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**On Homogeneous Convex Cones, Carathéodory
Number, and Duality Mapping**

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Abstract Using three simple examples, we answer three questions related to homogeneous convex cones, the Carathéodory number of convex cones and self-concordant barriers for convex cones. First, we show that if the convex cone is not homogeneous then the duality mapping does not have to be an involution. Next we show that there are very elementary convex cones that are not homogeneous, but have invariant Carathéodory number in the interior. Third, we show that the invariance of the Carathéodory number in the interior of the convex cone does not suffice to make the cone homogeneous even if the cone is self-dual. finally, we characterize the Carathéodory number of epi-graph of matrix norms.

Keywords Homogeneous cones, symmetric cones, Carathéodory number, barrier functions, self-concordance

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