Hypercube

Abstract: For any distance-regular graph Γ, two useful objects associated to it are its Terwilliger algebra and its generalized Terwilliger algebra. The former is defined combinatorially, while the latter is defined algebraically. There is a natural, surjective homomorphism from the Generalized Terwilliger algebra to the Terwilliger algebra, but in general, the map is not an isomorphism. However in some cases, this map is an isomorphism. I will show that this map is an isomorphism if Γ is a hypercube.

Speaker: Ian Seong

Title: The closure-complement-frontier problem in saturated polytopological spaces

Abstract: X be a space equipped with n topologies $\tau_1, \ldots, \tau_n$ which are pairwise comparable and saturated, and for each $1 \leq i \leq n$ let k_i and f_i be the associated topological closure and frontier operators, respectively. Inspired by the closure-complement theorem of Kuratowski, we prove that the monoid of set operators $\mathcal{KF}_n$ generated by $\{k_i, f_i : 1 \leq i \leq n\} \cup \{c\}$ (where c denotes the set complement operator) has cardinality no more than $2p(n)$ where $p(n) = \frac{5}{24}n^4 + \frac{37}{12}n^3 + \frac{79}{24}n^2 + \frac{101}{12}n + 2$. The bound is sharp in the following sense: for each n there exists a saturated polytopological space $(X, \tau_1, ..., \tau_n)$ and a subset $A \subseteq X$ such that repeated application of the operators k_i, f_i, c to A will yield exactly $2p(n)$ distinct sets. In particular, following the tradition for Kuratowski-type problems, we exhibit an explicit initial set in $\mathbb{R}$, equipped with the usual and Sorgenfrey topologies, which yields $2p(2) = 120$ distinct sets under the action of the monoid $\mathcal{KF}_2$.

Speaker: Lukas Nabergall

Title: The enumerative universe of chord diagrams

Abstract: We consider hereditary classes of chord diagrams (matchings) that satisfy one of several connectivity properties. Such classes are defined by a set of forbidden subdiagrams or patterns, and we focus on forbidding graphically-defined subdiagrams. We present a series of results enumerating several notable classes and then describe an equivalence between two infinite sets of these classes. We finish by reviewing a large number of numerically-generated conjectures for the counting sequences of many of these classes, underlining the rich combinatorial structure of these objects.

Based on joint work with Ali Assem Mahmoud.

Speaker: Aristotelis Chaniotis

Title: Minimal induced subgraphs of the class of 2-connected non-Hamiltonian wheel-free graphs

Abstract: Given a graph G and a property P we say that G is minimal with respect to P, if no proper induced subgraph of G has the property P. An HC-obstruction is a minimal 2-connected non-Hamiltonian graph. Given a graph H, a graph G is H-free if G has no induced subgraph isomorphic to H.

The main motivation for work presented in this talk originates from a theorem of Duffus, Gould, and Jacobson (1981), which characterizes all the minimal connected graphs with no Hamiltonian path. In 1998, Brousek, characterized all the claw-free HC-obstructions. On a similar note, Chiba, and Furuya (2021), characterized all (not only the minimal) 2-connected non-Hamiltonian $\{K_{1,3}, N_{3,1,1}\}$-free graphs. Recently, Cheriyan, Hajebi, Qu, and Spirkl (2022), characterized all triangle-free HC-obstructions and all the HC-obstructions which are split graphs.

A wheel is a graph obtained from a cycle by adding a new vertex with at least three neighbors in the cycle. We characterize all the HC-obstructions which are wheel-free graphs.

Joint work with: Zishen Qu and Sophie Spirkl.

Speaker: Manoj Belavadi

Title: Structural domination and coloring of some $(P7, C7)$-free graphs

Abstract: We show that every connected induced subgraph of a graph G is dominated by an induced connected split graph if and only if G is C-free, where C is a set of six graphs which includes $P7$ and $C7$, and each containing an induced $P5$. A similar characterization is shown for the class of graphs which are dominated by an induced connected complete split graph. Motivated by these results, we study structural descriptions of some classes of $(P7, C7)$-free graphs. In particular, we give structural descriptions for the class of $(P7, C7, C4$, gem)-free graphs and for the class of $(P7, C7, C4$, diamond)-free graphs. Using these results, we show that every $(P7, C7, C4$,gem)-free graph G satisfies $\chi(G) \leq 2\omega(G) - 1$, and that every $(P7, C7, C4$,diamond)-free graph H satisfies $\chi(H) \leq \max\{3, \omega(H)\}$.

Speaker: Ronen Wdowinski

Title: Linear arboricity of sparse multigraphs via orientations

Abstract: The linear arboricity $la(G)$ of a loopless multigraph G is the minimum number of colors required to edge-color the multigraph into linear forests, that is, forests whose components are all paths. The Linear Arboricity Conjecture of Akiyama, Exoo, and Harary asserts that the linear arboricity $la(G)$ of a simple graph G is at most

$\lceil(\Delta(G) + 1)/2\rceil$. We prove the conjecture when $\Delta(G) \geq 4pa(G) - 2$, where $pa(G)$ is the pseudoarboricity of G. This improves previously known results on the linear arboricity of sparse graphs, and our result holds more generally for loopless multigraphs. Our proof involves the study of edge-colorings of multigraphs into subgraphs that can be oriented so that every vertex v has indegree and outdegree at most some specified values $g(v)$ and $h(v)$. We also relate such orientation-based edge-colorings to the Goldberg-Seymour Conjecture for f-colorings.

Speaker: Mohabat Tarkeshian

Title: The geometry of random graphs with a Markov flavour

Abstract: Random graphs are at the intersection of probability and graph theory: it is the study of the stochastic process by which graphs form and evolve. In 1959, Erdős and Rényi defined the foundational model of random graphs on n vertices. Subsequently, Frank and Strauss (1986) added a Markov twist to this story by describing a topological structure on random graphs that encodes dependencies between local pairs of vertices. The general model that describes this framework is called the exponential random graph model (ERGM). It is used in social network analysis and appears in statistical physics as in the ferromagnetic Ising model. We characterize the parameters that determine when an ERGM has desirable properties using a well-developed dictionary between probability distributions and their corresponding generating polynomials.

Speaker: Alice Lacaze-Masmonteil

Title: Resolvable directed cycle decompositions of the complete symmetric digraph

Abstract: A $\overrightarrow{C}_m$-factorization (or resolvable $\overrightarrow{C}_m$ m-decomposition) of a digraph G is a decomposition of G into spanning subgraphs, each a disjoint union of directed cycles of length m. For positive integers α and m, it is conjectured that a $\overrightarrow{C}_m$-factorization of the complete symmetric digraph on αm vertices, $K^*_{\alpha m}$, exists if and only if $(m, \alpha) \notin \{(3, 2), (4, 1)\}$. This conjecture has been proven for $m \in \{3, 4\}$ and for the case m is even or α is odd. For $m \geq$ odd and α even, Burgess and Šajna have also shown that it suffices to find a $\overrightarrow{C}_m$-factorization of K^*_{2m} to completely settle the conjecture. In this talk, we take a major step towards a resolution of this problem by showing that, if m is odd and divisible by a prime congruent to 5 modulo 6, then K^*_{2m} admits a $\overrightarrow{C}_m$-factorization. This is joint work with Mateja Šajna.

Speaker: Lord Kavi

Title: Towards a proof of Haemers' toughness conjecture

Abstract: The toughness $t(G)$ of a graph $G = (V, E)$ is defined as $t(G) = \min\left\{\frac{|S|}{c(G-S)}\right\}$, in which the minimum is taken over all $S \subset V$ such that $G - S$ is disconnected, where $c(G - S)$ denotes the number of components of $G - S$. Haemers conjectured a Laplacian eigenvalue bound which would improve and generalize earlier spectral bounds by Alon, Brouwer and Gu. In this talk we present a framework of ideas and our progress in tackling this conjecture.

This is a work with my supervisor, Mike Newman.

Speaker: Sabrina Lato

Title: Spectral Moore Bounds

Abstract: In 1960, Moore posed the problem of asking which regular graphs have a maximal number of vertices for a given degree and diameter. Such graphs have been much studied since then, and a number of extensions of both the Moore bound and Moore graphs have proved fruitful. In 2014, Cioabă, Koolen, Nozaki, and Vermette developed a spectral Moore bound for regular graphs based on the degree and the second-largest eigenvalue of the adjacency matrix. They subsequently improved the bound for regular bipartite graphs, and more recently, I was able to extend their results to semiregular bipartite graphs. In this talk, I will give a brief overview of the Moore bounds, classical and spectral, and the graphs that meet the bounds.

Speaker: Maxwell Levit

Title: A survey of covering graphs

Abstract: A covering graph is a combinatorial specialization of the topological notion of a covering space. Or, if you prefer, a covering graph is an algebraic generalization of the combinatorial notion of a signed graph. I will address both perspectives, show some interesting examples, and explain some connections to group extensions, equiangular lines, and distance regular graphs.

Speaker: Behnaz Refahi

Title: Chromatic number of the partition graph $\mathcal{P}(3^3)$

Abstract: In 1978 Lovász used topological tools based on the Borsuk-Ulam theorem to prove the chromatic number of the Kneser graph $KG(\binom{[n]}{k})$ is $n - 2k + 2$. There exist different chromatic number lower bounds (Lovász, Sarkaria, Dol'nikov-Kříž, Bárány) ordered hierarchically for a graph G. Lovász bound makes use of the $\mathbb{Z}_2$-index of a box complex to provide the strongest lower bound. A box complex builds a functor between a graph with its homomorphisms and a $\mathbb{Z}_2$-space with its $\mathbb{Z}_2$-maps. To establish a new

and strong lower bound for the chromatic number of the partition graph $\mathcal{P}(3^3)$ we focus on the Lovász bound by studying the box complex of this graph.

Speaker: Hermie Monterde

Title: Strong cospectrality between twin vertices

Abstract: Strong cospectrality is a condition necessary for a pair of vertices in a graph to exhibit pretty good state transfer. In this presentation, we investigate when strong cospectrality occurs between twin vertices, and determine under which additional conditions does a vertex with a twin in a graph exihibit periodicity, perfect state transfer and pretty good state transfer.

Speaker: Weichen Xie

Title: Of Shadows and Gaps in Spatial Search

Abstract: Spatial search occurs in a connected graph if a continuous-time quantum walk on the adjacency matrix of the graph, suitably scaled, plus a rank-one perturbation induced by any vertex will unitarily map the principal eigenvector of the graph to the characteristic vector of the vertex. This phenomenon is a natural continuous-time analogue of Grover search. The spatial search is said to be optimal if it occurs with constant fidelity and in time inversely proportional to the shadow of the target vertex on the principal eigenvector. Extending a result of Chakraborty et al. (Physical Review A,102:032214, 2020), we prove a simpler characterization of optimal spatial search. Based on this characterization, we observe that some families of distance-regular graphs, such as Hamming and Grassmann graphs, have optimal spatial search. We also show a matching lower bound on time for spatial search with constant fidelity, which extends a bound due to Farhi and Gutmann for perfect fidelity.

Speaker: Tina Chen

Title: Bipartite Walks and the Hamiltonians

Abstract: The talk is going to introduce a discrete quantum walk model called the bipartite walk model. Bipartite walks generate many known and well-studied discrete quantum models, like the arc-reversal walks and vertex-face walks. We will show how the Hamiltonian of a bipartite walk builds a connection between a discrete quantum walk and a continuous quantum walk. We can use this connection to construct continuous walks with interesting but rare phenomena.

Speaker: Mariia Sobchuk

Title: Quantum isomorphisms

Abstract: You probably heard about isomorphisms of graphs. After this talk you will be able to say that you heard about quantum isomorphisms, and even some recent advances in this area.

Speaker: Bobby Miraftab

Title: Arc-disjoint hamiltonian paths in Cartesian products of directed cycles

Abstract: We show that if $C1$ and $C2$ are directed cycles (of length at least two), then the Cartesian product $C1\square C2$ has two arc-disjoint hamiltonian paths. (This answers a question asked by J. A. Gallian in 1985.) The same conclusion also holds for the Cartesian product of any four or more directed cycles (of length at least two), but some cases remain open for the Cartesian product of three directed cycles. We also discuss the existence of arc-disjoint hamiltonian paths in 2-generated Cayley digraphs on (finite or infinite) abelian groups.

Speaker: Daniel Carranza

Title: Cubical setting for discrete homotopy theory

Abstract: Discrete homotopy theory, introduced by H. Barcelo and collaborators, is a homotopy theory of (simple) graphs. Homotopy invariants of graphs have found numerous applications, for instance, in the theory of matroids, hyperplane arrangements, topological data analysis, and time series analysis. Discrete homotopy theory is also a special instance of a homotopy theory of simplicial complexes, developed by R. Atkin, to study social and technological networks.

I will report on the joint work with C. Kapulkin (arXiv:2202.03516) on developing a new foundation for discrete homotopy theory, based on the homotopy theory of cubical sets. We use this foundation to prove the conjecture of Babson, Barcelo, de Longueville, and Laubenbacher from 2006 relating homotopy groups of a graph to the homotopy groups of a certain cubical complex associated to it, as well as a discrete homotopy theory analogue of the Hurewicz theorem.

Peter Rodney Prize

The best talk given by a student at the Ontario Combinatorics Workshop will be awarded the Peter Rodney Memorial Book Prize. This prize of a book and memorial book plate is given in honour of Dr. Peter Rodney (1965-1995).

Dr. Peter Rodney graduated from McGill University, and then obtained his Masters and Ph.D degrees in Combinatorics, from the University of Toronto. He graduated in 1993 and went on to do postdoctoral research at the University of Vermont. He then continued to work in both the public and private sectors, using discrete mathematics, cryptography, and number theory. He had a deep appreciation for probability theory and was an avid card player. He died unexpectedly at the age of 30. His friends and family created a fund in his memory whose proceeds finance this book prize.

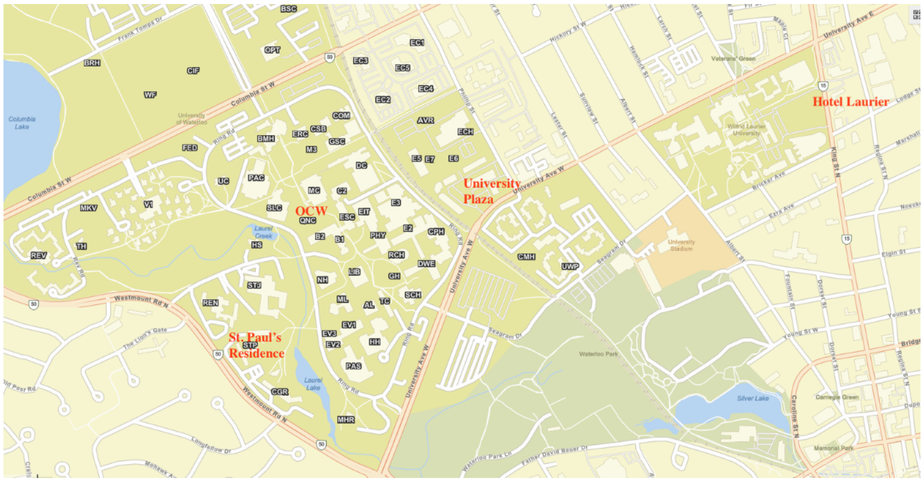

Figure 1: Map with important locations

Getting around the University of Waterloo

Here is an interactive campus map that allows you to find names of the buildings and make only food locations visible. If the link does not work, try our conference homepage. There is also a list of on-campus food options and hours.

In addition, there are numerous food options at University Plaza, which is marked on the map above.

Finding QNC 0101

If you enter from the Ring Road entrance, take a left in the vestibule, before passing through the centre doors to the main part of the building. If you enter through another entrance, you'll want to walk past the main office for IQC and take the smaller door to the right of the exit. The main entrance to the lecture hall has its own room number, 1103A. Signs will be posted.