CORR 99-32

New Quadratic Polynomials with High Densities of Prime Values

Michael J. Jacobson, Jr. and Hugh C. Williams

Abstract Hardy and Littlewood's Conjecture **F** [10] implies that the asymptotic density of prime values of the polynomials $f_A(x) = x^2+x+A$, $A \in \mathbb{Z}$ is related to the discriminant $\triangle = 1 - 4A$ of $f_A(x)$ via a quantity $C(\triangle)$. The larger $C(\triangle)$ is, the higher the asymptotic density of prime values for any quadratic polynomial of discriminant $\triangle$. A technique of Bach [2] allows one to estimate accurately $C(\triangle)$ for any $\triangle < 0$ given the class number of the imaginary quadratic order with discriminant $\triangle$. The Manitoba Scalable Sieve Unit (MSSU) [18, 17] affords us with the ability to rapidly generate many discriminants $\triangle$ which $C(\triangle)$ is potentially large, and new methods for evaluating class numbers and regulators of quadratic orders [12] allow us to compute accurate estimates of $C(\triangle)$ efficiently, even for values of $\triangle$ with as many as 70 decimal digits. Using these methods we were able to find a number of discriminants for which $C(\triangle)$ is larger than any previously known examples.